

Exiting from QE

Fumio Hayashi[†] and Junko Koeda^{§*}

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Abstract

We propose an empirical framework for analyzing the macroeconomic effects of quantitative easing (QE) and apply it to Japan. The framework is a regime-switching structural vector autoregression in which the monetary policy regime, chosen by the central bank responding to economic conditions, is endogenous and observable. QE is modeled as one of the regimes. The model incorporates an exit condition for terminating QE. We find that higher reserves at the effective lower bound raise inflation and output, and that terminating QE may be contractionary or expansionary, depending on the state of the economy at the point of exit.

JEL Classification: E58; E52; C54; C32; C13

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[†]National Graduate Institute for Policy Studies, Japan. E-mail: fumio.hayashi@gmail.com.

[§]School of Political Science and Economics, Waseda University, 1-6-1 Nishiwaseda, Shinjuku-ku, Tokyo 169-8050, Japan. E-mail: jkoeda@waseda.jp. Tel: +81-3-3208-0752. Fax: +81-3-3203-9816.

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1 Introduction and summary

Quantitative easing (QE) combines forward guidance, positive excess reserves held by depository institutions at the central bank, and targeted asset purchases at an effective lower bound (ELB), where the difference between the nominal policy rate and the ELB (hereinafter referred to as the net policy rate) is very close to zero. We study the macroeconomic effects of QE and exiting from QE using a regime-switching structural vector autoregression (SVAR) model. The model incorporates forward guidance in the form of an exit condition for terminating QE, the supply of excess reserves by the central bank, and an ELB. The data are drawn from Japan, a country with a relatively long history of QE. Japan's experience of multiple QE spells allows us to study exits from QE. Note that during our sample period, which ends in 2012, targeted asset purchases were not a focus of the Bank of Japan (BOJ).¹

We start by documenting that the net policy rate was effectively zero whenever the BOJ's stated policy was to guide the policy rate to "as low a level as possible." Thus, the ELB regime of a zero net policy rate is observable. We identify the following three ELB spells: March 1999 to July 2000, March 2001 to June 2006, and December 2008 to date. During these spells, the BOJ made a stated commitment of not exiting QE if inflation remained below a certain threshold.

Our baseline SVAR has two monetary policy regimes: an ELB regime and a regime of positive net policy rates. There are four variables: inflation, output (measured by the output gap), the policy rate, and excess reserves. The model's first two equations are reduced-form equations describing inflation and output dynamics. The reduced-form coefficients can be regime dependent. The third equation is the Taylor rule. The policy rate cannot be set to the Taylor rate (the rate prescribed by the Taylor rule) if it lies below the ELB. The fourth equation specifies the central bank's supply of excess reserves under QE. The exit condition requires that the central bank end QE only if the Taylor rate is positive and inflation exceeds a given threshold. Thus, regime endogeneity (where the regime's occurrence depends on inflation and output) arises not only from the ELB, but also from the exit condition.

¹ We end our sample period in 2012 because the BOJ under Governor Kuroda since 2013 appears to have embarked on a regime that is very different from that observed in our sample period. Under the pre-2013 regime, the BOJ held mostly short- and medium-term Japanese government bonds. It was not until April 2013 that the BOJ under Governor Kuroda started to address the maturity structure of long-term Japanese government bond holdings. As a result, reserves began to increase at a far higher rate in 2013.

We conduct nonlinear impulse-response and counterfactual analyses, in which nonlinearity arises from multiple regimes and the nonnegativity constraints on excess reserves. The regime and associated inflation and output dynamics change endogenously over the horizon. We find the following:

- QE is expansionary. When the current regime is an ELB regime, the response of output and inflation to an increase in excess reserves is positive. However, the statistical significance of this result depends on our measure of the output gap.
- Policy-induced exits from QE can be expansionary or contractionary, depending on the history.² We consider an alternative and counterfactual history for the July 2006 exit. Because the alternative history is chosen judiciously such that it differs from the baseline history solely in terms of policy shocks, the response is policy induced. An exit would have been expansionary for May or June 2006, and nearly so for April 2006. However, it would have been contractionary until March 2006 because of the higher level of excess reserves and the weaker macroeconomic conditions at exit.

The remainder of the paper is organized as follows. Section 2 reviews the related literature and states the paper's contributions. Section 3 documents the case for the monetary policy regime's observability. Section 4 describes the SVAR model. Section 5 explains the estimation strategy and the results. Section 6 provides the nonlinear impulse-response and counterfactual analyses. Section 7 provides extension and robustness checks. Section 8 concludes the paper.

2 Relation to the literature

Theoretical explanations of the QE effect, as surveyed, for example, in Woodford (2012, Section 3), include the signaling and portfolio-balance channels.³ As discussed in Section 7, we do not find strong empirical evidence for the macroeconomic effects of these channels over our sample period, which ends in 2012. However, other transmission channels could exist through central bank purchases of a risky asset

² To address possible concerns of spurious causality, where the monetary policy regime appears to cause the output (if the inflation–output dynamics is not adequately captured by our model), we have estimated a hidden-state Markov-switching model of Hamilton (1989) with a censored Taylor rule. We find that such a model does not generate the sort of impulse response profiles found in Section 6.1.

³ See Gagnon, Raskin, Remache, and Sack (2010) for a discussion on these channels and Chen, Cúrdia, and Ferrero (2012) and Farmer and Zabczyk (2016) for general-equilibrium models of the portfolio-balance channel.

in the presence of collateral constraints (Araújo, Schommer, and Woodford, 2015) and capital requirements for commercial banks (Ennis, 2014).⁴

An exit can be expansionary if it triggers the economy to move to a “better” economic state. The severity of the ELB is discussed in the seminal work of Eggertsson and Woodford (2003). Benhabib, Schmitt-Grohe, and Uribe (2001) show that the Taylor rule with the ELB has multiple equilibria, one of which is a liquidity trap. Aruoba, Cuba-Borda, and Schorfheide (2018) compute the sunspot equilibrium of a nonlinear New Keynesian model with the ELB, in which an economy can move between a targeted inflation regime and a deflation regime. Lansing (2017) develops a New Keynesian model with learning about regime transitions, and illustrates a case in which exit is expansionary. Gust, Herbst, López-Salido, and Smith (2017) quantify the cost of the ELB by estimating a nonlinear dynamic stochastic general-equilibrium model with the ELB, though their model does not converge to a deflationary steady state for the U.S. economy. However, the triggers for a shift from one regime to another vary; for example, it can be a rise in the equilibrium real rate (Eggertsson and Woodford, 2003), a sunspot (Aruoba et al., 2018), or changes in perceived transition probabilities (Lansing, 2017). In our case, the trigger is a combination of macroeconomic conditions and policy shocks.

The empirical literature on the macroeconomic effects of QE is growing rapidly. Relevant to our study are those that use quantities as a QE measure.⁵ The impulse-response analyses in all studies reviewed here find positive QE effects, albeit with varying magnitudes and statistical significance. The identification assumption employed in most of these studies is that inflation and output are predetermined.⁶

Several works exploit the observability of the ELB regime by estimating single-regime SVAR models for a sample deemed to be under the regime. Honda, Kuroki, and Tachibana (2013)⁷ and Schenkelberg and Watzka (2013) find a positive QE effect for Japan. Gambacorta, Hofmann, and

⁴ Ennis’s (2014) explanation of the QE effect can be aptly called the “quantity theory of bank capital.” Commercial banks’ assets expand as the central bank increases the supply of reserves. A leverage requirement forces commercial banks to provide more bank capital, but the real supply of bank capital is fixed. Therefore, prices must rise.

⁵ Studies with price-based QE measures include those of Kapetanios, Mumtaz, Stevens, and Theodoridis (2012) and Baumeister and Benati (2013), who use the yield spread in their vector autoregressions (VARs), and Wu and Xia (2014), who use a properly defined shadow policy rate. They all report expansionary QE effects.

⁶ Weale and Wieladek (2016) report an expansionary QE effect under four different identification assumptions, including that of predetermined inflation and output.

⁷ The work of Honda et al. (2013), which was originally written in Japanese in 2007, was perhaps the earliest QE study for Japan. Their QE measure is reserves, which was the target used by the BOJ during the ELB period of 2001 through 2006. Their recursive VAR of prices, output, and reserves, estimated on monthly data for the ELB period, shows that the impulse response of output to an increase in reserves is positive.

Peersman (2014) utilize a panel of countries for the period January 2008 to June 2011. They overcome the shortness of the sample by utilizing data from eight advanced economies, including that of Japan. Weale and Wieladek (2016) use U.S. and U.K. data from March 2009. The shortness of their sample is addressed through a Bayesian method. Consistent with studies for Japan, the authors find that asset purchases by the central bank raise inflation and output. This is also consistent with the findings of event studies of large-scale asset purchases in the two countries, surveyed in Woodford (2012), where announcements of such purchases raised asset prices.

Another way of addressing the small-sample problem is to include a period of positive policy rates, but to allow the model parameters to vary over time in specific ways. One strand of the literature employs regime switching. The regime-switching SVAR that has been used to study U.S. monetary policy (Bernanke and Mihov, 1998; Sims and Zha, 2006) assumes the regime to be unobservable and exogenous. Fujiwara (2006) and Inoue and Okimoto (2008) apply this line of approach with Japanese monthly data, and find that the probability of one of the regimes becomes very high from the late 1990s onward.⁸ For those months, the impulse response of output to an increase in the base money is positive and persistent. Another strand of literature applies a time-varying parameter approach. Kimura and Nakajima (2016) use quarterly Japanese data from 1981, and assume two QE spells (2001:Q1–2006:Q1 and 2010:Q1 on). Their time-varying parameter VAR takes the ELB into account by forcing the variance of the coefficient in the policy rate equation to shrink during QEs.

With the ELB, the policy rate is a censored variable, which renders the regime endogenous. Therefore, restricting the sample to the ELB period entails a sample selection bias. Iwata and Wu (2006) add the period of positive policy rates to the sample and estimate their SVAR while treating the policy rate as a censored variable. They assume that the inflation and output dynamics under positive policy rates is the same as that under the ELB regime.

Relative to the literature reviewed here, our study makes three contributions.

- Our analysis of the QE effect is more general in two respects. First, unlike the studies cited above, we allow the regime and associated inflation–output dynamics to change endogenously over the horizon

⁸ Aruoba et al. (2016) can be viewed as providing an underlying model to this type of regime-switching model. Their structural model is the (nonlinear) New Keynesian model with an ELB on the nominal interest rate. It has two equilibria, one of which is the liquidity trap with deflation. A nonlinear filtering technique reveals that the inflation–output dynamics during the period of near-zero policy rates in Japan was very likely generated by the deflationary equilibrium.

of the impulse response function. Second, our estimation takes into account regime endogeneity that is due not only to the ELB, but also to the exit condition.

- Our estimated SVAR can generate ELB spells comparable in length to those experienced in Japan and the United States, whereas recent structural New Keynesian models with an ELB have difficulty doing so. Thus, our study fills a gap in the literature by providing a model-free characterization of how inflation and output interact with the ELB.
- Our study is (to the best of our knowledge) the first to analyze the macroeconomic effects of exiting from QE. We provide a detailed discussion in Section 6.2.

3 Identifying the ELB regime

Three Spells of the ELB Regime

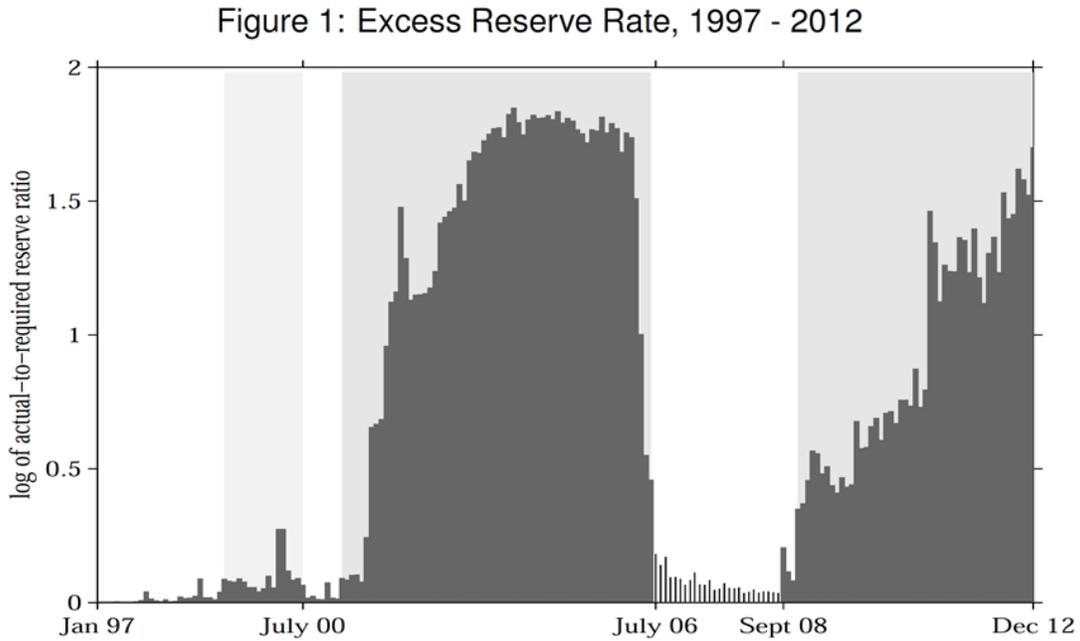
If the policy rate is the overnight interbank rate, its ELB is the interest rate on reserves (IOR), that is, the interest rate paid on reserves held by depository institutions at the central bank.⁹ We say that the monetary policy regime s_t in period t is the ELB regime, denoted by $s_t = \mathbf{Z}$, if the *net policy rate* $r_t - \bar{r}_t$, defined as the excess of the policy rate r_t over the IOR \bar{r}_t , is “effectively zero.” We can identify the months under regime \mathbf{Z} based on the monetary-policy statements made by the BOJ that guide the policy rate to a level near the lower bound.¹⁰ There are three spells of ELB regimes in Japan: March 1999 to July 2000 (Spell 1), March 2001 to June 2006 (Spell 2), and December 2008 onward (Spell 3). As it turns out, during those spells, the average over the reserve maintenance period (from the 16th of the month to the 15th of the next month) of the net rate $r_t - \bar{r}_t$ is less than five annual basis points (bps), thus providing an operational meaning to the phrase “effectively zero.”¹¹ Outside these spells, with a net rate above 5 bps, the regime s_t is the *positive-net-policy-rate regime* (\mathbf{P}).

⁹ The IOR in Japan was zero until November 2008, when it was raised to 0.1%.

¹⁰ The relevant statements are the announcements (“Statement on Monetary Policy”) made after the BOJ’s Monetary Policy Meetings (the Japanese equivalent of the U.S. Federal Open Market Committee, held every month and, sometimes, more often). They can be accessed at <https://www.boj.or.jp/en/mopo/mpmdeci/index.htm/>.

¹¹ The policy announcement made on February 12, 1999, includes a passage, reproduced in Table 1, that an ELB regime will be initiated. However, this passage is followed by the statement: “To avoid excessive volatility in the short-term financial markets, the Bank of Japan will initially aim to guide the call rate to move around 0.15%, and subsequently induce

We define the *excess reserve rate* (m_t) as the logarithm of the ratio of the actual to required levels of reserves for month t . Therefore, excess reserves are positive whenever $m_t > 0$. Because this definition involves required reserves, the excess reserve rate m_t applies to reserve maintenance periods, from the 16th of month t to the 15th of month $t + 1$. Figure 1 plots m_t since 1997. The shaded areas in the figure indicate the three spells of **Z**.



Note: The shades indicate the three ELB (effective lower bound) spells.

Barring unusual events (e.g., financial crises), excess reserves would be zero under **P**. Outside the shaded areas in Figure 1 and before 1997, incidents of positive excess reserves are rare. Among those incidents, the largest m occurs, not surprisingly, in September 2008. It is true that the months before September 2008 and after the second **Z** spell have positive excess reserves (see the thin bars in Figure 1), but there is good reason to ignore these months of positive excess reserves.¹² After setting $m_t = 0$ for those

further decline ...” Indeed, for the first reserve maintenance period after the announcement (February 16 to March 15, 1999), the average policy rate was above 5 bps, at 0.075%.

¹² Industry sources indicate that, after several years of a near-zero interbank rate with large excess reserves, the response by smaller-scale banks when the policy rate turned positive from essentially zero was to delay re-entry to the interbank market. A breakdown of excess reserves by type of financial institution since 2005, available from the BOJ’s homepage, shows that large banks quickly reduced their excess reserves after the termination in July 2006 of the ELB policy, whereas other banks (regional banks, foreign banks, and trust banks) were slow to adjust. The average of excess reserves for the period from July

months with thin bars, there are only a handful of months of positive excess reserves outside the three **Z** spells.

QE

The excess reserve rate m_t in the second and third **Z** spells, which is far higher than in September 2008, would be supply-determined. Indeed, during the second spell (between March 2001 and June 2006), the BOJ set a target range for reserves (see “Guideline for Money Market Operations” in their statements on monetary policy). More debatable is the first **Z** spell (March 1999–July 2000). Our reading of the minutes of the BOJ policy meetings at that time is that it supplied just enough reserves to prevent the interbank rate from rising above zero.¹³ The Japanese financial crisis of the late 1990s had not subsided during the first **Z** spell. Reserves had to exceed required reserves for the policy rate to stay at zero.

If we use the term *QE* to refer to the special case of the ELB regime **Z** in which reserves are supply determined, the discussion thus far can be summarized as follows: QE is in place during the second and third **Z** spells, whereas reserves are demand determined not only under regime **P**, but also during the first **Z** spell.

Exit Condition

Several authors have noted that the BOJ’s zero-interest-rate policy is a combination of the net zero-interest-rate policy and a stated commitment to a condition on inflation for exiting from the ELB regime.¹⁴ The BOJ statements collected in Table 1 indicate that during the three **Z** spells, the BOJ repeatedly expressed its commitment to an exit condition, stated in terms of the year-on-year (i.e., 12-month) consumer price index (CPI) inflation rate. For example, in the very first reserve maintenance period under

2006 to August 2008 is only 0.1% of the average for the period from January 2005 to June 2006 for large banks and 5.4% for other banks. To exploit the arbitrage opportunity presented by the positive interbank rates, banks need to train their employees anew. The reason commonly cited for the slow adjustment (e.g., Kato, 2010) is that medium- to small-scale banks, after several years of near-zero overnight rates, did not find it profitable to return immediately to the interbank market by incurring this re-entry cost.

¹³ Proposals by one board member to supply far more reserves were repeatedly voted down. The minutes can be accessed at https://www.boj.or.jp/en/mopo/mpmsche_minu/.

¹⁴ See, for example, Okina and Shiratsuka (2004) and Ueda (2012).

Z (March 16, 1999–April 15, 1999), the BOJ governor pledged to continue the zero-interest-rate policy rate “until the deflationary concern is dispelled” (see the April 13, 1999, announcement in Table 1).

Table 1: Policy Statements by the Bank of Japan, 1999-2012

date	quotes
1999.2.12	“The Bank of Japan will provide more ample funds and encourage the uncollateralized overnight call rate to move as low as possible.”
1999.4.13	“(The Bank of Japan will) continue to supply ample funds until the deflationary concern is dispelled.” (A remark by governor Hayami in a Q & A session with the press. Translation by authors.) http://www.boj.or.jp/announcements/press/kaiken_1999/kk9904a.htm/
1999.9.21	“The Bank of Japan has been pursuing an unprecedented accommodative monetary policy and is explicitly committed to continue this policy until deflationary concerns subside.”
2000.8.11	“... the downward pressure on prices ... has markedly receded. ... deflationary concern has been dispelled, the condition for lifting the zero interest rate policy.”
2001.3.19	“The main operating target for money market operations be changed from the current uncollateralized overnight call rate to the outstanding balance of the current accounts at the Bank of Japan. Under the new procedures, the Bank provides ample liquidity, and the uncollateralized overnight call rate will be determined in the market ... The new procedures for money market operations continue to be in place until the consumer price index (excluding perishables, on a nationwide statistics) registers stably a zero percent or an increase year on year.”
2003.10.10	“The Bank of Japan is currently committed to maintaining the quantitative easing policy until the consumer price index (excluding fresh food, on a nationwide basis) registers stably a zero percent or an increase year on year.”
2006.3.9	“... the Bank of Japan decided to change the operating target of money market operations from the outstanding balance of current accounts at the Bank to the uncollateralized overnight call rate... The Bank of Japan will encourage the uncollateralized overnight call rate to remain at effectively zero percent. ... The outstanding balance of current accounts at the Bank of Japan will be reduced towards a level in line with required reserves. ... the reduction in current account balance is expected to be carried out over a period of a few months.... Concerning prices, year-on-year changes in the consumer price index turned positive. Meanwhile, the output gap is gradually narrowing. ... In this environment, year-on-year changes in the consumer price index are expected to remain positive. The Bank, therefore, judged that the conditions laid out in the commitment are fulfilled.”
2006.7.14	“... the Bank of Japan decided ... to change the guideline for money market operations... The Bank of Japan will encourage the uncollateralized overnight call rate to remain at around 0.25 percent.”
2008.12.19	“... it (author note: meaning the policy rate) will be encouraged to remain at around 0.1 percent (author note: which is the rate paid on reserves)...”
2009.12.18	“The Policy Board does not tolerate a year-on-year rate of change in the CPI equal to or below 0 percent.”
2010.10.5	“The Bank will maintain the virtually zero interest rate policy until it judges, on the basis of the ‘understanding of medium- to long-term price stability’ that price stability is in sight...”
2012.2.14	“The Bank will continue pursuing the powerful easing until it judges that the 1 percent goal is in sight...”

Note: Accessible from <https://www.boj.or.jp/en/mopo/mpmdeci/index.htm/> except for the 13 April 1999 statement.

4 Regime-switching SVAR

This section presents our regime-switching SVAR model for four variables: the inflation rate p , output gap x , policy rate r , and excess reserve rate m , defined as the logarithm of the actual-to-required reserve ratio. Strictly for expositional clarity, the baseline model presented here makes the following two simplifying assumptions: (i) m is zero under the regime of positive net policy rates (**P**); and (ii) m is supply-determined by the central bank under the ELB regime (**Z**) (so that the ELB regime **Z** can be equated with the QE). A more general model without these assumptions is developed in Appendix A.

The Standard Three-Variable SVAR

We start on familiar ground by considering the standard three-variable SVAR in the review conducted by Stock and Watson (2001). The three variables are the monthly inflation rate from months $t - 1$ to t (p_t), the output gap (x_t), and the policy rate (r_t).¹⁵ The inflation and output gap equations are reduced-form equations, where the regressors are the (constant and) lagged values of all three variables. The third equation is the Taylor rule, which relates the policy rate to the contemporaneous values of the year-on-year inflation rate and the output gap.

As in a standard block-recursive SVAR (see Christiano, Eichenbaum, and Evans, 1999), the identifying assumption is that inflation and output are predetermined. This assumption is plausible with our data alignment. We measure reserves for the month as the average of daily values over the reserve maintenance period (from the 16th of the month to the 15th of the following month). We measure the policy rate for the month in a similar manner. If there is a half-month lag for the policy instruments to have macroeconomic effects, the inflation and output of the month cannot respond to the policy instruments of the same month. Another issue is whether there exist underlying models that admit this SVAR representation. Appendix D provides two examples of such models.¹⁶

¹⁵ The slack measure in Stock and Watson (2001) is the unemployment rate, not the output gap. We use the output gap because Okun's Law does not seem to apply to Japan.

¹⁶ In these examples, the structural model has two variables (inflation and the policy rate) with two equations (the Fisher equation and the Taylor rule). In the first example, agents are forward-looking. Nevertheless, in its reduced form, inflation is predetermined owing to a one-period information lag. Inflation is predetermined in the second example because agents are backward-looking.

As is standard in the literature, we consider the Taylor rule with interest-rate smoothing. That is, the Taylor rule sets the policy rate r_t to the *Taylor rate*, defined as

$$\underbrace{(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}}_{\text{Taylor rate}}, \text{ where } r_t^* \equiv \underbrace{\alpha_r^* + \underbrace{\beta_r^{*'} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}}_{(1 \times 2)}}_{\text{desired Taylor rate}}, \quad v_{rt} \sim \mathcal{N}(0, \sigma_r^2), \quad (4.1)$$

where π_t , defined as $\pi_t \equiv \frac{1}{12}(p_t + \dots + p_{t-11})$, is the inflation rate over the previous 12 months. The speed of adjustment is $1 - \gamma_r$. If this is equal to unity, the equation reduces to $r_t = r_t^* + v_{rt}$. We call r_t^* the *desired Taylor rate*.¹⁷

Introducing the Regimes

The aforementioned three-variable SVAR ignores the lower bound on the policy rate. Given the IOR rate \bar{r}_t paid on reserves, the lower bound is \bar{r}_t . Therefore, the Taylor rule involves censoring, as in

$$r_t = \max \left[\underbrace{(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}}_{\text{Taylor rate}}, \bar{r}_t \right], \quad v_{rt} \sim \mathcal{N}(0, \sigma_r^2). \quad (4.2)$$

In preparation for incorporating the exit condition below, we rewrite this in the following *equivalent* manner. Define the monetary policy regime s_t by

$$s_t = \begin{cases} \mathbf{P} & \text{if } \underbrace{(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}}_{\text{Taylor rate}} > \bar{r}_t, \\ \mathbf{Z} & \text{otherwise.} \end{cases} \quad (4.3)$$

Then the censored Taylor rule (4.2) can be written as a regime-dependent rule:

$$r_t = \begin{cases} \underbrace{(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}}_{\text{Taylor rate}}, v_{rt} \sim \mathcal{N}(0, \sigma_r^2) & \text{if } s_t = \mathbf{P}, \\ \bar{r}_t & \text{if } s_t = \mathbf{Z}. \end{cases} \quad (4.4)$$

¹⁷ The desired Taylor rate is typically expressed as $r_t^* = \rho + \beta_\pi^* (\pi_t - \bar{\pi}) + \beta_x^* x_t$, where ρ is the equilibrium real interest rate, and $\bar{\pi}$ is the target inflation rate. See, for example, p. 342 of Walsh's (2010) textbook. The intercept α_r^* in the desired Taylor rate in equation (4.1) is related to the equilibrium real interest rate ρ and the target inflation $\bar{\pi}$ as $\alpha_r^* \equiv \rho - \beta_\pi^* \bar{\pi}$, $\beta_r^{*'} \equiv (\beta_\pi^*, \beta_x^*)'$.

Here, we assume that the excess reserve rate m_t (the logarithm of the actual-to-required reserve ratio) is zero under \mathbf{P} and is supply-determined by the central bank under \mathbf{Z} . The supply of m_t is a censored variable because excess reserves cannot be negative. Therefore, if m_{st} is the (underlying) supply of excess reserves, the actual m_t is determined as

$$m_t = \begin{cases} 0 & \text{if } s_t = \mathbf{P}, \\ \max[m_{st}, 0] & \text{if } s_t = \mathbf{Z}. \end{cases} \quad (4.5)$$

With partial adjustment, the excess reserve supply m_{st} is given by

$$m_{st} \equiv \alpha_s + \underset{(1 \times 2)}{\boldsymbol{\beta}'_s} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \gamma_s m_{t-1} + v_{st}, \text{ where } v_{st} \sim \mathcal{N}(0, \sigma_s^2). \quad (4.6)$$

We expect the inflation (π_t) and output (x_t) coefficients to be negative (i.e., $\boldsymbol{\beta}_s < \mathbf{0}$), because the central bank would increase excess reserves when deflation worsens or output declines. Our specification of the excess-reserve supply equation follows Eggertsson and Woodford (2003) in that it depends on the current value of inflation and output, and is required to be non-negative.

Shifts in the monetary policy regime could alter the inflation and output dynamics described by the reduced form. We allow the reduced-form coefficients to depend on the monetary-policy regime. Because inflation and output are predetermined, the dependence of the reduced form on the regime for period t is through the lagged regime s_{t-1} .

Incorporating the Exit Condition

As documented in the previous subsection, attached to the ELB policy is the exit condition that the year-on-year inflation rate be greater than or equal to some threshold in order for the ELB regime to end. Incorporating the condition requires us to redefine the regime s_t recursively by modifying (4.3) as

$$\begin{cases}
\text{If } s_{t-1} = \mathbf{P}, s_t = \begin{cases} \mathbf{P} & \text{if } \underbrace{(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}}_{\text{Taylor rate}} > \bar{r}_t, \\ \mathbf{Z} & \text{otherwise.} \end{cases} \\
\text{If } s_{t-1} = \mathbf{Z}, s_t = \begin{cases} \mathbf{P} & \text{if } \underbrace{(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}}_{\text{Taylor rate}} > \bar{r}_t \text{ and } \pi_t \geq \underbrace{\bar{\pi} + v_{\bar{\pi}t}}_{\text{period } t \text{ threshold}}, \quad v_{\bar{\pi}t} \sim \mathcal{N}(0, \sigma_{\bar{\pi}}^2), \\ \mathbf{Z} & \text{otherwise,} \end{cases}
\end{cases} \tag{4.7}$$

where $\bar{\pi}$ is the target inflation rate. The threshold inflation rate for exit, $\bar{\pi} + v_{\bar{\pi}t}$, is allowed to deviate from the target. We assume that the threshold-inflation shock $v_{\bar{\pi}t}$ is i.i.d. over time.¹⁸

To Recapitulate

This completes our exposition of the regime-switching SVAR on four variables: p_t (monthly inflation), x_t (the output gap), r_t (policy rate), and m_t (the excess reserve rate). The sequence of events within the period that justifies our SVAR model of the four variables can be described as follows:

- (i) (p_t, x_t) determined. At the beginning of period t and given the previous period's regime s_{t-1} , nature draws two reduced-form shocks, one for inflation and the other for output, from a bivariate distribution. The reduced-form coefficients and the error variance–covariance matrix could depend on s_{t-1} . This determines (p_t, x_t) and, hence, the 12-month inflation rate $\pi_t \equiv \frac{1}{12}(p_t + \dots + p_{t-11})$.
- (ii) s_t determined. The central bank then draws three monetary-policy shocks $(v_{rt}, v_{\bar{\pi}t}, v_{st})$ from $\mathcal{N}(\mathbf{0}_{(3 \times 1)}, \text{diag}(\sigma_r^2, \sigma_{\bar{\pi}}^2, \sigma_s^2))$. It can now calculate the Taylor rate $((1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt})$, the inflation threshold $(\bar{\pi} + v_{\bar{\pi}t})$, shown in (4.7), and the excess reserve supply (m_{st}) , given in (4.6). Suppose the previous regime was the \mathbf{P} regime (so $s_{t-1} = \mathbf{P}$). Then the central bank picks $s_t = \mathbf{P}$ if the Taylor rate is above the IOR \bar{r}_t , and $s_t = \mathbf{Z}$ otherwise. Suppose next that $s_{t-1} = \mathbf{Z}$. Then the bank terminates QE and picks $s_t = \mathbf{P}$ only if the Taylor rate is above the IOR and $\pi_t \geq \bar{\pi} + v_{\bar{\pi}t}$.

¹⁸ If we introduced serial correlation by allowing $v_{\bar{\pi}t}$ to follow the first-order autoregressive process (AR(1)), for example, we would have to deal with an unobservable state variable ($v_{\bar{\pi}t-1}$ for the AR(1) case) appearing only in an inequality. Thus, the usual filtering technique would not be applicable.

- (iii) (r_t, m_t) determined. If $s_t = \mathbf{P}$, the central bank sets r_t at the Taylor rate and the market sets m_t to zero; if $s_t = \mathbf{Z}$, it sets r_t to \bar{r}_t and m_t to $\max[m_{st}, 0]$.

We can write the model as the following mapping:¹⁹

$$(s_t, p_t, x_t, r_t, m_t) = f_t \left(s_{t-1}, p_{t-1}, x_{t-1}, r_{t-1}, m_{t-1}, \dots, \begin{pmatrix} \boldsymbol{\varepsilon}_t \\ (2 \times 1) \end{pmatrix}, v_{rt}, v_{\bar{\pi}t}, v_{st} \right); \boldsymbol{\theta}_A, \boldsymbol{\theta}_B, \boldsymbol{\theta}_C, \quad (4.8)$$

where $\boldsymbol{\varepsilon}_t$ is the bivariate reduced-form shock, v_{rt} is the Taylor-rate shock, $v_{\bar{\pi}t}$ is the threshold-inflation shock, and v_{st} is the reserve-supply shock. The model parameters are $(\boldsymbol{\theta}_A, \boldsymbol{\theta}_B, \boldsymbol{\theta}_C)$. The first subset of parameters, $\boldsymbol{\theta}_A$, consists of the reduced-form parameters for inflation and output. Because we allow the reduced form to depend on the (lagged) regime, the parameter vector $\boldsymbol{\theta}_A$ consists of two sets of parameters, one for \mathbf{P} and the other for \mathbf{Z} . The second subset, $\boldsymbol{\theta}_B$, consists of the parameters of the Taylor rule (equation (4.4) with regime evolution (4.7)), whereas the third subset, $\boldsymbol{\theta}_C$, describes the excess reserve supply equation (4.6). More precisely,

$$\boldsymbol{\theta}_B = \left(\alpha_r^*, \begin{pmatrix} \boldsymbol{\beta}_r^* \\ (2 \times 1) \end{pmatrix}, \gamma_r, \sigma_r, \bar{\pi}, \sigma_{\bar{\pi}} \right), \quad \boldsymbol{\theta}_C = \left(\alpha_s, \begin{pmatrix} \boldsymbol{\beta}_s \\ (2 \times 1) \end{pmatrix}, \gamma_s, \sigma_s \right).$$

5 Estimating the model

This section has three parts: a derivation of the likelihood function for the baseline model, a summary of the data described in Appendix B, and our estimation results. As already remarked, the baseline model abstracts from two features, namely, active excess reserve demand and the other ELB regime with demand-determined excess reserves. These are incorporated in the more general model developed in Appendix A, which also derives the likelihood function for the more general model. In addition, Appendix A covers the estimation of excess reserve demand and the reduced form under the other ELB regime.

¹⁹ Although it is not made explicit, 11 lags of p are needed because the current 12-month inflation rate π_t depends on $(p_t, p_{t-1}, \dots, p_{t-11})$. For lagged regimes, only the most recent regime, s_{t-1} , is relevant in the mapping. The mapping f is time-dependent and has subscript t because it also depends on \bar{r}_t . However, these are minor details that do not matter in the remainder of the paper.

Likelihood Function (Summary of Appendix A)

Owing to the block-recursive SVAR structure, the model's log-likelihood function has the convenient property of additive separability in the partition of the parameter vector $(\boldsymbol{\theta}_A, \boldsymbol{\theta}_B, \boldsymbol{\theta}_C)$. The ML estimator of the parameters in each partition can be obtained by maximizing the corresponding part of the function.

The estimation of the reduced-form parameters $\boldsymbol{\theta}_A$ is entirely standard, with the maximum-likelihood (ML) estimator given by the ordinary least squares (OLS); that is, the reduced-form parameters for \mathbf{P} can be obtained by OLS on the subsample for which the lagged regime s_{t-1} is \mathbf{P} , and the same for \mathbf{Z} . There is no need to correct for regime endogeneity because the reduced-form errors for period t are independent of the lagged regime.

The excess reserve supply equation parameters $\boldsymbol{\theta}_C$ are estimated on observations with $s_t = \mathbf{Z}$. The censoring implicit in the “max” operator in (4.5) calls for Tobit with m_t as the limited dependent variable. There is no need to correct for regime endogeneity because the current regime s_t is independent of the reserve-supply shock v_{st} .

Regime endogeneity *is* an issue for the Taylor-rule parameters $\boldsymbol{\theta}_B$, because the Taylor-rate shock v_{rt} and the threshold-inflation shock $v_{\bar{\pi}t}$ in the exit condition affect the evolution of the regime. If the exit condition were absent, such that the censored Taylor rule (4.2) were applicable, then the ML estimator of $\boldsymbol{\theta}_B$ that controls for regime endogeneity would be Tobit on the whole sample composed of \mathbf{P} and \mathbf{Z} . Here, subsample \mathbf{P} , on which $r_t > \bar{r}_t$, provides “non-limit observations,” and subsample \mathbf{Z} , on which $r_t = \bar{r}_t$, provides “limit observations.” The exit condition complicates the expression of the probability of the limit observations. See Appendix A for further detail.

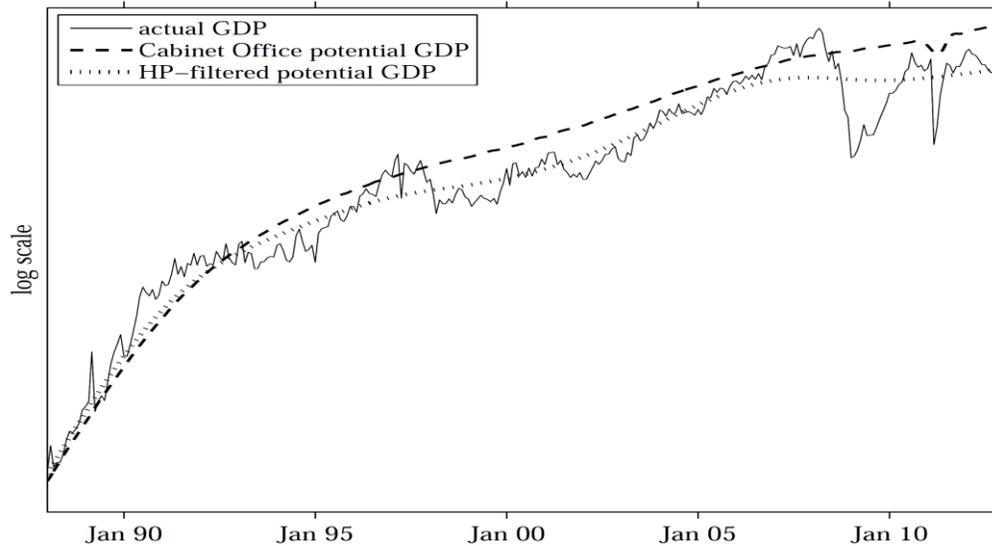
Data (Summary of Appendix B)

The excess reserve rate (m) is the logarithm of the actual-to-required reserves ratio. The level of actual reserves for month t is the average of daily balances over the reserve maintenance period of the 16th of month t to the 15th of the following month. This is to be consistent with how required reserves are defined in practice. The policy rate (r) is calculated similarly: its value for month t is the average of the daily rates over the reserve maintenance period. The graph of m is shown in Figure 1. Recall that we define the ELB

regime \mathbf{Z} as those months for which the net policy rate, $r_t - \bar{r}_t$, is less than 5 bps. We ignore the variations of r during the regime and set $r_t - \bar{r}_t$ to zero for all observations in subsample \mathbf{Z} .

The output measure underlying the output gap (x) is a monthly GDP series obtained by combining the quarterly GDP and a monthly comprehensive index of industry activities, which is available from January 1988 only. This determines the first month of the sample period. For the potential GDP, we use the official estimate of the (economic analysis division of the) Cabinet Office of the Japanese government (the equivalent of the U.S. Bureau of Economic Analysis). The potential GDP is based on the Cobb–Douglas production function with the Hodrick–Prescott (HP) filtered Solow residual. The output gap is defined as 100 times the log difference between the actual and potential GDP. The monthly GDP and potential GDP are shown in Figure 2. This figure shows the well-known decline in the trend growth rate that occurred in the early 1990s, often described as the (ongoing) “lost decade(s).” It also shows that the output gap has rarely been above zero during the lost decades. The fluctuations in potential output toward the end of the sample period reflect the earthquake and the tsunami of March 2011. The figure also shows the HP-filtered GDP. Use of it as the potential GDP measure will make some difference in our results (see Section 7).

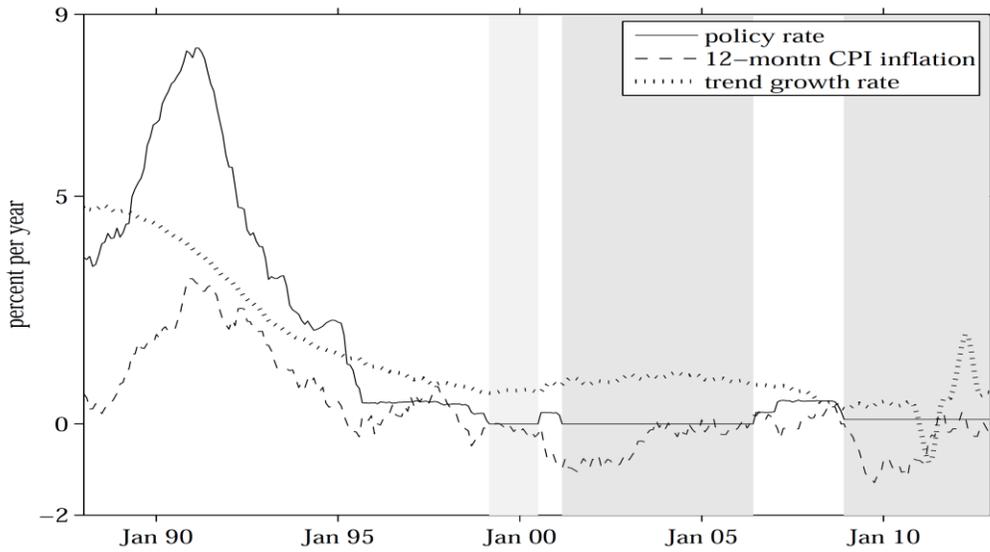
Figure 2: Actual and Potential Monthly GDP, 1988 - 2012



The monthly inflation rate (p) is constructed from the CPI. The relevant CPI component is the so-called “core” CPI (the CPI excluding fresh food), which, as documented in Table 1, is the price index

most often mentioned in BOJ announcements. (Confusingly, the core CPI in the U.S. sense, which excludes food and energy, is called the “core-core” CPI.) Figure 3 shows the year-on-year inflation rate (defined as $\pi_t \equiv \frac{1}{12}(p_t + \dots + p_{t-11})$) since 1988, along with the policy rate r_t and the trend growth rate, defined as the 12-month growth rate of the potential output series shown in Figure 2. It is noteworthy that the three variables fell rather sharply in the early to mid-1990s.

Figure 3: Policy Rate, Inflation, and Trend Growth Rate, 1988-2012



Note: The shades indicate the three ELB (effective lower bound) spells.

Table 2 shows the simple statistics of the selected variables. As mentioned, we set the net policy rate $r_t - \bar{r}_t$ to zero under \mathbf{Z} . Because $\bar{r}_t = 0$ during the first two \mathbf{Z} spells and $\bar{r}_t = 0.1\%$ during the third spell, the policy rate r_t is 0% during the first two \mathbf{Z} spells and 0.1% during the third spell.

Table 2: Simple Statistics, January 1988 - December 2012

	π (12-month inflation rate, %)	x (output gap, %)	r (policy rate, % per year)	m (excess reserve rate)
subsample P (1/1988-2/1999, 8/2000-2/2001, 7/2006-11/2008, sample size = 170)				
mean	0.845	-0.224	2.640	0.006
std. dev.	1.008	1.940	2.582	0.021
First Zero-Rate Spell (March 1999-July 2000, sample size= 17)				
mean	-0.118	-2.998	0.0	0.098
std. dev.	0.095	0.917	0.0	0.069
Second Zero-Rate Spell (March 2001-June 2006, sample size= 64)				
mean	-0.399	-2.184	0.0	1.379
std. dev.	0.382	1.158	0.0	0.545
Third Zero-Rate Spell (December 2008-December 2012, sample size= 49)				
mean	-0.495	-3.764	0.1	0.941
std. dev.	0.461	2.111	0.0	0.417

Note: See the data description in Section 5 for how data are constructed.

Parameter Estimates

Having described the estimation method and the data, we are now ready to report the parameter estimates.

We start with θ_B .

Taylor rule with the exit condition (θ_B)

Two issues are relevant to the specification of the Taylor rule.

- The Mieno disinflation. It is widely agreed that the rapid rate hike from December 1989, when Yasushi Mieno became the BOJ governor, to June 1991, when the policy rate peaked, was specifically to respond to asset bubbles.²⁰ We view this as a prolonged deviation from the Taylor rule.
- Variable equilibrium real interest rates. We have been treating the intercept in the desired Taylor rate r_t^* (α_r^* in (4.1)) as a constant because of the assumption of the constant real interest rate. This

²⁰ See, for example, the booklet on popular consumption by Okina (2013), who was a director of the BOJ's research arm.

assumption does not seem appropriate for Japan, given the well-documented decline in the trend growth rate since around 1990.²¹

If we ignore these issues and estimate the Taylor rule on the full sample from 1988 to 2012, then the estimate of γ_r (the coefficient of the lagged policy rate in the Taylor rate; see equation (4.1)) is about one. This yields a very imprecise estimate of the inflation and output coefficients in the desired Taylor rate (β_r^* in equation (4.1)). Therefore, we exclude the so-called bubble period of 1988–1991, which includes the Mieno disinflation and the sharp initial decline in the trend growth rate shown in Figure 3.²² Because trend growth kept declining after 1991, it is included in the desired Taylor rate to control for movements in the equilibrium real interest rate.²³ In Section 7, we describe how our results change if trend growth is dropped from the model.

Table 3 reports our ML estimates for the sample period from 1992 to 2012. The positive trend-growth coefficient estimate of 0.62 implies that the equilibrium real rate indeed kept declining with the trend growth after the bubble period. The estimated speed of adjustment per month, $1 - \gamma_r$, is about 10%. The target inflation rate $\bar{\pi}$ is mere 0.53% per year. The inflation coefficient is estimated to be 0.69. The relatively small output coefficient of 0.05 means that inflation, rather than output stability, was the BOJ's primary concern.

²¹ For example, Hayashi and Prescott (2002) document that both the total factor productivity and the rate of return on capital declined in the early 1990s.

²² If we include the bubble period of 1988–1991, the parameter estimates are similar to those reported in Table 3, provided that both the trend growth rate and a dummy for the Mieno disinflation are included. The inflation and output coefficients in the desired Taylor rate are 0.64 (t -value = 3.6) and 0.08 (t -value = 1.3), respectively.

²³ Okina and Shiratsuka (2002) and Braun and Waki (2006) use trend growth to control for the equilibrium real interest rate.

Table 3: Taylor Rule, January 1992 - December 2012 (sample size = 252)

coefficients in the desired Taylor rate			
trend growth rate	inflation	output gap	
0.62 [1.5]	0.69 [2.5]	0.05 [0.7]	

speed of adjustment ($1 - \gamma_r$), % per month	std. dev. of error (σ_r), % per year	target inflation ($\bar{\pi}$), % per year	std. dev. of threshold ($\sigma_{\bar{\pi}}$), % per year
9.8 (2.8)	0.113 (0.0073)	0.53 (0.45)	0.33 (0.27)

Note: Estimation by the ML (maximum likelihood) method described briefly in the text and more fully in Appendix 2. t -values in brackets and standard errors in parentheses. The Taylor rule is the regime-dependent rule defined in (4.4), with the desired Taylor rate r_t^* given in (4.1) and the regime s_t defined in (4.7). The intercept α_r^* in the desired Taylor rate is a linear function of the trend growth rate.

The Taylor principle is violated because the inflation coefficient is less than one. As shown by Lubik and Schorfheide (2004), if the underlying model is the New Keynesian model with forward-looking agents, the violation means that the dynamics described by the SVAR could contain sunspot fluctuations. Sunspots, however, would not affect the equilibrium of the underlying model if agents are backward-looking. This point is illustrated by the second example in Appendix D. Even if agents are forward-looking, Davig and Leeper (2007) and Farmer, Waggoner, and Zha (2009) show that the prospect of monetary policy becoming active (in the sense of the inflation coefficient exceeding one) sometime in the future could eliminate sunspot equilibria. More recently, Hagedorn (2017) shows that violating the Taylor principle does not lead to price indeterminacy in a large class of incomplete market models.

Excess reserve supply equation (θ_C)

As already noted, the ML estimation is by Tobit on subsample \mathbf{Z} that excludes the first ELB spell. Since m is well above zero on the subsample, Tobit reduces to OLS. The estimates are presented in Table 4. Both the inflation and output coefficients pick up the expected signs.

Table 4: Excess Reserve Supply Equation

t is in	coefficient of				R^2	σ_s (%)
	const	π_t	x_t	m_{t-1}		
QE (113 obs.)	-0.009 [-0.1]	-0.007 [-0.2]	-0.018 [-2.2]	0.98 (0.033)	0.94	0.132 (0.0088)

Note: Estimation by OLS. t -values in brackets and standard errors in parentheses. The sample of 113 observations is the QE months, which consists of the second and third ELB (effective lower bound) spells (March 2001 - June 2006 and December 2008 - December 2012). m_t is the excess reserve rate, π_t is the 12-month inflation rate to month t in percent, x_t is the output gap in percent, σ_s (standard deviation of the error) is estimated as $\widehat{\sigma}_s = \sqrt{SSR/n}$ where n is the sample size. The standard error of $\widehat{\sigma}_s$ is calculated as $\frac{\widehat{\sigma}_s}{\sqrt{2n}}$.

Inflation and output reduced-form equations (θ_A)

The ML estimate of the reduced form is OLS for two separate subsamples: the “lagged” subsample \mathbf{P} (i.e., those t ’s with $s_{t-1} = \mathbf{P}$) and the “lagged” subsample \mathbf{Z} (with $s_{t-1} = \mathbf{Z}$), excluding the first ELB spell. We include the trend growth rate in the set of regressors. For the lagged subsample \mathbf{P} , we exclude lagged m in order to be consistent with the model’s current assumption that $m = 0$ under regime \mathbf{P} . The Bayesian information criterion (BIC) instructs us to set the lag length to one for both subsamples.²⁴

The reduced-form estimates are shown in Table 5. First, we consider the reduced form for the lagged subsample \mathbf{P} . Andrews’ (1993) sup F -test finds no structural break for the inflation equation, but does find a structural break for the output equation in March 1995.²⁵ Therefore, we show the reduced-form estimates for the sample beginning March 1995. The monthly inflation (p) equation exhibits two notable features. First, inflation-persistence is non-existent, as indicated by the lagged p -coefficient of almost zero. Second, the lagged r -coefficient has the wrong sign, but its magnitude is very small.

²⁴ Given the moderate sample size, we set the maximum lag length to six and start the sample from July 1988 when we choose the lag length. Under the Akaike information criterion (AIC), we choose a lag length of two for the lagged subsample \mathbf{P} and one for \mathbf{Z} .

²⁵ The trimming parameter is 15%. The F -statistic peaks twice, in October 1994 and in March 1995. We choose the latter because it is the recent months of the sample that are relevant in the counterfactual analysis. Miyao’s (2002) VAR analysis finds that a break occurs in 1995. Fujiwara (2006) and Inoue and Okimoto (2008) estimate the regime-switching VAR and find that the probability of one of the regimes becomes very high after the late 1990s.

We turn now to the lagged ELB subsample **Z**, excluding the first **Z** spell. The regressors include r_{t-1} because, although it is constant in each of the two QE spells, it differs across spells. The positive lagged m -coefficients under QE imply that both inflation and output rise as excess reserves are increased. These effects are statistically significant. The coefficient of 0.46 in the output (x) equation, for example, means that the impulse response of x to a unit increase in m is 0.46 percentage points in the subsequent period.

Table 5: Inflation and Output Reduced Form

lagged subsample P								
$t - 1$ is in	dependent variable	coefficient of						R^2
		const.	g_t	p_{t-1}	x_{t-1}	r_{t-1}	m_{t-1}	
P (85 obs.)	inflation (p_t)	-0.11 [-0.2]	0.25 [0.4]	-0.05 [-0.4]	0.16 [1.8]	0.04 [0.1]		0.04
	output (x_t)	-0.70 [-2.3]	0.83 [2.1]	-0.04 [-0.5]	0.94 [16]	-0.31 [-0.9]		0.77
lagged subsample Z								
$t - 1$ is in	dependent variable	coefficient of						R^2
		const.	g_t	p_{t-1}	x_{t-1}	r_{t-1}	m_{t-1}	
Z/QE (112 obs.)	inflation (p_t)	-0.57 [-1.1]	-0.26 [-0.8]	-0.02 [-0.2]	0.10 [1.3]	-0.03 [-0.0]	0.55 [2.2]	0.09
	output (x_t)	-1.07 [-2.8]	0.01 [0.0]	0.02 [0.3]	0.80 [15]	-0.72 [-0.4]	0.46 [2.5]	0.79

Note: Estimation by OLS. t -values in brackets. The “lagged subsample **P**” of 85 observations refers to t 's for which $t \geq$ March 1995 and $t - 1$ is not in the three ELB (effective lower bound) spells. The “lagged subsample **Z**” of 112 observations refers to t 's for which $t - 1$ is in the second or third ELB spells (March 2001 - June 2006 and December 2008 - December 2012). p is the monthly inflation rate in percent per year, x is the output gap in percent, r is the policy rate in percent per year, m is the excess reserve rate (defined as the log of the ratio of actual to required reserves), and g is the trend growth rate in percent (the 12-month growth rate of potential output). The value of r_{t-1} is 0% during the second ELB spell and 0.1% during the third ELB spell.

6 Nonlinear impulse-response and counterfactual analyses

For counterfactual analyses involving nonlinear models, such as ours, we find it more transparent to designate histories in terms of model variables (as in Gallant, Rossi, and Tauchen, 1993) rather than in

terms of shocks. The shock-based translations of all the histories considered in this section, which are far more cumbersome to write down, are provided in Appendix C.

The model we use for the impulse-response and counterfactual analyses in this section is the baseline model with two regimes, \mathbf{P} and \mathbf{Z} . This means that the other ELB regime, with demand-determined excess reserves, is assumed not to arise in the underlying simulations. Section 7 will show that incorporating this other ELB regime with active excess reserve demand does not substantially alter the results presented here.

6.1 Transmission channels of monetary policy at and away from the ELB

Policy-Rate Effect

We start with the familiar case where the only difference between the two histories lies in the Taylor-rate shock v_{rt} . Our counterfactual analysis asks what the response of the variables would be if the shock were of a different size. The response is given by the difference

$$\begin{aligned} & E \left(y_{t+k} \middle| s_t = \mathbf{P}, \underbrace{(p_t, x_t, r_t + \delta_r, 0)}_{(p_t, x_t, r_t, m_t) \text{ in the alternative history}}, \underbrace{\dots}_{\text{lagged information}} \right) \\ & - E \left(y_{t+k} \middle| s_t = \mathbf{P}, \underbrace{(p_t, x_t, r_t, 0)}_{(p_t, x_t, r_t, m_t) \text{ in the baseline history}}, \underbrace{\dots}_{\text{lagged information}} \right), y = p, x, r, m, \end{aligned} \tag{6.1}$$

where the lagged information indicated by “...” consists of the lagged values of (s, p, x, r, m) , and the conditional expectations are defined by the mapping given in equation (4.8).²⁶ Not only the history up to $t - 1$ but also the *current* values of (p_t, x_t) are the same in both the baseline and the alternative histories. Thus, we control for the reduced-form shocks for inflation and output. The Taylor-rate shock v_{rt} is the only relevant policy shock here, because both m_t and s_t are the same across the histories as well. This point is made more fully in Appendix C (see equation (C.22)). The response profile, that is, the difference

²⁶ Because the mapping is time-dependent, the expectations operator should have a subscript (E_t rather than E). However, we omit this subscript t for notational simplicity. We compute the conditional expectations by simulating 10,000 sample paths of (s, p, x, r, m) generated by the mapping, and then taking the average of the simulated sample paths.

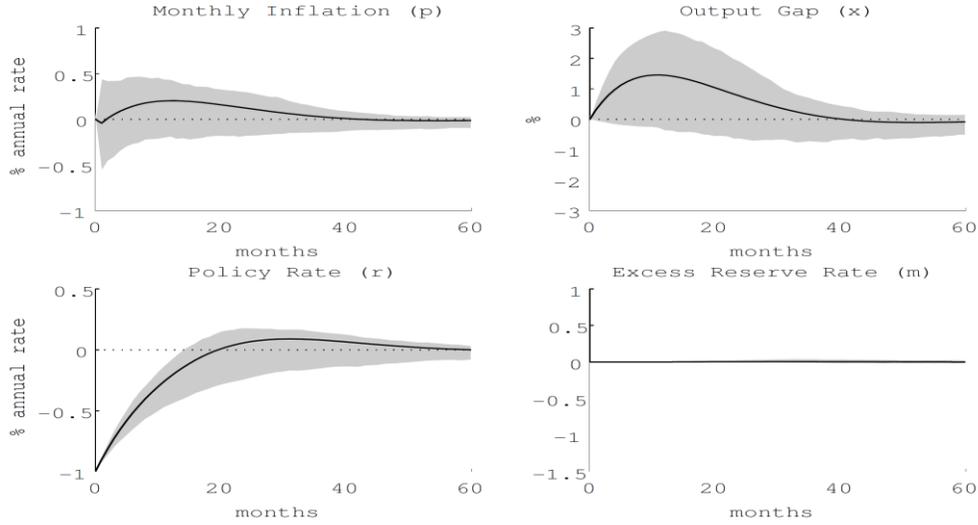
in the conditional expectation at various horizons (k), reduces to the standard impulse-response function if the two histories differ in the value for the base period of only one policy variable (as here) and the model is linear with only one regime.²⁷

Figure 4a shows the policy-rate effect, that is, the response profiles given in (6.1) for horizons $k = 0, 1, 2, \dots, 60$ months.²⁸ The interest-rate shock is $\delta_r = -1$, that is, a policy rate cut of 100 bps. In contrast to the linear case, the difference in conditional expectations depends not only on how the alternative history differs from the baseline, but also on the baseline history itself. Therefore, the base period needs to be specified. However, in order to calculate the response profiles of the policy rate cut, the base period has to be June 1995 or before, when the policy rate was above 1 percent. We take the base period t to be the earliest month after the structural break, March 1995, when the policy rate, at $r_t = 2.0\%$, was comfortably above zero.

²⁷ There are two exogenous variables in the system: \bar{r} (the IOR rate paid on reserves) and the trend growth rate (the 12-month growth rate of the potential GDP). Each simulated sample path of (s, p, x, r, m) from the base period t depends on the projected path from t onward for those exogenous variables. This affects the difference (6.1) because our model is nonlinear. We assume static expectations, in that the projected path from t onward is constant at the value at t .

²⁸ The error bands are obtained by drawing parameter vectors from the asymptotic distribution and picking the 84 and 16 percentiles for each horizon (such that the coverage rate is 68%, corresponding to one standard-error bands). For further detail, see Appendix E.

Figure 4a: The Policy-Rate Effect, the base period is March 1995



Note: The policy-rate effect is defined in (6.1). The 68% probability bands in shades.

That the rate cut is 100 bps (i.e., 1 percentage point) can be read from the intercept of the profile in the lower-left panel of Figure 4a, which shows the response profile of r to r . The response profile of p , shown in the upper-left panel, is not significantly different from zero in that the error band includes the horizontal axis. The same is true for the profile for x in the upper-right panel. Because of the high initial policy rate of 2.0%, the system rarely switches to \mathbf{Z} in the simulations, which explains the almost no response of m , as shown in the lower-right panel of the figure.

The QE Effect

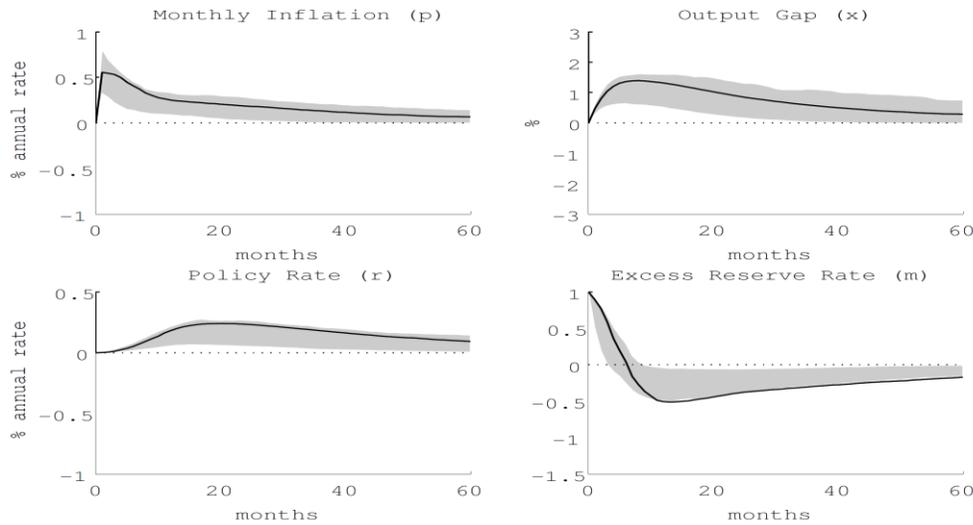
We now turn to the response to supply-determined changes in the excess reserve rate m . To study the effect of changes in the excess reserve rate m under the ELB regime, we set $s_t = \mathbf{Z}$ and $r_t = \bar{r}_t$ in both the baseline and the alternative histories. The response to the reserve-supply shock v_{st} is given by

$$\begin{aligned}
 & E \left(y_{t+k} \middle| s_t = \mathbf{Z}, \underbrace{(\underbrace{p_t, x_t, \bar{r}_t, m_t + \delta_m}_{(p_t, x_t, r_t, m_t) \text{ in the alternative history}}, \dots)}_{\text{lagged information}} \right) \\
 & - E \left(y_{t+k} \middle| s_t = \mathbf{Z}, \underbrace{(\underbrace{p_t, x_t, \bar{r}_t, m_t}_{(p_t, x_t, r_t, m_t) \text{ in the baseline history}}, \dots)}_{\text{lagged information}} \right), y = p, x, r, m.
 \end{aligned} \tag{6.2}$$

The only difference between the two histories is that the reserve-supply shock v_{st} is higher in the alternative history by δ_m . This point is made formally in equation (C.23).

Figure 4b shows the QE effect, that is, the profiles of the response to m for the base period of February 2004 (the peak QE month) when $m_t = 1.849$, of about 6.4 ($= \exp(1.849)$) times the required reserves, or about 6% of the GDP. The lower-right panel shows the response profile of m to m , so its intercept at horizon $k = 0$ (the base period) is equal to the perturbation δ_m . We set $\delta_m = 1.0$.²⁹ The perturbation is about 10% of the GDP.

Figure 4b: The QE Effect, the base period is February 2004



Note: The QE effect is defined in (6.2). The 68% probability bands in shades.

The response profile for x is shown in the upper-right panel of Figure 4b. Its next-period response (the response at $k = 1$) is about 0.46% (the lagged m -coefficient in the output equation of 0.46 shown in Table 5 multiplied by $\delta_m = 1$). Because of the persistence in the output dynamics exhibited in the estimated reduced form, the response feeds into the next-period response and increases to about 1.4% in

²⁹ The perturbation size is chosen so that its ratio to the estimated standard deviation of the reserve-supply shock v_{st} (0.132 in Table 4) is roughly equal to the ratio of $-\delta_r$ (the size of the interest-shock) to the estimated standard deviation of the Taylor-rate shock v_{rt} (0.113 in Table 3). We have already set $\delta_r = -1$ (100bps).

about 10 months. For p , the next-period response (at $k = 1$) is greater, at 0.55, but the effect tapers off owing to the lack of persistence in monthly inflation.

Because both output and inflation increase, the initial regime of \mathbf{Z} is more likely to switch to \mathbf{P} earlier under the alternative scenario. This is why the response of r gradually rises from zero, whereas m 's response turns negative. The mean duration of the initial regime \mathbf{Z} is indeed shorter with the positive shock to reserves: about 26 months under the baseline, and about nine months under the alternative. Interestingly, for the QE spell from March 2001 to June 2006, the mean duration at the time of QE entry is about 64 months, which is approximately equal to the actual duration of the spell. Thus, our estimated SVAR generates ELB spells comparable in length to those experienced in Japan and the United States, whereas recent structural New Keynesian models with the ELB have difficulty doing so. Gust et. al. (2017) find the average duration for a lower bound spell is just over 3.5 quarters for the U.S. economy. Boneva, Braun, and Waki (2016) further discuss the failure of the New Keynesian models in this respect.

6.2 Timing of exit

A more interesting analysis can be conducted by allowing the two histories to differ in more than one policy shock. To illustrate this, we examine the exit from the March 2001–July 2006 QE spell. The relevant statistics are shown in Table 6. The last row shows that the desired Taylor rate. Hence, the Taylor rate³⁰ was already positive before the exit. What kept the BOJ from exiting was the low inflation rate.

³⁰ Recall that the Taylor rate is $(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}$. During ELB spells, the net rate is zero: $r_t - \bar{r}_t = 0$. During the second ELB spell, $r_t = \bar{r}_t = 0$, which means the Taylor rate is proportional to the desired rate plus noise: $(1 - \gamma_r)r_t^* + v_{rt}$.

Table 6: Months Leading Up to the Exit in July 2006

	March	April	May	June	July
regime	Z	Z	Z	Z	P
m , log of actual-to-required reserve ratio	1.51	1.00	0.55	0.46	0
actual reserves relative to GDP (%)	4.2	2.6	1.6	1.4	0.9
π , year-on-year inflation rate (% per year)	0.1	-0.1	0.0	0.2	0.2
x , output gap (%)	-0.7	-0.4	-0.7	-0.4	-0.9
r , the policy rate (% per year)	0.0	0.0	0.0	0.0	0.26
g , trend growth rate (% per year)	0.9	0.9	0.9	0.9	0.9
r^* , desired Taylor rate (% per year)	0.5	0.4	0.4	0.5	0.5

Note: The desired Taylor rate r^* is defined in (4.1).

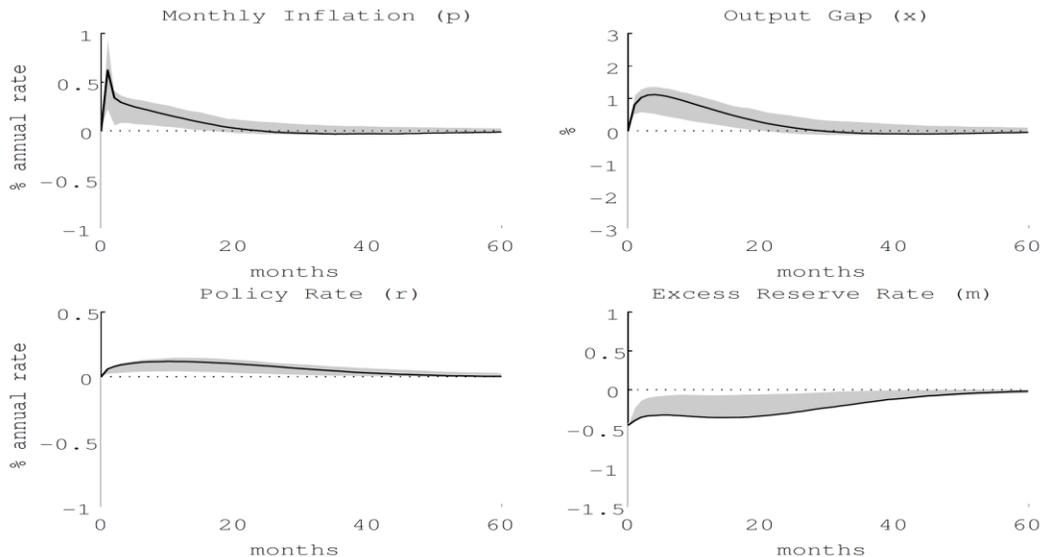
What would the difference have been if the exit had occurred a month earlier? We can answer this question by setting the base period to $t = \text{June 2006}$ (when the regime was $s_t = \mathbf{Z}$) and then considering the difference³¹

$$E(y_{t+k} | s_t = \mathbf{P}, (p_t, x_t, \bar{r}_t, 0), \dots) - E(y_{t+k} | s_t = \mathbf{Z}, (p_t, x_t, \bar{r}_t, m_t), \dots). \quad (6.3)$$

Thus, the perturbation occurs for not just one, but two variables: m_t and s_t . Because the regime is different between the baseline and alternative histories, the inflation–output dynamics in the following period will be different as well. The response profiles of the difference (6.3) are shown in Figure 4c. The perturbations to m of $\delta_m = 0.46$ (the value of m in June 2006, see Table 6) can be read from the intercepts in the lower-right panel. Surprisingly, despite the decrease in m , both inflation and output increase. Thus, exiting from QE in June 2006 would have been *expansionary*.

³¹ Under \mathbf{P} , $r_t > \bar{r}_t$. We define $E(y_{t+k} | s_t = \mathbf{P}, (p_t, x_t, \bar{r}_t, 0), \dots)$ as $\lim_{r \downarrow \bar{r}_t} E(y_{t+k} | s_t = \mathbf{P}, (p_t, x_t, r, 0), \dots)$. Alternatively, we could set r_t to some low value (e.g., 0.25%, which is the policy rate for July 2006) instead of \bar{r}_t ; however, the profiles would look very similar.

Figure 4c: The Effect of Exiting QE in June 2006



Note: The effect is defined in (6.3). The 68% probability bands in shades.

To see why, decompose the (overall) response (6.3) as the difference between two components:

$$\begin{aligned}
 (6.3) = & \underbrace{[E(y_{t+k}|s_t = \mathbf{P}, (p_t, x_t, \bar{r}_t, 0), \dots) - E(y_{t+k}|s_t = \mathbf{Z}, (p_t, x_t, \bar{r}_t, 0), \dots)]}_{\text{transitional effect of an exit from } \mathbf{Z} \text{ to } \mathbf{P}} \\
 & - \underbrace{[E(y_{t+k}|s_t = \mathbf{Z}, (p_t, x_t, \bar{r}_t, m_t), \dots) - E(y_{t+k}|s_t = \mathbf{Z}, (p_t, x_t, \bar{r}_t, 0), \dots)]}_{\text{the QE effect}}.
 \end{aligned}
 \tag{6.4}$$

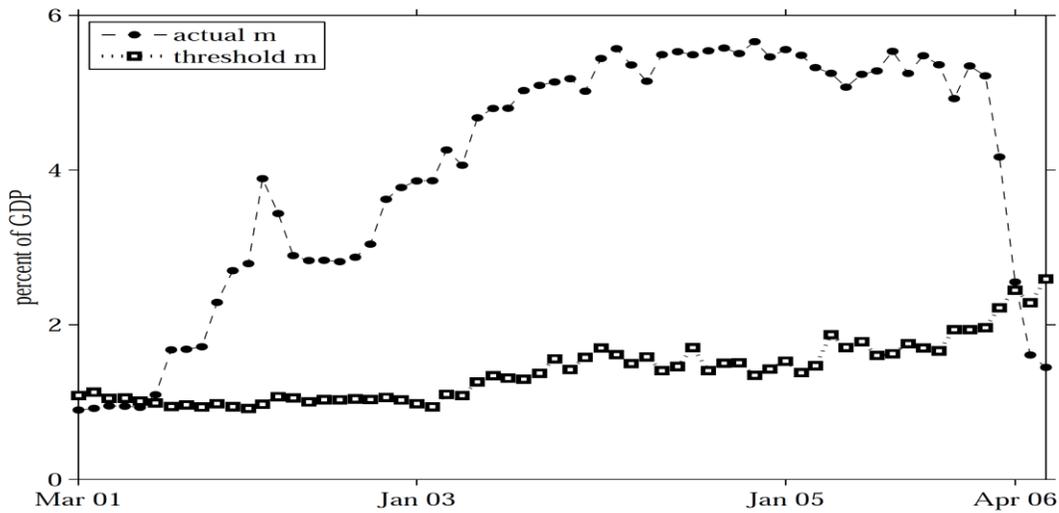
The culprit is the first component, which we call the “transitional effect.” The configuration of the three monetary policy shocks (v_{rt} , $v_{\pi t}$, v_{st}) underlying the difference is shown in equation (C.24). Its response profile (not shown here) exhibits an expansionary effect for inflation and output. The second component is the QE effect, which (as can be surmised from Figure 4b) is expansionary for $t = \text{June 2006}$.

Whether or not the overall response (6.3) is positive depends on the relative strength of the second component, which, in turn, depends on the size of m_t . The threshold value of m_t under which an exit would be expansionary³² is plotted for all months of the second \mathbf{Z} spell in Figure 5, along with the actual

³² An exit is deemed “expansionary” if the impulse response of output adds up to a positive number, that is, if the sum of (6.3) over $k = 1, 2, \dots, 60$ is positive for $y = x$.

value of m_t , as a percentage of GDP. The figure shows that an exit would have been expansionary for May 2006 as well, and nearly so for April 2006. However, an exit would have been contractionary for March 2006, or earlier. Turning to the other end of the spell, the March 2001 value of the actual m is less than the threshold m , implying that continuing **P** would have been expansionary. The central bank, if it wishes to stimulate the economy by entering the ELB regime, needs to expand reserves aggressively upon entry.

Figure 5: Actual and Threshold m , March 2001 - June 2006



Note: If actual m_t is less (greater) than threshold m_t , then an exit is expansionary (contractionary). The value is expressed as percent of GDP.

When is the Transitional Effect Expansionary?

The positive transitional effect from **Z** to **P** for $t = \text{June 2006}$ arises because the reduced form is more conducive to inflation and output under **P**. For both inflation and output, the intercept in the reduced-form equation is higher under **P** than it is under **Z**. The difference in the intercept is 0.9 ($t\text{-value} = 1.7$) for inflation and 1.1 ($t = 3.0$) for output. Here, the intercept includes the effect of trend growth. The trend growth rate for June 2006 is 0.9% (see Table 6). The difference in the intercept for output, for example, can be calculated for June 2006 from Table 5 as $(-0.70 + 0.83 \times 0.9) - (1.07 + 0.01 \times 0.9) = 1.1$. Thus, a higher trend growth (which represents the time-varying real rate) is more conducive to output

under **P**. This positive transitional effect from **Z** to **P** is consistent with Lansing's (2017) finding that the output response to the real rate gap is larger in a deflation equilibrium.³³

The transitional effect can be negative with low trend growth, depending on the initial conditions. The average trend growth has been around zero during the 2000s in Japan. To illustrate this point, consider an exit for the following two cases with zero trend growth: $\pi_t = p_t = 2, x_t = 0, \bar{r}_t = 0$ (Case 1), and $\pi_t = p_t = 0, x_t = 2, \bar{r}_t = 0$ (Case 2). The only difference between the two cases arises from the macroeconomic conditions upon exit; that is, Case 1 has higher inflation, but lower output than Case 2. We find that an exit is contractionary in Case 1, but it is expansionary in Case 2. This is mainly because the policy rate is more sensitive to inflation than output after exit (because the inflation coefficient in the Taylor rule is higher than the output coefficient). As a result, Case 1 leads to policies that are more contractionary than in Case 2 after the exit. In either case, the probability of the economy remaining under **P** for at least 12 months after the exit is less than 10 percent. In other words, the economy cannot enjoy the benefit of conducive inflation and output reduced-form dynamics under **P** for long.

7 Alternative specifications

In this section, we examine how the results from our counterfactual analysis are affected by various alterations of the baseline model. We show that (a) failure to control for the equilibrium real interest rate in the Taylor rule results in the price puzzle, (b) the QE effect is no longer statistically significant if the measure of potential GDP is the HP-filtered GDP, (c) allowing for active excess reserve demand and two types of the ELB regime hardly changes our results, and (d) the macroeconomic effects of signaling and portfolio rebalance channels are weak during our sample period.

Dropping Trend Growth

The baseline model includes the trend growth rate of potential GDP in the desired Taylor rate (defined in (4.1)), as well as in the inflation and output reduced forms. If trend growth is dropped from the model, the inflation coefficient in the desired Taylor rate rises from 0.69 (as shown in Table 3) to 0.96 (t -value = 3.6).

³³ For theoretical examples of expansionary policy-induced exits, see Hayashi (2018).

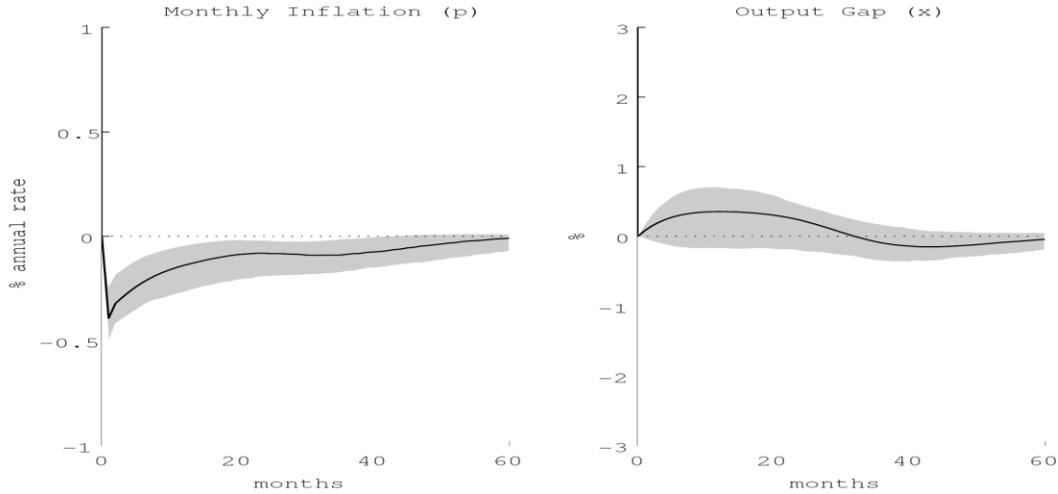
This can be understood as an omitted-variable bias. Recall from Figure 3 the simultaneous fall in inflation, trend growth, and the policy rate in the early to mid-1990s. The effect on the policy rate of the fall in the equilibrium real rate is now picked up by inflation, which is correlated with trend growth. For the reduced form, the break date detected by the sup F -statistic for the output equation, which was March 1995 when trend growth was included, is now May 1993. If the reduced form is estimated without trend growth on the post-break sample, the lagged r -coefficient in the inflation equation, which was virtually zero in Table 5, rises to 0.39 ($t = 2.6$). This is because the longer sample period for the reduced form now includes the period of the rapid fall of both the policy rate and inflation.

As a result of this change in the lagged r -coefficient, the policy-rate effect exhibits the price puzzle, as shown in the left panel of Figure 6: the rate cut is followed by deflation. Our interpretation of this evidence is that the price puzzle is spurious because it arises from the failure to control for the fall in the equilibrium real rate. Still, the QE effect and the June 2006 exit effect are very similar to those in Figures 4b and 4c for the baseline model.

HP-Filtered GDP as Potential GDP

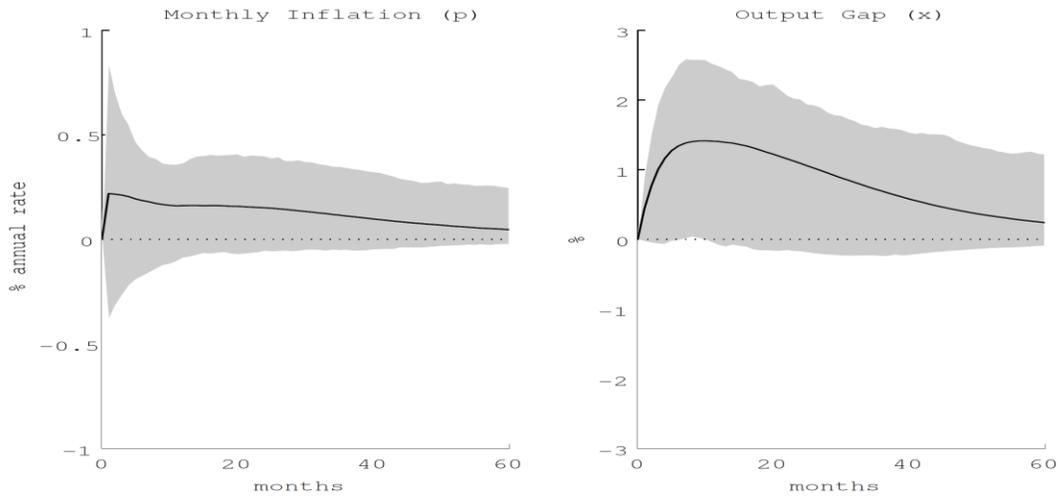
We used the official estimate by the Cabinet Office as the measure of potential GDP that underlies the output gap and the trend growth rate. If we switch the measure to the HP-filtered GDP, the main consequence is that the positive lagged m -coefficients in the inflation and output equations under QE, shown in Table 5 for the case of the Cabinet Office potential GDP, become statistically insignificant. As a result, the error bands for the QE effect and for the June 2006 exit effect become wider than those shown in Figures 4b and 4c for the baseline model. The QE effect is no longer significant because the error bands include the horizontal axes, as shown in Figure 7. The expansionary effect of the June 2006 exit remains significant.

Figure 6: The Policy-Rate Effect, March 1995, Trend Growth Dropped



Note: The trend growth rate is dropped from the Taylor rule and the reduced-form equations. The policy-rate effect is defined in (6.1). The perturbation size is -100 basis points. The 68% probability bands in shades.

Figure 7: The QE Effect, February 2004, HP-Filtered Potential Output



Note: The potential GDP measure that underlies the output gap and the trend growth rate is the HP (Hodrick-Prescott) filtered GDP. The QE effect is defined in (6.2). The perturbation size is 1. The 68% probability bands in shades.

Active Excess Reserve Demand and Allowing for Two Types of Z

The two assumptions we have assumed for the baseline model are that (i) m is zero under \mathbf{P} and (ii) m is supply-determined under \mathbf{Z} . We now relax the both assumptions. A formal statement of this generalized model is given in Appendix A.³⁴

The response profiles and error bands are very similar to those for the baseline model, except for the policy-rate effect, where the initial regime is \mathbf{P} in both the baseline and the alternative histories. During the initial months, m is occasionally positive in the underlying simulations. This is itself contractionary because the lagged m -coefficient in the reduced form under \mathbf{P} (shown in the upper panel of Appendix Table A.1) is negative for both inflation and output. The contractionary effect is greater under the alternative scenario because m is larger owing to the lower policy rates. Thus, the expansionary effect of the rate cut is somewhat weakened. However, as in the baseline model, the effect is not statistically significant.

Expanded SVAR

As in Koeda (2018), we do not find macroeconomic effects of signaling and portfolio-rebalance effects over our sample period. When we include lagged asset-price variables (e.g., term spread) as additional regressors in the reduced form for inflation and output, the corresponding coefficients are statistically insignificant. Given this weak empirical evidence for the macro effects, we decide not to include the long-term rate or term spread in our baseline SVAR, because doing so would not change the main results.

³⁴ Relaxing (i) entails three changes. First, replace the zero excess reserve under \mathbf{P} in equation (4.5) with $\max[m_{dt}, 0]$, where m_{dt} is the demand for excess reserves specified in equation (A.6) of Appendix A. Second, include lagged m in the reduced-form equations for the lagged subsample \mathbf{P} . Third, the definition of the policy-rate effect in (6.1) and that of the effect of an exit in (6.3) need to be modified as follows: (a) in (6.1), replace the zero in the baseline history by m_t (the actual m for period t), and (b) in both (6.1) and (6.3), replace the zero in the alternative history by m_t^e (the expected value of $\max[m_{dt}, 0]$, given the information specified in the respective alternative history). Relaxing (ii) requires that we distinguish between two ELB regimes. Under the “strong” ELB regime, as in the second ELB spell (March 2001–June 2006) and the third ELB spell (December 2008 onward), the net policy rate is zero and m is supply-determined. This regime is what we have been referring to as QE. Under the “weak” ELB regime, as in the first ELB spell, the net policy rate is zero but m is demand-determined. We assume that the central bank chooses between the weak and strong ELB regimes randomly. We set the probability for the weak regime to 1/3. We do not allow the regime to change from weak to strong or from strong to weak. See equation (A.4) for a precise formulation.

8 Conclusion

We have constructed a regime-switching SVAR in which the observable regime is determined by the central bank responding to economic conditions. The model was used to study the dynamic effect of not only changes in the policy rate and the reserve supply, but also of shifts in the regime chosen by the central bank.

Our nonlinear counterfactual analyses show that whether a QE exit is expansionary or contractionary depends on the history. The exit bonus of the transitional effect arises from conducive macroeconomic reduced-form dynamics under **P**. However, the transitional effect can be negative if the economy after the exit does not remain under **P** for long, or if the policy rate hikes after exit are sufficiently aggressive. Such a situation can occur with a combination of relatively high inflation, a low output gap, and low trend growth at the exit. The flip side of the exit bonus of transitional effects is the entry cost to QE. We find an entry cost to QE in Japan; that is, entering QE with no significant increase in the reserve supply would be contractionary. Thus, the central bank would wish to raise the reserve supply aggressively upon entry.

Although this study provides an estimate of the effect of a change in the IOR under QE, it is not precise. Because the policy rate changes in tandem with the IOR under QE, the effect is captured by the lagged policy-rate coefficient in the reduced form for QE. The very limited variability of the IOR in the sample is responsible for the imprecise estimates. This is pertinent because the “exit” by the Fed in the fall of 2015 is, in our definition, a continuation of QE with a zero net policy rate, but with a higher IOR.

APPENDIX A: THE GENERALIZED MODEL

This appendix has three parts:

- A self-contained exposition of the model that is more general than the one in the text in that: (i) excess reserves are not constrained to be zero and (ii) there are two types of the ELB (effective lower bound) regime, one with demand-determined and the other with supply-determined excess reserves.
- A derivation of the likelihood function for the generalized model.
- Parameter estimates by maximum likelihood

A.1 The model

The model's state variables are (s_t, \mathbf{y}_t) . The continuous state \mathbf{y}_t has the following elements:

$$\mathbf{y}_t \equiv \begin{bmatrix} \mathbf{y}_{1t} \\ r_t \\ m_t \end{bmatrix}, \quad \mathbf{y}_{1t} \equiv \begin{bmatrix} p_t \\ x_t \end{bmatrix}, \quad (\text{A.1})$$

(4×1) (2×1)

where p = monthly inflation rate, x = output gap, r = policy rate, and m = excess reserve rate. The model's discrete state variable s_t is either = **P**, **W**, or **S** (**W** for a “weak” ELB regime and **S** for a “strong” ELB regime). Regime **S** is what is referred to as “QE” in the text. In both **S** and **W**, the policy rate (r_t) equals the rate paid on reserves (\bar{r}_t). The difference, as indicated below in (A.5), is that m_t is supply-determined under **S**, while it is demand-determined under **W**.

The model also involves a vector of exogenous variables, \mathbf{x}_t . It includes \bar{r}_t , the rate paid on reserves. It can include other variables (such as the trend growth rate), but the identity of those other exogenous variables is immaterial in the derivation of the likelihood function below.

The model is a mapping from

$$(s_{t-1}, \mathbf{y}_{t-1}, \dots, \mathbf{y}_{t-11}, \mathbf{x}_t, \underbrace{\boldsymbol{\varepsilon}_t}_{(2 \times 1)}, v_{rt}, v_{\bar{r}t}, v_{st}, v_{dt})$$

to (s_t, \mathbf{y}_t) . Here, $(\boldsymbol{\varepsilon}_t, v_{rt}, v_{\bar{r}t}, v_{st}, v_{dt})$ are mutually and serially independent shocks. We need to include 11 lags of \mathbf{y} because of the appearance of the 12-month inflation rate in the model, see (A.3) below. The mapping is defined as follows.

- (i) (\mathbf{y}_{1t} determined) ε_t is drawn from $\mathcal{N}(\mathbf{0}, \mathbf{\Omega}(s_{t-1}))$ and \mathbf{y}_{1t} (the first two elements of \mathbf{y}_t) is given by the inflation and output reduced form:

$$\mathbf{y}_{1t} = \mathbf{c}(s_{t-1}) + \mathbf{A}(s_{t-1})\mathbf{x}_t + \mathbf{\Phi}(s_{t-1})\mathbf{y}_{t-1} + \varepsilon_t. \quad (\text{A.2})$$

(2×1) (2×1) (2×4) (4×1) (2×1)

Here, only one lag is allowed, strictly for expositional purposes; more lags can be included without any technical difficulties. The matrix $\mathbf{A}(s_{t-1})$ has two rows.

- (ii) (s_t determined) The central bank draws $(v_{rt}, v_{\bar{\pi}t})$ from $\mathcal{N}(\mathbf{0}, \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_{\bar{\pi}}^2 \end{bmatrix})$. Given this, \mathbf{y}_{1t} , and $(\mathbf{y}_{t-1}, \dots, \mathbf{y}_{t-11})$, the central bank calculates the 12-month inflation rate and the *Taylor rate* given by:

$$\pi_t \equiv \frac{1}{12} (p_t + \dots + p_{t-11}) \quad \text{and} \quad \underbrace{(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}}_{\text{Taylor rate}}, \quad \text{where } r_t^* \equiv \alpha_r^* + \delta_r^* \mathbf{x}_t + \beta_r^* \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}. \quad (\text{A.3})$$

The central bank determines s_t as

$$\left\{ \begin{array}{l} \text{If } s_{t-1} = \mathbf{P}, \quad \text{then } s_t = \begin{cases} \mathbf{P} & \text{if } \underbrace{(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}}_{\text{Taylor rate}} > \bar{r}_t, \\ \left\{ \begin{array}{l} \mathbf{W} \text{ with probability } q \\ \mathbf{S} \text{ with probability } 1 - q \end{array} \right. & \text{otherwise.} \end{cases} \\ \\ \text{If } s_{t-1} = \mathbf{W}, \quad \text{then } s_t = \begin{cases} \mathbf{P} & \text{if } \underbrace{(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}}_{\text{Taylor rate}} > \bar{r}_t \text{ and } \pi_t \geq \underbrace{\bar{\pi} + v_{\bar{\pi}t}}_{\text{period } t \text{ threshold}}, \\ \mathbf{W} & \text{otherwise,} \end{cases} \\ \\ \text{If } s_{t-1} = \mathbf{S}, \quad \text{then } s_t = \begin{cases} \mathbf{P} & \text{if } \underbrace{(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}}_{\text{Taylor rate}} > \bar{r}_t \text{ and } \pi_t \geq \underbrace{\bar{\pi} + v_{\bar{\pi}t}}_{\text{period } t \text{ threshold}}, \\ \mathbf{S} & \text{otherwise.} \end{cases} \end{array} \right. \quad (\text{A.4})$$

The *exit condition* is the extra condition “ $\pi_t \geq \bar{\pi} + v_{\bar{\pi}t}$ ” in the above.

- (iii) ((r_t, m_t) determined) The central bank draws v_{st} from $\mathcal{N}(0, \sigma_s^2)$ and the market draws v_{dt} from $\mathcal{N}(0, \sigma_d^2)$. They are independently distributed. Given (v_{st}, v_{dt}) , the Taylor rate, and

s_t , the central bank determines (r_t, m_t) as follows.

$$\left\{ \begin{array}{l} \text{If } s_t = \mathbf{P}, \quad \text{then } (r_t, m_t) = \left(\underbrace{(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}}_{\text{Taylor rate}}, \max[m_{dt}, 0] \right), \\ \text{If } s_t = \mathbf{W}, \quad \text{then } (r_t, m_t) = \left(\bar{r}_t, \max[m_{dt}, 0] \right), \\ \text{If } s_t = \mathbf{S}, \quad \text{then } (r_t, m_t) = \left(\bar{r}_t, \max[m_{st}, 0] \right), \end{array} \right. \quad (\text{A.5})$$

where m_{dt} and m_{st} are given by the excess reserve demand and supply equations:

$$m_{dt} = m_{dt}^e + v_{dt}, \quad m_{dt}^e \equiv \alpha_d + \beta'_d \begin{bmatrix} \pi_t \\ x_t \\ r_t - \bar{r}_t \end{bmatrix} + \gamma_d \max[m_{d,t-1}, 0], \quad (\text{A.6})$$

$$m_{st} = m_{st}^e + v_{st}, \quad m_{st}^e \equiv \alpha_s + \beta'_s \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \gamma_s m_{t-1}. \quad (\text{A.7})$$

Note that the specification of the lagged term differs between m_{dt} and m_{st} . For m_{st} , the lagged term is m_{t-1} , which means that the central bank decides on m_{st} taking the last month's actual m , whether it is demand- or supply-determined, as the base for current decision. Having m_{t-1} as the lagged term may not be plausible for m_{dt} , which is determined by the private sector. A more plausible specification would be to have $m_{d,t-1}$ as the lagged term. However, as will be discussed below, evaluating the likelihood function for this specification is very difficult. Instead, the specification we employ is to have $\max[m_{d,t-1}, 0]$ as the lagged term.

Let θ be the model's parameter vector. It consists of q (the probability that \mathbf{W} is chosen when the previous regime is \mathbf{P} and the current Taylor rate is below \bar{r}_t , see (A.4)) and the following four groups:

$$\left\{ \begin{array}{ll} \text{(reduced-form)} & \theta_A = (\mathbf{c}(j), \mathbf{A}(s_{t-1}), \Phi(s_{t-1}), \Omega(s_{t-1}), s_{t-1} = \mathbf{P}, \mathbf{W}, \mathbf{S}), \\ \text{(Taylor rule)} & \theta_B = (\alpha_r^*, \delta_r^*, \beta_r^*, \gamma_r, \sigma_r, \bar{\pi}, \sigma_{\bar{\pi}}), \\ \text{(excess reserve supply)} & \theta_C = (\alpha_s, \beta_s, \gamma_s, \sigma_s), \\ \text{(excess reserve demand)} & \theta_D = (\alpha_d, \beta_d, \gamma_d, \sigma_d). \end{array} \right. \quad (\text{A.8})$$

A.2 Derivation of the likelihood function

The likelihood of the data is (with its dependence on the parameter vector left implicit)

$$\mathcal{L} \equiv p(s_1, \dots, s_T, \mathbf{y}_1, \dots, \mathbf{y}_T | \mathbf{x}, \mathcal{Z}_0). \quad (\text{A.9})$$

Here, $\mathbf{x} \equiv (\mathbf{x}_T, \mathbf{x}_{T-1}, \dots)$, $\mathcal{Z}_t \equiv (s_t, s_{t-1}, \dots, \mathbf{y}_t, \mathbf{y}_{t-1}, \dots)$, and $p(\cdot|\cdot)$ is the joint density-distribution function of $(s_1, \dots, s_T, \mathbf{y}_1, \dots, \mathbf{y}_T)$ conditional on $(\mathbf{x}, \mathcal{Z}_0)$. The usual sequential factorization yields

$$\mathcal{L} = \prod_{t=1}^T p(s_t, \mathbf{y}_t | \mathbf{x}, \mathcal{Z}_{t-1}). \quad (\text{A.10})$$

Consider the likelihood for date t , $p(s_t, \mathbf{y}_t | \mathbf{x}, \mathcal{Z}_{t-1})$ in (A.10). Since $\{\mathbf{x}_t\}$ is exogenous, it can be written as

$$p(s_t, \mathbf{y}_t | \mathbf{x}, \mathcal{Z}_{t-1}) = p(s_t, \mathbf{y}_t | \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1}). \quad (\text{A.11})$$

Recalling that $\mathbf{y}_t = (\mathbf{y}_{1t}, r_t, m_t)$, we rewrite this date t likelihood as

$$\begin{aligned} p(s_t, \mathbf{y}_t | \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1}) &= p(m_t | r_t, s_t, \mathbf{y}_{1t}, \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1}) \\ &\quad \times p(r_t | s_t, \mathbf{y}_{1t}, \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1}) \\ &\quad \times \text{Prob}(s_t | \mathbf{y}_{1t}, \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1}) \\ &\quad \times p(\mathbf{y}_{1t} | \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1}). \end{aligned} \quad (\text{A.12})$$

In what follows, we rewrite each of the four terms on the right-hand side of this equation in terms of the model parameters. To facilitate the notation, define

$$r_t^e \equiv (1 - \gamma_r)r_t^* + \gamma_r r_{t-1} \quad \text{where (as in (A.3)) } r_t^* \equiv \alpha_r^* + \delta_r^{*'} \mathbf{x}_t + \beta_r^{*'} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}, \quad (\text{A.13})$$

so that the Taylor rate $(1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}$ equals $r_t^e + v_{rt}$.

The Fourth Term, $p(\mathbf{y}_{1t} | \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1})$

This term is entirely standard:

$$p(\mathbf{y}_{1t} | \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1}) = b\left(\mathbf{y}_{1t} - (\mathbf{c}(s_{t-1}) + \mathbf{A}(s_{t-1})\mathbf{x}_t + \mathbf{\Phi}(s_{t-1})\mathbf{y}_{t-1}); \mathbf{\Omega}(s_{t-1})\right), \quad (\text{A.14})$$

where $b(\cdot; \mathbf{\Omega})$ is the density of the bivariate normal distribution with mean $\mathbf{0}$ and variance-covariance matrix $\mathbf{\Omega}$.

The Third Term, $\text{Prob}(s_t | \mathbf{y}_{1t}, \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1})$

This is the transition probability matrix for $\{s_t\}$. The probabilities depend on $(r_t^e, \pi_t, \bar{r}_t)$ (which in term can be calculated from $(\mathbf{y}_{1t}, \mathbf{x}_t, \mathcal{Z}_{t-1})$, see (A.3) and (A.13)). They are easy to derive from (A.4):

$s_{t-1} \backslash s_t$	P	W	S
P	P_{rt}	$(1 - P_{rt})q$	$(1 - P_{rt})(1 - q)$
W	$P_{rt}P_{\pi t}$	$1 - P_{rt}P_{\pi t}$	0
S	$P_{rt}P_{\pi t}$	0	$1 - P_{rt}P_{\pi t}$

Here,

$$P_{rt} \equiv \text{Prob}(r_t^e + v_{rt} > \bar{r}_t | r_t^e, \bar{r}_t) = \Phi\left(\frac{r_t^e - \bar{r}_t}{\sigma_r}\right), \quad (\text{A.15})$$

$$P_{\pi t} \equiv \text{Prob}(\pi_t \geq \bar{\pi} + v_{\pi t} | \pi_t) = \Phi\left(\frac{\pi_t - \bar{\pi}}{\sigma_{\bar{\pi}}}\right), \quad (\text{A.16})$$

where $\Phi(\cdot)$ is the cdf of $\mathcal{N}(0, 1)$.

The First Term, $p(m_t | r_t, s_t, \mathbf{y}_{1t}, \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1})$

m_t is given by (A.5) where m_{dt} and m_{st} are defined in (A.6) and (A.7). The right-hand-side variables in those definitions, including $\max[m_{d,t-1}, 0]$ and m_{t-1} , are functions of $(r_t, s_t, \mathbf{y}_{1t}, \mathbf{x}_t, \mathcal{Z}_{t-1})$. So this term is the Tobit distribution-density function given by

$$h_{jt} \equiv \left[\frac{1}{\sigma_j} \phi\left(\frac{m_t - m_{jt}^e}{\sigma_j}\right) \right]^{1(m_t > 0)} \times \left[1 - \Phi\left(\frac{m_{jt}^e}{\sigma_j}\right) \right]^{1(m_t = 0)}, \quad (\text{A.17})$$

$j = d$ if $s_t = \mathbf{P}$ or \mathbf{W} ; $j = s$ if $s_t = \mathbf{S}$,

where $1(\cdot)$ is the indicator function, $\phi(\cdot)$ and $\Phi(\cdot)$ are the density and the cdf of $\mathcal{N}(0, 1)$.

The Second Term, $p(r_t | s_t, \mathbf{y}_{1t}, \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1})$

If $s_t = \mathbf{W}$ or \mathbf{S} , then $r_t = \bar{r}_t$ with probability 1, so this term can be set to 1. If $s_t = \mathbf{P}$, there are two cases to consider due to the exit condition. In either case, the second term turns out to be the same, as shown below:

- For $s_{t-1} = \mathbf{P}$,

$$\begin{aligned}
& p(r_t | s_t = \mathbf{P}, \mathbf{y}_{1t}, \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1}) \\
&= p(r_t^e + v_{rt} | r_t^e + v_{rt} > \bar{r}_t, r_t^e, \bar{r}_t) \\
&\quad \text{(by (A.4) and (A.5), and since } (r_t^e, \bar{r}_t) \text{ is a function of } (\mathbf{y}_{1t}, \mathbf{x}_t, \mathcal{Z}_{t-1})) \\
&= \frac{p(r_t^e + v_{rt} | r_t^e)}{\text{Prob}(r_t^e + v_{rt} > \bar{r}_t | r_t^e, \bar{r}_t)} \\
&= \frac{\frac{1}{\sigma_r} \phi\left(\frac{v_{rt}}{\sigma_r}\right)}{\text{Prob}(r_t^e + v_{rt} > \bar{r}_t | r_t^e, \bar{r}_t)} \quad \text{(b/c } r_t^e + v_{rt} \sim \mathcal{N}(r_t^e, \sigma_r^2)) \\
&= \frac{\frac{1}{\sigma_r} \phi\left(\frac{r_t - r_t^e}{\sigma_r}\right)}{P_{rt}} \quad \text{(by definition (A.15) of } P_{rt}) \tag{A.18}
\end{aligned}$$

- For $s_{t-1} = \mathbf{W}$ or \mathbf{S} ,

$$\begin{aligned}
& p(r_t | s_t = \mathbf{P}, \mathbf{y}_{1t}, \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1}) \\
&= p(r_t^e + v_{rt} | r_t^e + v_{rt} > \bar{r}_t, \pi_t \geq \bar{\pi} + v_{\bar{\pi}t}, r_t^e, \bar{r}_t, \pi_t) \\
&\quad \text{(by (A.4) and (A.5), and since } (r_t^e, \bar{r}_t, \pi_t) \text{ is a function of } (\mathbf{y}_{1t}, \mathbf{x}_t, \mathcal{Z}_{t-1})) \\
&= p(r_t^e + v_{rt} | r_t^e + v_{rt} > \bar{r}_t, r_t^e, \bar{r}_t) \quad \text{(b/c } v_{rt} \text{ and } v_{\bar{\pi}t} \text{ are independent)} \\
&= \frac{\frac{1}{\sigma_r} \phi\left(\frac{r_t - r_t^e}{\sigma_r}\right)}{P_{rt}} \quad \text{(as above).} \tag{A.19}
\end{aligned}$$

Putting All Pieces Together

Putting all those pieces together, the components of the likelihood for date t , defined in (A.12), can be written as (with \mathcal{X}_t here denoting $(\mathbf{x}_t, \mathbf{x}_{t-1}, \dots)$ for brevity)

$s_t s_{t-1}$	$p(m_t r_t, s_t, \mathbf{y}_{1t}, \mathcal{X}_t, \mathcal{Z}_{t-1})$	$p(r_t s_t, \mathbf{y}_{1t}, \mathcal{X}_t, \mathcal{Z}_{t-1})$	$\text{Prob}(s_t \mathbf{y}_{1t}, \mathcal{X}_t, \mathcal{Z}_{t-1})$	$p(\mathbf{y}_{1t} \mathcal{X}_t, \mathcal{Z}_{t-1})$
P P	h_{dt}	$\frac{g_t}{P_{rt}}$	P_{rt}	f_{Pt}
P W	h_{dt}	$\frac{g_t}{P_{rt}}$	$P_{rt}P_{\pi t}$	f_{Wt}
P S	h_{dt}	$\frac{g_t}{P_{rt}}$	$P_{rt}P_{\pi t}$	f_{St}
W P	h_{dt}	1	$(1 - P_{rt})q$	f_{Pt}
W W	h_{dt}	1	$1 - P_{rt}P_{\pi t}$	f_{Wt}
S P	h_{st}	1	$(1 - P_{rt})(1 - q)$	f_{Pt}
S S	h_{st}	1	$1 - P_{rt}P_{\pi t}$	f_{St}

Here,

$$f_{jt} \equiv b(\mathbf{y}_{1t} - \mathbf{c}(j) - \mathbf{A}(j)\mathbf{x}_t - \mathbf{\Phi}(j)\mathbf{y}_{t-1}; \mathbf{\Omega}(j)) \text{ for } j = \mathbf{P}, \mathbf{W}, \mathbf{S},$$

$$g_t \equiv \frac{1}{\sigma_r} \phi\left(\frac{r_t - r_t^e}{\sigma_r}\right), \quad P_{rt} \equiv \Phi\left(\frac{r_t^e - \bar{r}_t}{\sigma_r}\right) \text{ (as in (A.15))}, \quad P_{\pi t} \equiv \Phi\left(\frac{\pi_t - \bar{\pi}}{\sigma_{\bar{\pi}}}\right) \text{ (as in (A.16))},$$

h_{jt} is defined in (A.17) and $b(\cdot; \mathbf{\Omega})$ is the density function of the bivariate normal distribution with mean $\mathbf{0}_{(2 \times 1)}$ and variance-covariance matrix $\mathbf{\Omega}_{(2 \times 2)}$.

Dividing it into Pieces

Taking the log of both sides of (A.10) while taking into account (A.11) and (A.12) and substituting the entries in the table, we obtain the log likelihood of the sample:

$$L \equiv \log(\mathcal{L}) = \sum_{t=1}^T \log[p(s_t, \mathbf{y}_t | \mathbf{x}_t, \mathbf{x}_{t-1}, \dots, \mathcal{Z}_{t-1})] = L_A + L_1 + L_2 + L_D + L_q$$

where

$$L_A = \sum_{s_{t-1}=\mathbf{P}} \log[f_{Pt}] + \sum_{s_{t-1}=\mathbf{W}} \log[f_{Wt}] + \sum_{s_{t-1}=\mathbf{S}} \log[f_{St}], \quad (\text{A.20})$$

$$L_1 = \sum_{s_t=\mathbf{P}} \log[P_{rt}] + \sum_{s_t|s_{t-1}=\mathbf{P}|\mathbf{W}, \mathbf{P}|\mathbf{S}} \log[P_{\pi t}]$$

$$+ \sum_{s_t|s_{t-1}=\mathbf{W}|\mathbf{P}, \mathbf{S}|\mathbf{P}} \log[1 - P_{rt}] + \sum_{s_t|s_{t-1}=\mathbf{W}|\mathbf{W}, \mathbf{S}|\mathbf{S}} \log[1 - P_{rt}P_{\pi t}], \quad (\text{A.21})$$

$$L_2 = \sum_{s_t=\mathbf{P}} [\log(g_t) - \log(P_{rt})] + \sum_{s_t=\mathbf{S}} \log[h_{st}], \quad (\text{A.22})$$

$$L_D = \sum_{s_t=\mathbf{P}, \mathbf{W}} \log [h_{dt}], \quad (\text{A.23})$$

$$L_q = \sum_{s_t | s_{t-1}=\mathbf{W} | \mathbf{P}} \log [q] + \sum_{s_t | s_{t-1}=\mathbf{S} | \mathbf{P}} \log [1 - q]. \quad (\text{A.24})$$

The terms in $L_1 + L_2$ can be regrouped into L_B and L_C , as in

$$L = L_A + \underbrace{L_B + L_C}_{=L_1+L_2} + L_D + L_q, \quad (\text{A.25})$$

where

$$\begin{aligned} L_B = & \sum_{s_t=\mathbf{P}} \log [g_t] + \sum_{s_t | s_{t-1}=\mathbf{P} | \mathbf{W}, \mathbf{P} | \mathbf{S}} \log [P_{\pi t}] \\ & + \sum_{s_t | s_{t-1}=\mathbf{W} | \mathbf{P}, \mathbf{S} | \mathbf{P}} \log [1 - P_{rt}] + \sum_{s_t | s_{t-1}=\mathbf{W} | \mathbf{W}, \mathbf{S} | \mathbf{S}} \log [1 - P_{rt}P_{\pi t}], \end{aligned} \quad (\text{A.26})$$

$$L_C = \sum_{s_t=\mathbf{S}} \log [h_{st}]. \quad (\text{A.27})$$

L_A, L_B, L_C, L_D , and L_q can be maximized separately, because L_j ($j = A, B, C, D$) depends only on θ_j ($j = A, B, C, D$) ($(\theta_A, \theta_B, \theta_C, \theta_D)$ was defined in (A.8) above) and L_q depends only on q .

As a special case, consider simplifying step (ii) of the mapping above by dropping the exit condition “ $\pi_t \geq \bar{\pi} + v_{\pi t}$ ”. This is equivalent to constraining $P_{\pi t}$ to be 1, so L_B becomes

$$L_B = \sum_{s_t=\mathbf{P}} \log [g_t] + \sum_{s_t=\mathbf{W}, \mathbf{S}} \log [1 - P_{rt}], \quad (\text{A.28})$$

which is the Tobit log likelihood function.

A.3 Parameter estimates

For estimation, we need to designate the set of months for each regime. As we argued in Section 3, excess reserves were demand-determined during the first ELB (effective lower bound) spell (March 1999 - July 2000) and supply-determined during the second and third ELB spells (March 2001-June 2006 and December 2008-December 2012). Thus:

- $s_t = \mathbf{W}$ if March 1999 $\leq t \leq$ July 2000 (17 months),

- $s_t = \mathbf{S}$ (i.e., QE) if March 2001 $\leq t \leq$ June 2006 or December 2008 $\leq t \leq$ December 2012 (113 months),
- $s_t = \mathbf{P}$ for all other months since January 1988 (170 months).

The model's parameters are listed in (A.8). Of these,

- the ML estimate of θ_A for $s_{t-1} = \mathbf{S}$ (on the sample of t 's such that $s_{t-1} = \mathbf{S}$; 112 months) is shown in the lower panel of Table 5 (\mathbf{Z} in the table corresponds to \mathbf{S} or QE),
- the ML estimate of θ_B (on the sample of t 's such that $s_t = \mathbf{P}, \mathbf{W}$, or \mathbf{S} , $t \geq$ May 1995; 252 months) in Table 3,
- the ML estimate of θ_C (for $s_t = \mathbf{S}$; 113 months) in Table 4.

The ML estimate of θ_A for $s_{t-1} = \mathbf{P}$ is in the upper panel of Table 5, but m_{t-1} is excluded as a regressor because in the text m is constrained to be zero under \mathbf{P} . With active excess reserve demand, we need to include m_{t-1} , which is occasionally positive even under \mathbf{P} , as a regressor. Thus, the remaining parameters to be estimated are: θ_A for $s_{t-1} = \mathbf{P}$, θ_A for $s_{t-1} = \mathbf{W}$, and θ_D (for the excess reserve demand equation)

θ_A for $s_{t-1} = \mathbf{P}$: The upper panel of Table A.1 has the reduced-form estimates for the post-break period beginning March 1995. The lagged m coefficient comes in with a negative sign in both the inflation and output equations, perhaps because positive excess reserves act as a signal of shocks disrupting financial intermediation that would have a contractionary effect in the next period.

θ_A for $s_{t-1} = \mathbf{W}$: The sample, which is composed of t 's for which $s_{t-1} = \mathbf{W}$, has only 17 observations. Because r is constant (at zero), the lagged r coefficient cannot be identified. We constrain it to be zero. There is not much variation in the trend growth rate, which creates near multi-collinearity between trend growth and the constant. We subsume the effect of trend growth in the constant by dropping it from the reduced form.

The lower panel of Table A.1 has the reduced-form estimates for \mathbf{W} . The positive lagged m coefficient in the output equation is not consistent with our interpretation, given above for the negative lagged m coefficients in the upper panel, that a positive excess reserve is a signal of a dis-intermediation shock. This positive coefficient will not materially affect our

counterfactual analysis because regime **W** does not occur very often and also because excess reserves are positive only rarely under **W**.

TABLE A.1. Reduced form, with occasionally positive excess reserve demand

lagged subsample P								
$t - 1$ is in	dependent variable	coefficient of						R^2
		const.	g_t	p_{t-1}	x_{t-1}	r_{t-1}	m_{t-1}	
P (85 obs.)	inflation (p_t)	0.04 [0.1]	0.09 [0.1]	-0.05 [-0.4]	0.13 [1.4]	0.07 [0.1]	-3.3 [-0.6]	0.05
	output (x_t)	-0.23 [-0.7]	0.33 [0.8]	-0.03 [-0.4]	0.87 [14]	-0.23 [-0.7]	-10.2 [-3.0]	0.80
lagged subsample W								
$t - 1$ is in	dependent variable	coefficient of						R^2
		const.	g_t	p_{t-1}	x_{t-1}	r_{t-1}	m_{t-1}	
W (17 obs.)	inflation (p_t)	0.62 [0.8]		-0.29 [-1.1]	0.12 [0.6]		-5.3 [-2.2]	0.31
	output (x_t)	-2.0 [-2.6]		0.26 [0.9]	0.49 [2.5]		6.4 [2.6]	0.63

Note: Estimation by OLS. t -values in brackets. The “lagged subsample **P**” of 85 observations refers to t ’s for which $t \geq$ March 1995 and $t - 1$ is not in the three ELB spells. The “lagged subsample **W**” of 17 observations refers to t ’s for which $t - 1$ is in the first ELB spell (March 1999 - July 2000). p is the monthly inflation rate in percent per year, x is the output gap in percent, r is the policy rate in percent per year, m is the excess reserve rate (defined as the log of the ratio of actual to required reserves), and g is the trend growth rate (the 12-month growth rate in percent of potential output). For lagged subsample **W**, the trend growth rate g_t is excluded to avoid near-multicollinearity with the constant, and r_{t-1} is excluded because its value is 0 for all months.

θ_D : The specification of m_{dt} is given in (A.6). The equation is to be estimated on those months for which m_t is demand-determined, i.e., on the sample of t ’s such that s_t is either **P** (170 months) or **W** (17 months). So the sample size is 187.

The lagged term in (A.6) is not the lagged dependent variable $m_{d,t-1}$ but its censored value $\max[m_{d,t-1}, 0]$. If the lagged term were the former, then the likelihood function would be very difficult to evaluate, see Lee (1999). When the lagged regime is not **S**, the lagged term equals m_{t-1} because excess reserves are demand-determined (see (A.5)). When the lagged regime is **S**, which in our data occurs only for $t =$ July 2006 when the

regime changed from **S** to **P**, the lagged term is unobservable because excess reserves were supply-determined in $t - 1 = \text{June } 2006$. We set the lagged term to zero, which amounts to assuming that the excess reserve demand in June 2006 was nonnegative.

The estimation method is Tobit because of the censoring of $\max[m_{dt}, 0]$ (recall that $m_t = \max[m_{dt}, 0]$ when $s_t = \mathbf{P}$ or \mathbf{W} , see (A.5)). We define the limit observations as the months for which $m < 0.5\%$. There are 144 such months in the sample of 187 months. Recall that we have set $m_t = 0$ for months between the second and the third ELB spells (except the Lehman crisis months of September to November 2008), on the ground that banks postponed re-entry to the interbank market and held on to excess reserves. So those months, indicated by the thin bars in Figure 1, are limit observations.

The estimated excess reserve demand equation is (t -values in brackets)

$$m_{dt} = 0.003 + 0.008\pi_t - 0.013x_t - 0.14(r_t - \bar{r}_t) + 0.59 \max[m_{d,t-1}, 0],$$

$$\begin{array}{ccccccc} [0.1] & [0.4] & [-2.1] & [-2.6] & [4.1] & & \end{array} \quad (\text{A.29})$$

estimated standard deviation of the error = 0.055 (s.e. = 0.0061),
sample size = 187, number of limit observations = 144.

The output coefficient is negative, probably because commercial banks desire more excess reserves in proportion to the severity of recessions.

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