Supplemental Material for "Bounds on Treatment Effects in Regression Discontinuity Designs with a Manipulated Running Variable"

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C. Identification: Further Results and Extensions

The results in Section 3 in the main body of the paper can be extended in various ways. In this section, we show how our results can be extended to quantile treatment effects, how our bounds on Γ change if we allow for non-continuously distributed outcomes, that additional behavioral assumptions can lead to narrower bounds on Γ, that covariates can be used to tighten the bounds as well, and that the distribution of covariates among always-assigned and potentially-assigned units is point identified in our model.

C.1. **Quantile Treatment Effects.** It is straightforward to generalize our identification analysis for average treatment effects to their quantile counterparts. For example, instead of the parameter Γ that we focus on in Section 3, one could consider the quantile treatment effect among compliers at the cutoff, formally defined as

$$\Psi(u) \equiv Q_{Y(1)|X=c^-,D^+>D^-}(u) - Q_{Y(0)|X=c^-,D^+>D^-}(u).$$

This extension is straightforward because our general strategy in Section 3 is to first obtain sharp lower and upper bounds, in a first-order stochastic dominance sense, on the c.d.f.s $F_{Y(d)|X=c,C_0}$ for $d \in \{0,1\}$. Once these have been obtained, it follows from Stoye (2010, Lemma 1) that sharp upper and lower bounds on on any functional of the form $\theta(F_{Y(d)|X=c,C_0})$ are given, respectively, by $\theta(F_d^U(y))$ and $\theta(F_d^L(y))$ as long as $\theta(\cdot)$ increases with first-order stochastic dominance. Quantiles are easily seen to fall into this class. To define the resulting

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bounds on $\Psi(u)$ in the general context of Theorem 2, for example, let

$$\Psi^U_{FRD}(u,t_1,t_0) = Q^U_{1,FRD}(u,\tau_1,\tau_0) - Q^L_{0,FRD}(u,\tau_1,\tau_0) \quad \text{and}$$
$$\Psi^L_{FRD}(u,t_1,t_0) = Q^L_{1,FRD}(u,\tau_1,\tau_0) - Q^U_{0,FRD}(u,\tau_1,\tau_0),$$

where $Q^U_{1,FRD}(\cdot,\tau_1,\tau_0)$ is the inverse of $F^U_{1,FRD}(\cdot,\tau_1,\tau_0)$, and the other terms in the previous equation are defined similarly. Then sharp lower and upper bounds on $\Psi(u)$ are given by

$$\Psi^L_{FRD}(u) = \inf_{(t_1,t_0)\in\mathcal{T}} \Psi^L_{FRD}(u,t_1,t_0) \quad \text{and} \quad \Psi^U_{FRD}(u) = \sup_{(t_1,t_0)\in\mathcal{T}} \Psi^U_{FRD}(u,t_1,t_0),$$

respectively. Bounds on quantile treatment effects in sharp designs, or under the various refinements studied in the remainder of this section, can be obtained analogously.

C.2. **Non-Continuously Distributed Outcomes.** Theorem 1 and 2 are stated for the case in which the outcome variable is continuously distributed. This is for notational convenience only, and our results immediately generalize to the case of a discrete outcome variable, which occurs frequently in empirical applications. Suppose that $\text{supp}(Y)$ is a finite set. Then in the case of a Sharp RD design our sharp upper and lower bounds on $F_{Y(1)|X=c,C_0}$ are

$$F^U_{1,SRD}(y) = (1-\theta^U)F_{Y|X=c^+,Y>Q_{Y|X=c^+}(\tau)}(y) + \theta^U \mathbb{I}\left\{y \geq Q_{Y|X=c^+}(\tau)\right\} \quad \text{and}$$
$$F^L_{1,SRD}(y) = (1-\theta^L)F_{Y|X=c^+,Y<Q_{Y|X=c^+}(1-\tau)}(y) + \theta^L \mathbb{I}\left\{y \geq Q_{Y|X=c^+}(1-\tau)\right\},$$

where

$$\theta^L = \frac{\mathrm{P}(Y \geq Q_{Y|X=c^+}(1-\tau)|X=c^+) - \tau}{1-\tau} \quad \text{and} \quad \theta^U = \frac{\mathrm{P}(Y \leq Q_{Y|X=c^+}(\tau)|X=c^+) - \tau}{1-\tau}.$$

The following Corollary uses these bounds to obtain explicit sharp bounds on the local average treatment effect Γ.

Corollary 1. *Suppose that the assumptions of Theorem 1 hold, and that supp(Y) is a finite set. Then sharp lower and upper bounds on Γ are given by*

$$\begin{aligned}
\Gamma^L_{SRD} &= (1-\theta^L)\mathrm{E}(Y|X=c^+, Y < Q_{Y|X}(1-\tau|c^+)) + \theta^L Q_{Y|X}(1-\tau|c^+) \\
&\quad - \mathrm{E}(Y|X=c^-) \quad \text{and} \\
\Gamma^U_{SRD} &= (1-\theta^U)\mathrm{E}(Y|X=c^+, Y > Q_{Y|X}(\tau|c^+)) + \theta^U Q_{Y|X}(\tau|c^+) \\
&\quad - \mathrm{E}(Y|X=c^-),
\end{aligned}$$

respectively.

In a Fuzzy RD design, we modify the expressions for the sharp upper and lower bounds on $F_{Y(1)|X=c,C_0}$ and $F_{Y(0)|X=c,N_0}$ for known values of τ_1 and τ_0 as follows:

$$F^U_{1,FRD}(y,\tau_1,\tau_0) = (1-\theta^U_1) G_{Y|Y>Q_G\left(\frac{\tau_1}{1-\kappa_1}\right)}(y) + \theta^U_1 \mathbb{I}\left\{y \geq Q_G\left(\frac{\tau_1}{1-\kappa_1}\right)\right\} \quad \text{and}$$
$$F^L_{1,FRD}(y,\tau_1,\tau_0) = (1-\theta^L_1) G_{Y|Y<Q_G\left(1-\frac{\tau_1}{1-\kappa_1}\right)}(y) + \theta^L_1 \mathbb{I}\left\{y \geq Q_G\left(1-\frac{\tau_1}{1-\kappa_1}\right)\right\},$$

where

$$\theta^U_1 = \frac{\mathrm{P}_G\left(Y \leq Q_G\left(\frac{\tau_1}{1-\kappa_1}\right)\right) - \frac{\tau_1}{1-\kappa_1}}{1-\frac{\tau_1}{1-\kappa_1}} \quad \theta^L_1 = \frac{\mathrm{P}_G\left(Y \geq Q_G\left(1-\frac{\tau_1}{1-\kappa_1}\right)\right) - \frac{\tau_1}{1-\kappa_1}}{1-\frac{\tau_1}{1-\kappa_1}}.$$

The modified expressions for bounds on $F_{Y(0)|X=c,N_0}$ are given by

$$F^U_{Y(0)|X=c,N_0}(y) = \int_{-\infty}^{y} s(t,\tau_0)\mathbb{I}\left\{t \leq q_U(\tau_0)\right\} dt + \theta^U_0 \mathbb{I}\left\{y > q_U(\tau_0)\right\} \quad \text{and}$$
$$F^L_{Y(0)|X=c,N_0}(y) = \int_{-\infty}^{y} s(t,\tau_0)\mathbb{I}\left\{t \geq q_L(\tau_0)\right\} dt + \theta^L_0 \mathbb{I}\left\{y > q_L(\tau_0)\right\},$$

where

$$\theta^U_0 = 1 - \int_{-\infty}^{q_U(\tau_0)} s(t,\tau_0)\mathbb{I}\left\{t \leq q_U(\tau_0)\right\} dt,$$
$$\theta^L_0 = 1 - \int_{q_L(\tau_0)}^{\infty} s(t,\tau_0)\mathbb{I}\left\{t \geq q_L(\tau_0)\right\} dt,$$
$$q_L(\tau_0) = \inf\{y \in \mathrm{supp}(Y) : \int_{y}^{\infty} s(t,\tau_0)dt \leq 1\}, \text{ and}$$
$$q_U(\tau_0) = \sup\{y \in \mathrm{supp}(Y) : \int_{-\infty}^{y} s(t,\tau_0)dt \leq 1\}.$$

We then obtain the following expressions for sharp bounds on the local average treatment effect Γ given knowledge of τ_1 and τ_0:

$$\Gamma^U_{FRD}(\tau_1,\tau_0) \equiv \int y dF^U_{1,FRD}(y,\tau_1,\tau_0) - \int y dF^L_{0,FRD}(y,\tau_1,\tau_0),$$
$$\Gamma^U_{FRD}(\tau_1,\tau_0) \equiv \int y dF^L_{1,FRD}(y,\tau_1,\tau_0) - \int y dF^U_{0,FRD}(y,\tau_1,\tau_0).$$

The following Corollary finally states the sharp bounds on Γ given that the values of τ_1 and τ_0 are only partially identified.

Corollary 2. *Suppose that the assumptions of Theorem 2 hold, and that supp(Y) is a finite set. Then sharp lower and upper bounds on Γ are given by*

$$\Gamma^L_{FRD} = \inf_{(t_1,t_0)\in\mathcal{T}} \Gamma^L_{FRD}(t_1,t_0) \quad \text{and} \quad \Gamma^U_{FRD} = \sup_{(t_1,t_0)\in\mathcal{T}} \Gamma^U_{FRD}(t_1,t_0),$$

respectively.

C.3. **Adding Behavioral Assumptions in Fuzzy RD Designs.** The bounds in Theorem 2 can be narrowed by imposing stronger assumptions on the units' behavior, which relate to behavioral restrictions that arise naturally in certain empirical contexts. Consider for instance settings where always-assigned units obtain values of the running variable to the right of the cutoff by taking conscious actions. Since such units actively choose to be eligible for the treatment, it seems plausible to assume that their probability of actually receiving the treatment conditional on being eligible is relatively high in some appropriate sense.

First, one might be willing to assume that always-assigned units are at least as likely to get treated as eligible potentially-assigned units, implying the following corollary:

Corollary 3. *Suppose that the conditions of Theorem 2 hold, and that* $\mathrm{E}(D|X=c^+, M=1) \geq \mathrm{E}(D|X=c^+, M=0)$. *Then sharp lower and upper bounds on* Γ *are given by*

$$\Gamma^L_{FRD(a)} = \inf_{(t_1,t_0)\in\mathcal{T}_a} \Gamma^L_{FRD}(t_1,t_0) \quad \text{and} \quad \Gamma^U_{FRD(a)} = \sup_{(t_1,t_0)\in\mathcal{T}_a} \Gamma^U_{FRD}(t_1,t_0),$$

respectively, where $\mathcal{T}_a \equiv \{(t_1,t_0) : (t_1,t_0) \in \mathcal{T} \text{ and } t_1 \geq \tau\}$.

We see that the additional restriction of Corollary 3 relative to Theorem 2 increases the lowest possible value of τ_1 from $\max\{0, 1+(\tau-1)/\mathrm{E}(D|X=c^+)\}$ to τ, and correspondingly decreases the largest possible value for τ_0 from $\min\{1, \tau/(1-\mathrm{E}(D|X=c^+))\}$ to τ. This follows from a simple application of Bayes' Rule, and means that $\mathcal{T}_a \subset \mathcal{T}$. We then obtain bounds on Γ that are (weakly) narrower, as optimization is carried out over a smaller set.

Second, in some cases, it may be reasonable to drive this line of reasoning further and assume that always-assigned units *always* receive the treatment. This implies the following corollary:

Corollary 4. *Suppose that the conditions of Theorem 2 hold, and that* $\mathrm{E}(D|X=c^+, M=1) = 1$. *Then* $\tau_1 = \tau/\mathrm{E}(D|X=c^+)$ *and* $\tau_0 = 0$ *are point identified; and sharp lower and upper bounds on* Γ *are given by*

$$\Gamma^L_{FRD(b)} = \Gamma^L_{FRD}\left(\frac{\tau}{\mathrm{E}(D|X=c^+)}, 0\right) \quad \text{and} \quad \Gamma^U_{FRD(b)} = \Gamma^U_{FRD}\left(\frac{\tau}{\mathrm{E}(D|X=c^+)}, 0\right),$$

respectively.

Under the conditions of Corollary 4, the set of feasible values of (τ_1, τ_0) shrinks to a singleton, which means that sharp bounds on our parameter of interest can be defined without invoking an optimization operator. Moreover, we can see from Table 1 that due to the absence of always-assigned untreated units the distributions $F_{Y(0)|X=c,N_0}$ and $F_{Y(0)|X=c,C_0}$ are point identified in this case.

C.4. **Using Covariates to Tighten the Bounds.** Following arguments similar to those in Lee (2009), covariates that are measured prior to treatment assignment can also be used to narrow the bounds on Γ that we derived above. Let W be a vector of such covariates, and denote its support by $\mathcal{W}$. The idea is that, if the outcome distribution or the proportion of always-assigned units varies with W, trimming units based on their position in the outcome distribution conditional on W leads to units with less extreme values in the overall outcome distribution being trimmed, which narrows the bounds.

For the sharp RD design, the sharp upper and lower bounds on $F_{Y(1)|X=c,C_0}$ become:

$$F^U_{1,SRD(W)}(y) = \int F_{Y|X=c^+,W=w,Y\geq Q_{Y|X=c^+,W=w}(\tau(w))}(y)dF_{W|X=c^-}(w) \quad \text{and}$$
$$F^L_{1,SRD(W)}(y) = \int F_{Y|X=c^+,W=w,Y\leq Q_{Y|X=c^+,W=w}(1-\tau(w))}(y)dF_{W|X=c^-}(w),$$

where $\tau(w) = \mathrm{P}(M = 1|X = c^+, W = w)$ is a conditional version of τ defined as in (3.1), which is point identified as $\tau(w) = 1 - f_{X|W}(c^-, w)/f_{X|W}(c^+, w)$ through arguments analogous to those used in the proof of Lemma 1, conditioning on $W = w$ throughout. The next corollary gives the resulting sharp lower and upper bounds on Γ.

Corollary 5. *Suppose that the assumptions of Theorem 1 hold, mutatis mutandis, with conditioning on the covariates W. Then sharp lower and upper bounds on Γ are given by*

$$\begin{aligned}
\Gamma^L_{SRD(W)} &= \int \mathrm{E}(Y|X = c^+, W = w, Y \leq Q_{Y|X=c^+,W=w}(1 - \tau(w)))dF_{W|X=c^-}(w) \\
&\quad - \mathrm{E}(Y|X = c^-) \quad \text{and} \\
\Gamma^U_{SRD(W)} &= \int \mathrm{E}(Y|X = c^+, W = w, Y_i \geq Q_{Y|X=c^+,W=w}(\tau(w)))dF_{W|X=c^-}(w) \\
&\quad - \mathrm{E}(Y|X = c^-),
\end{aligned}$$

respectively.

To state a similar result for the fuzzy RD design, we need to define conditional versions of τ_1, τ_0, $\mathcal{T}$, κ_1 and κ_0 in the same fashion. We denote the resulting quantities by $\tau_1(w)$, $\tau_0(w)$, $\mathcal{T}(w)$, $\kappa_1(w)$ and $\kappa_0(w)$, respectively. We then define conditional versions of $F^U_{d,FRD}(y, \tau_1, \tau_0)$ and $F^L_{d,FRD}(y, \tau_1, \tau_0)$, denoted by $F^U_{d,FRD|W=w}(y, \tau_1(w), \tau_0(w))$ and $F^L_{d,FRD|W=w}(y, \tau_1(w), \tau_0(w))$, respectively, for $d \in \{0, 1\}$. These objects are constructed following the steps in the previous section by conditioning on $W = w$ throughout. We also define the set $\mathcal{T}_{\mathcal{W}} = \{(t_1(\cdot), t_1(\cdot)) : (t_1(w), t_1(w)) \in \mathcal{T}(w) \text{ for all } w \in \mathcal{W}\}$. Finally, we denote the proportion of potentially-assigned compliers (C_0) conditional on $W = w$ just to the left of the cutoff by

$$\begin{aligned}\mathrm{P}(C_0|X = c^-, W = w) &= \frac{1-\tau_1(w)}{1-\tau(w)}\mathrm{E}(D|X = c^+, W = w) - \mathrm{E}(D|X = c^-, W = w)\\ &\equiv \Pi_{W=w}(\tau_1(w), \tau_0(w)).\end{aligned}$$

With this notation, we can then construct sharp upper and lower bounds on $F_{Y(1)|X=c,C_0}$ and $F_{Y(0)|X=c,C_0}$ given (hypothetical) knowledge of the function $w \mapsto (\tau_1(w), \tau_0(w))$. These bounds are given by

$$\begin{aligned}F^U_{d,FRD(W)}(y, \tau_1(\cdot), \tau_0(\cdot)) &= \int F^U_{d,FRD|W=w}(y, \tau_1(w), \tau_0(w))\omega(w, \tau_1(w), \tau_0(w))dF_{W|X=c^-}(w)\\ F^L_{d,FRD(W)}(y, \tau_1(\cdot), \tau_0(\cdot)) &= \int F^L_{d,FRD|W=w}(y, \tau_1(w), \tau_0(w))\omega(w, \tau_1(w), \tau_0(w))dF_{W|X=c^-}(w),\end{aligned}$$

for $d \in \{0, 1\}$, where

$$\omega(w, \tau_1(w), \tau_0(w)) \equiv \frac{\Pi_{W=w}(\tau_1(w), \tau_0(w))}{\int \Pi_{W=w}(\tau_1(w), \tau_0(w))dF_{W|X=c^-}(w)}.$$

The resulting sharp upper and lower bounds on the local average treatment effect Γ given (hypothetical) knowledge of the function $w \mapsto (\tau_1(w), \tau_0(w))$ are given by

$$\begin{aligned}&\Gamma^U_{FRD(W)}(\tau_1(\cdot), \tau_0(\cdot))\\ &\quad\equiv \int y dF^U_{1,FRD(W)}(y, \tau_1(\cdot), \tau_0(\cdot)) - \int y dF^L_{0,FRD(W)}(y, \tau_1(\cdot), \tau_0(\cdot)) \quad \text{and}\\ &\Gamma^L_{FRD(W)}(\tau_1(\cdot), \tau_0(\cdot))\\ &\quad\equiv \int y dF^L_{1,FRD(W)}(y, \tau_1(\cdot), \tau_0(\cdot)) - \int y dF^U_{0,FRD(W)}(y, \tau_1(\cdot), \tau_0(\cdot)),\end{aligned}$$

respectively. The following corollary gives the feasible sharp bounds on Γ, using the fact that the function $w \mapsto (\tau_1(w), \tau_0(w))$ is partially identified.

Corollary 6. *Suppose that the assumptions of Theorem 2 hold, mutatis mutandis, with conditioning on the covariates W. Then sharp lower and upper bounds on Γ are given by*

$$\begin{aligned}\Gamma^L_{FRD(W)} &= \inf_{(t_1(\cdot),t_0(\cdot))\in\mathcal{T}_{\mathcal{W}}} \Gamma^L_{FRD}(t_1(\cdot), t_0(\cdot)) \quad \text{and}\\ \Gamma^U_{FRD(W)} &= \sup_{(t_1(\cdot),t_0(\cdot))\in\mathcal{T}_{\mathcal{W}}} \Gamma^U_{FRD}(t_1(\cdot), t_0(\cdot)),\end{aligned}$$

respectively.

C.5. **Characteristics of Always- and Potentially-Assigned Units.** It is not possible to determine whether any given unit belongs to the group of always-assigned or potentially-assigned units in our model. This does not mean, however, that it is impossible to give any further characterization of these two groups. In particular, if the data include a vector W of covariates that are measured prior to treatment assignment, and whose conditional distribution given the running variable does not change discontinuously at c among potentially-assigned units, one can identify the distribution of these covariates among both always-assigned and potentially-assigned units. This information could be useful, for instance, for targeting policies aimed at mitigating manipulation. The following corollary formally states this result.

Corollary 7. *Suppose that Assumptions 1–2 hold, that* $\mathrm{P}(W \leq w|X = x, M = m)$ *is continuous in* x *at* c *for* $m \in \{0, 1\}$*. Then*

$$\begin{aligned}\mathrm{P}(W \leq w|X = c, M = 1) &= \frac{1}{\tau}(\mathrm{P}(W \leq w|X = c^+) - \mathrm{P}(W \leq w|X = c^-)) \\ &\quad + \mathrm{P}(W \leq w|X = c^-) \quad \text{and} \\ \mathrm{P}(W \leq w|X = c, M = 0) &= \mathrm{P}(W \leq w|X = c^-).\end{aligned}$$

Of course, identification of the distribution of W immediately implies identification of moments, quantiles, and related summary statistics.

D. Applicability of our Model

Our model, developed in Section 2 of the main paper, is able to capture a wide range of empirical scenarios of manipulation by appropriately assigning the labels of always-assigned and potentially-assigned to specific groups of units. To illustrate this point, consider a transfer program for which eligibility is based on a cutoff value of a poverty score, and the formula that creates the score takes as inputs household characteristics and assets recorded during home visits by local administrators. There might also be other criteria that make a household (in-)eligible irrespective of the poverty score, so that the resulting RD design could in principle be fuzzy. These types of programs are common in developing countries, and various types of manipulation have been documented for them (e.g., Camacho and Conover, 2011).

The following examples illustrate how various empirical scenarios are accommodated by our model. They also show why it may be necessary to allow always-assigned units to be treated or untreated in some settings, while in others it can be reasonable to assume that all of them are treated. One can easily construct further variants of these examples that also fit into our model, and these examples also have natural analogues in other contexts. For instance, Example 3 and Example 4 below are similar to the two manipulation scenarios for the financial aid example in the introduction, respectively.

Example 1 ("Unsystematic" Misreporting)**.** There might be concerns of manipulation whenever a running variable can be affected by some agents' behaviors. Running variables are commonly endogenous, misreported, or mismeasured in the empirical literature, and this may certainly affect the composition of the units observed around the cutoff. However, it is not sufficient to create a manipulated running variable in the sense used in this paper. Suppose for example that the formula for the poverty score is not publicly known. Then, even if households might misreport or genuinely modify their input variables (within reasonable bounds), they may not be able to ensure program assignment. All households are potentially-assigned in this case; households just above and below the cutoff are still comparable; and a standard RD analysis could estimate causal parameters for those households with realized poverty scores at the cutoff. This is a special case of our model in which always-assigned units are absent.

Example 2 ("Systematic" Misreporting)**.** Suppose that some households know the poverty score formula, and local administrators are unwilling or unable to recognize whether a household reports inaccurate information as long as it is within reasonable bounds. Some households with knowledge of the formula, and whose poverty score would otherwise fall to the left of the cutoff, may then be able to misreport their inputs such that their score is to the right of the cutoff. The assumption of one-sided manipulation is likely to hold, e.g., if program assignment is weakly desirable for all households (they can always refuse to participate). They might also have an incentive to report data that put them barely above the cutoff but not exactly at the cutoff, e.g., in order to avoid detection. This makes the assumption of a continuously distributed running variable among always-assigned units palatable. If these misreporting households are systematically different from the other households with poverty scores in the vicinity of the cutoff, the distribution of potential outcomes may be discontinuous at the cutoff, and conventional RD analysis is invalid. In our model, the households with knowledge of the formula that would be willing to misreport their inputs in order to avoid having a poverty score below the cutoff are always-assigned: all of them will have a score above the cutoff, either by misreporting inputs if their score would otherwise fall to the left of the cutoff, or by scoring above the cutoff even without misreporting (they would have misreported inputs if needed, but it is simply not needed in that case). In that sense, they are "always assigned". All other households are potentially-assigned. Given that always-assigned households are willing to actively violate the rules of the program to ensure that their poverty score is above the cutoff, it may be reasonable to assume that all of them end up treated.

Example 3 ("Systematic" Misreporting with Partial Verification Checks)**.** Suppose that the same households as above misreport their data to try to ensure program assignment, but that some local administrators now thoroughly verify the information provided to them. As a result, only a fraction of the households is able to carry out its intended misreporting. Those households with knowledge of the formula that would be willing to misreport their inputs in order to avoid having a poverty score below the cutoff, and that would be successful in doing so, are now the always-assigned units in our setup. The households with knowledge of the formula that would be willing to misreport their inputs in order to avoid having a poverty score below the cutoff, but that would be unsuccessful in doing so, are classified as potentially-assigned along with all other households, provided that local administrators simply enter the correct information if they detect misreporting. Indeed, this type of households – that would unsuccessfully misreport their information if their correct score fell to the left of the cutoff – also exists on the right of the cutoff; they just did not need to try to misreport any data given that they were already on the right of the cutoff. Suppose instead that local administrators apply a penalty by removing households from the data if they detect misreporting. In that case, the same type of households will not be observed on the left of the cutoff anymore; it will only exist on the right of the cutoff and will thus be classified as always-assigned. In both cases, it seems reasonable to assume again that all always-assigned units are treated.

Example 4 ("Systematic" Misreporting by Administrators)**.** Suppose that all households report their information truthfully, but that local administrators sometimes misreport the information that they receive. This may lead to a manipulated running variable even though the observational units, i.e., the households, do not engage in any manipulation themselves. For instance, local administrators may increase the score of households who support the local government to ensure their program assignment in case their score would otherwise fall to the left of the cutoff. Conventional RD analysis is invalid in this case too, e.g., if a household's political leanings correlates with the effect of program participation. Our general model also likely applies. Manipulation is likely to be one-sided and local administrators are unlikely to misreport information such that the modified scores are equal to the cutoff (e.g., to avoid detection by central administrators). Households who support the local government are then always-assigned, and all others are potentially-assigned. Note that some always-assigned households might now refuse to participate in the program (e.g., if it comes with social stigma), or might have qualified even with a lower poverty score. Alternatively, suppose that local administrators also decrease the score of political opponents whose score would otherwise fall to the right of the cutoff. This would be a situation in which our model does not apply because of two-sided manipulation.

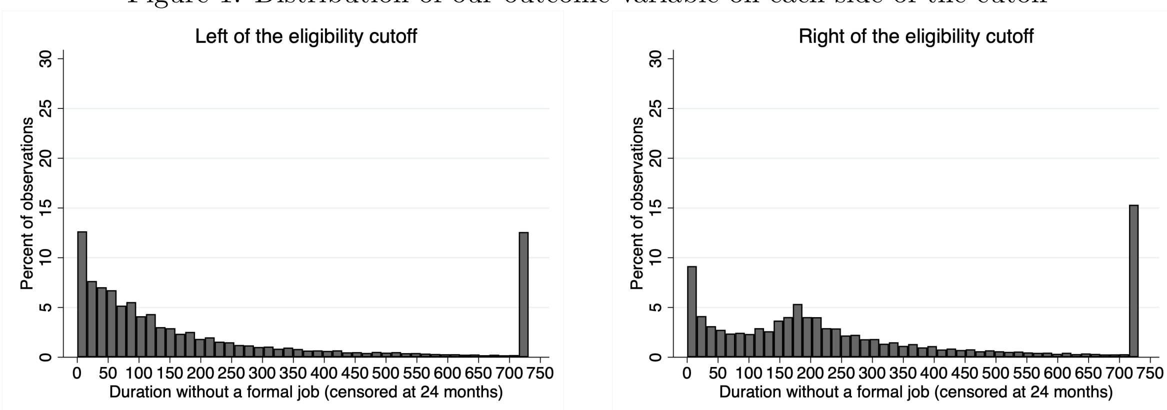
Example 5 (Manipulation through Location Selection)**.** Manipulation of the running variable does not require that any agent engages in some form of wrongdoing. Suppose that there is no misreporting whatsoever, but that the program only exists in some localities. Households in other localities may then choose to move to become eligible for the program. If the formula is known, the probability of moving may increase discontinuously for households whose poverty score would fall above the cutoff conditional on living in an eligible locality. As a result, the density of the poverty score in eligible localities may be discontinuous at the cutoff and, to the extent that the potential outcomes of movers differ from those of incumbent residents observed around the cutoff, a conventional RD analysis may be invalid. Moreover, the assumptions of one-sided manipulation and of a continuously distributed running variable among always-assigned units are reasonable if the program is weakly desirable. Those households who move because they know that they are eligible for the program at destination are then the always-assigned units in our model (they are responsible for the discontinuity in the moving probability) and they are all likely to be treated in this setting.

Example 6 (Second Home Visit)**.** Finally, here is another example in which manipulation of the running variable does not require that any agent engages in some form of wrongdoing. Suppose that households' information is measured with some error in any given home visit, and that households can request a second home visit after learning the value of their score by arguing that their information was mismeasured in the first visit. Additionally, only the score based on the most recent visit, which determines program eligibility, is observed by the econometrician. Let X_{ji} be the poverty score for household i based on visit j, which is assumed to be smoothly distributed at the cutoff, and suppose that households request a second visit if and only if they were ineligible based on the first visit. The observed poverty score is then: $X_i = X_{1i} \cdot \mathbb{I}\left(X_{1i} \geq c\right) + X_{2i} \cdot \mathbb{I}\left(X_{1i} < c\right)$. Its density is discontinuous at the cutoff as long as error terms are imperfectly correlated across visits. The excess density is due to households whose score fell on the right side of the cutoff in the first visit; those are the always-assigned units in our model. Moreover, to the extent that their potential outcomes differ from those of households observed on the left of the cutoff (whose poverty score fell on the left in both visits), a conventional RD analysis is invalid. In contrast, if feasible, a RD analysis based on X_{1i} or $X_{2i}|X_{1i} < c$ could be valid in this setting. Depending on the details of the program, this is also a case in which it may be reasonable to allow always-assigned households to be treated or untreated.

E. Additional Tables and Graphs for the Empirical Application

We present here some supporting graphs for our empirical application. Figure 1 displays the distribution of our outcome variable (duration without a formal job, censored at two years after layoff) on the left and on the right of the cutoff (30-day window around the cutoff). Figure 2 displays the full schedule for the statutory UI benefit level, which is a function of a worker's average monthly wage in the three years prior to her layoff. Figure 3 displays the mean of different covariates on each side of the cutoff by day between the layoff and eligibility dates. Finally, Table 1 displays estimates of the change in the average value of these covariates at the cutoff, including the statutory UI replacement rate, and of the average value of these covariates among potentially-assigned and always-assigned units, based on the results in Corollary 7.

Figure 1: Distribution of our outcome variable on each side of the cutoff

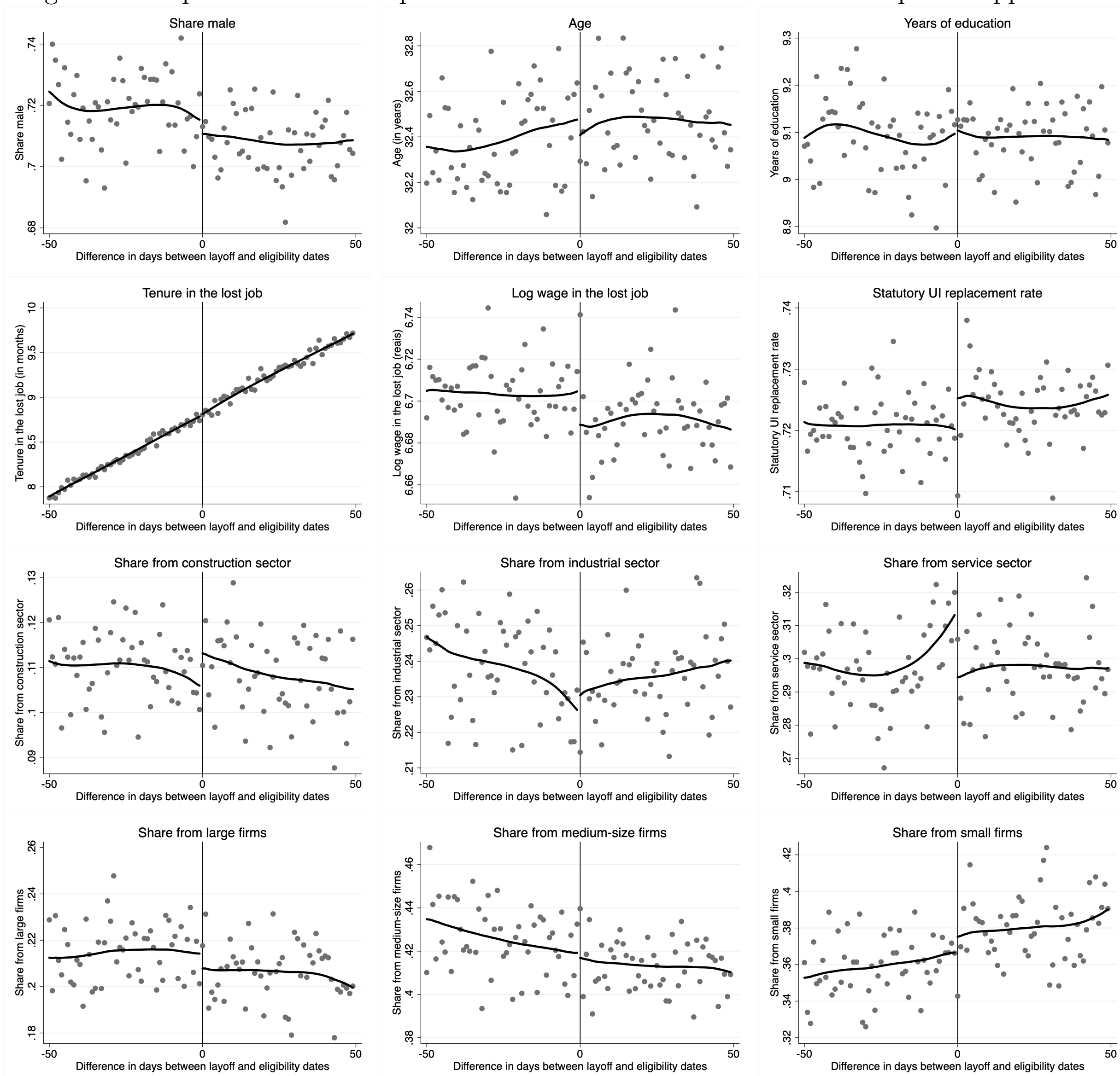


Notes: The figure displays the distribution of our outcome variable (duration without a formal job, censored at two years after layoff) on the left and on the right of the cutoff (30-day window on each side of the cutoff). The figure is based on a sample of 102,791 displaced formal workers whose layoff date fell within 30 days of the eligibility date.

Figure 2: Monthly UI benefit amount

The figure displays the relationship between a worker's average monthly wage in the three months prior to her layoff and her monthly statutory UI benefit level. All monetary values are indexed to the federal minimum wage, which changes every year. The replacement rate is 100% at the bottom of the wage distribution as the minimum benefit level is equal to one minimum wage. The graph displays a slope of 0% until 125% of the minimum wage, then of 80% until 165% of the minimum wage, and finally of 50% until 275% of the minimum wage. The maximum benefit level is equal to 187% of the minimum wage.

Figure 3: Graphical evidence of potential selection at the cutoff for our empirical application



Notes: The figure displays the mean of different covariates on each side of the cutoff by day between the layoff and eligibility dates, as well as local linear regressions on each side of the cutoff using an edge kernel and a bandwidth of 30 days. The figure is based on a sample of 169,575 displaced formal workers whose layoff date fell within 50 days of the eligibility date.

Table 1: Characteristics of always- and potentially-assigned workers

	Difference at the cutoff	Potentially-assigned	Always-assigned
Share male	-0.0031	0.714	0.665
	[-0.0163;0.0100]	[0.704;0.724]	[0.439;0.891]
Average age (in years)	-0.0729	32.475	31.330
	[-0.2842;0.1384]	[32.311;32.638]	[27.833;34.828]
Average years of education	0.0011	9.104	9.121
	[-0.0803;0.0825]	[9.047;9.162]	[7.770;10.472]
Average tenure (in months)	0.0103	8.802	8.963
	[-0.0389;0.0594]	[8.770;8.834]	[8.124;9.802]
Average log wage (R$2010)	-0.0160	6.704	6.453
	[-0.0312;-0.0009]	[6.693;6.716]	[6.211;6.695]
Average statutory UI replacement rate	0.0051	0.720	0.801
	[0.0004;0.0099]	[0.717;0.724]	[0.724;0.878]
Share from commercial sector	0.0071	0.355	0.466
	[-0.0067;0.0209]	[0.345;0.366]	[0.246;0.687]
Share from construction sector	0.0073	0.106	0.220
	[-0.0015;0.0160]	[0.099;0.112]	[0.077;0.363]
Share from industrial sector	0.0061	0.225	0.320
	[-0.0059;0.0181]	[0.216;0.234]	[0.124;0.516]
Share from service sector	-0.0204	0.314	-0.006
	[-0.0331;-0.0077]	[0.305;0.324]	[-0.215;0.202]
Share from small firm	0.0083	0.367	0.498
(<10 employees)	[-0.0050;0.0217]	[0.357;0.377]	[0.273;0.723]

Notes: Total number of observations within our bandwidth of 30 days around the cutoff: 102,791 displaced formal workers. Numbers in square brackets are 95% confidence intervals calculated by adding $\pm 1.96\times$standard error to the respective point estimate, where standard errors are calculated via the bootstrap with 500 replications. The characteristics of potentially-assigned and always-assigned units are obtained using the results in Corollary 7.

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