Additional Materials for "Simple and Honest Confidence Intervals in Nonparametric Regression"

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Appendix F Additional Monte Carlo results

In this appendix, we revisit the simulation study from Section 5 in the paper, and consider an additional method for constructing CIs, as well as a number of variations on the DGP.

In particular, we also consider a conventional CI based on the coverage-error optimal bandwidth $\hat{h}_{\text{CE}}$, which can be considered a form of undersmoothing, but without any bias correction. Table S1 reports the results for Designs 1–3 with this additional methods added. Using the bandwidth $\hat{h}_{\text{CE}}$ leads to better coverage of conventional CIs relative to $\hat{h}^*_{\text{PT,ROT}}$ when $M = 2$, but worse coverage when $M = 6$.

Next, we investigate the robustness of the results to a number of variations on the baseline design. Table S2 reports the results when x_i is drawn from a Beta$(2, 5)$ distribution. In Table S3, to consider the effects of heteroskedasticity, we draw the errors form the distribution $\mathcal{N}(0, 1/4(1 + \sqrt{|x_i|})^2)$, while x_i is drawn from a uniform distribution, as in the baseline. In Table S4, $x_i \sim$ Beta$(2, 5)$ distribution, and $u_i \sim \mathcal{N}(0, 1/4(1 + \sqrt{|x_i|})^2)$. In Table S5, we draw u_i from a log-normal distribution, scaled to have mean zero and variance $1/4$, while x_i is drawn from a uniform distribution. Table S6 reports the results for u_i drawn from a log-normal distribution, scaled to have mean zero and variance $1/4$, and $x_i \sim$ Beta$(2, 5)$. Table S7 returns to the baseline specification, but with $u_i \sim \mathcal{N}(0, 1/16)$. Finally, in Table S8 we consider a smooth approximation to the functions f_1, f_2, and f_3. In particular, we replace the function $\mathsf{s}(\cdot)$ in the definition of these functions by the function $s_\lambda(x) = -\operatorname{Li}_2(-e^{\lambda x})/\lambda^2$, where $\operatorname{Li}_2(x) =$

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$-\int_0^x \frac{\log(1-s)}{s} ds$ is the dilogarithm function. The function s_λ is analytic for any λ, and it converges to s as $\lambda \to \infty$. We set $\lambda = 40$.

The results in Table S8 are nearly identical to those in Table S1, indicating that the lack of differentiability is not driving the results. The FLCIs perform well for all designs in terms of coverage when the correct or conservative M is used, or when one uses $\hat{M}_{\text{ROT}}$. The coverage is at least 92.5% in all designs except Table S5, where the coverage, where the FLCIs undercover slightly for Design 3, with coverage around 90%. The RBC CIs with bandwidth chosen based on uniform-in-f asymptotics (either $\hat{h}^*_{\text{RMSE},2}$, $\hat{h}^*_{\text{RMSE},6}$, or $\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$) also perform well in terms of coverage, with coverage at least 93% for all designs, although they are longer than FLCI CIs. The remaining CIs, based on pointwise-in-f asymptotics, suffer from poor coverage in these alternative specifications, just like in the baseline specification in the main text.

References

Calonico, S., Cattaneo, M. D., and Farrell, M. H. (2018). On the effect of bias estimation on coverage accuracy in nonparametric inference. *Journal of the American Statistical Association*, 113(522):767–779.

Fan, J. and Gijbels, I. (1996). *Local Polynomial Modelling and Its Applications*. Monographs on Statistics and Applied Probability. Chapman & Hall/CRC, New York, NY.

Table S1: Monte Carlo simulation: baseline DGP

		$M=2$					$M=6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 1											
RBC	$h=\hat{h}^*_{\text{PT}},\ b=\hat{b}^*_{\text{PT}}$	0.063	0.035	0.75	55.6	0.73	0.157	0.036	0.62	0.1	0.61
RBC	$h=b=\hat{h}^*_{\text{PT}}$	0.025	0.042	0.75	93.1	0.88	0.042	0.047	0.62	89.1	0.78
RBC	$h=\hat{h}_{\text{CE}},\ b=\hat{b}_{\text{CE}}$	0.030	0.041	0.45	85.8	0.85	0.059	0.045	0.34	72.4	0.76
RBC	$h=b=\hat{h}^*_{\text{RMSE},2}$	0.001	0.061	0.36	94.5	1.27	0.002	0.061	0.36	94.5	1.01
RBC	$h=b=\hat{h}^*_{\text{RMSE},6}$	0.000	0.076	0.23	94.2	1.58	0.000	0.075	0.23	94.2	1.26
RBC	$h=b=\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.000	0.078	0.22	93.9	1.64	0.000	0.097	0.14	93.4	1.63
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	0.032	0.036	0.56	76.6	0.76	0.049	0.046	0.31	77.4	0.77
Conventional	$\hat{h}_{\text{CE}}$	0.029	0.039	0.45	85.2	0.81	0.058	0.044	0.34	72.3	0.74
FLCI, $M=2$	$\hat{h}^*_{\text{RMSE},2}$	0.021	0.043	0.36	94.9	1.00	0.065	0.043	0.36	75.2	0.80
FLCI, $M=6$	$\hat{h}^*_{\text{RMSE},6}$	0.009	0.054	0.23	96.6	1.25	0.028	0.053	0.23	94.7	1.00
FLCI, $M=\hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.008	0.056	0.22	95.6	1.29	0.010	0.069	0.14	96.3	1.30
Design 2											
RBC	$h=\hat{h}^*_{\text{PT}},\ b=\hat{b}^*_{\text{PT}}$	0.043	0.035	0.77	75.9	0.72	0.129	0.035	0.77	4.6	0.58
RBC	$h=b=\hat{h}^*_{\text{PT}}$	0.026	0.041	0.77	90.9	0.87	0.077	0.042	0.77	53.0	0.70
RBC	$h=\hat{h}_{\text{CE}},\ b=\hat{b}_{\text{CE}}$	0.028	0.040	0.49	87.4	0.83	0.074	0.041	0.44	54.1	0.69
RBC	$h=b=\hat{h}^*_{\text{RMSE},2}$	0.002	0.061	0.36	94.5	1.27	0.006	0.061	0.36	94.4	1.01
RBC	$h=b=\hat{h}^*_{\text{RMSE},6}$	0.000	0.076	0.23	94.2	1.58	0.000	0.075	0.23	94.2	1.26
RBC	$h=b=\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.001	0.068	0.30	94.0	1.43	0.000	0.083	0.20	93.8	1.38
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	0.032	0.032	0.78	74.4	0.67	0.073	0.040	0.44	53.0	0.66
Conventional	$\hat{h}_{\text{CE}}$	0.028	0.037	0.49	85.9	0.78	0.076	0.039	0.44	50.1	0.66
FLCI, $M=2$	$\hat{h}^*_{\text{RMSE},2}$	0.020	0.043	0.36	95.1	1.00	0.061	0.043	0.36	78.1	0.80
FLCI, $M=6$	$\hat{h}^*_{\text{RMSE},6}$	0.009	0.054	0.23	96.6	1.25	0.028	0.053	0.23	94.7	1.00
FLCI, $M=\hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.013	0.048	0.30	94.3	1.13	0.020	0.059	0.20	94.3	1.10

Monte Carlo simulation: baseline DGP (continued)

Method	Bandwidth	$M=2$ Bias	SE	$E[h]$	Cov	RL	$M=6$ Bias	SE	$E_m[h]$	Cov	RL
Design 3											
RBC	$h=\hat{h}^*_{\text{PT}}$, $b=\hat{b}^*_{\text{PT}}$	-0.043	0.035	0.77	75.7	0.72	-0.123	0.035	0.74	9.9	0.59
RBC	$h=b=\hat{h}^*_{\text{PT}}$	-0.024	0.042	0.77	90.8	0.87	-0.066	0.043	0.74	60.3	0.71
RBC	$h=\hat{h}_{\text{CE}}$, $b=\hat{b}_{\text{CE}}$	-0.026	0.040	0.49	88.1	0.83	-0.063	0.043	0.43	64.2	0.71
RBC	$h=b=\hat{h}^*_{\text{RMSE},2}$	-0.002	0.061	0.36	94.5	1.27	-0.007	0.061	0.36	94.4	1.01
RBC	$h=b=\hat{h}^*_{\text{RMSE},6}$	0.000	0.076	0.23	94.2	1.58	0.000	0.075	0.23	94.2	1.26
RBC	$h=b=\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.000	0.074	0.25	94.2	1.54	0.000	0.092	0.16	93.6	1.54
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	-0.032	0.033	0.72	74.7	0.69	-0.065	0.042	0.39	62.0	0.70
Conventional	$\hat{h}_{\text{CE}}$	-0.028	0.037	0.49	85.7	0.78	-0.074	0.040	0.43	52.0	0.66
FLCI, $M=2$	$\hat{h}^*_{\text{RMSE},2}$	-0.020	0.043	0.36	95.0	1.00	-0.060	0.043	0.36	78.1	0.80
FLCI, $M=6$	$\hat{h}^*_{\text{RMSE},6}$	-0.009	0.054	0.23	96.5	1.25	-0.027	0.053	0.23	94.7	1.00
FLCI, $M=\hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	-0.010	0.052	0.25	95.6	1.22	-0.013	0.065	0.16	96.1	1.22

Legend: SE—average standard error; $E[h]$—average (over Monte Carlo draws) bandwidth; Cov—coverage of CIs (in %); RL—relative (to optimal FLCI) length.

Bandwidth descriptions: $\hat{h}^*_{\text{PT}}$—plugin estimate of pointwise MSE optimal bandwidth (bw); $\hat{b}^*_{\text{PT}}$—analog for estimate of the bias; $\hat{h}_{\text{CE}}$—plugin estimate of coverage error optimal bw; $\hat{b}_{\text{CE}}$—analog for estimate of the bias; The implementation of Calonico et al. (2018) is used for all four bws. $\hat{h}^*_{\text{RMSE},2}$, $\hat{h}^*_{\text{RMSE},6}$—RMSE optimal bw, assuming $M=2$, and $M=6$, respectively. $\hat{h}^*_{\text{PT,ROT}}$—Fan and Gijbels (1996) rule of thumb; $\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$—RMSE optimal bw, using rule-of-thumb for M. 50,000 Monte Carlo draws.

Table S2: Monte Carlo simulation: beta distribution for x_i

		$M=2$					$M=6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 1											
RBC	$h=\hat{h}^*_{\mathrm{PT}}$, $b=\hat{b}^*_{\mathrm{PT}}$	0.030	0.037	0.56	85.6	0.83	0.056	0.041	0.43	64.8	0.74
RBC	$h=b=\hat{h}^*_{\mathrm{PT}}$	0.009	0.044	0.56	93.7	0.98	0.009	0.050	0.43	91.7	0.92
RBC	$h=\hat{h}_{\mathrm{CE}}$, $b=\hat{b}_{\mathrm{CE}}$	0.009	0.044	0.38	93.1	0.99	0.011	0.049	0.29	92.6	0.90
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},2}$	0.001	0.054	0.36	94.6	1.21	0.003	0.054	0.37	94.6	0.98
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},6}$	0.000	0.068	0.23	94.3	1.53	0.000	0.068	0.23	94.4	1.24
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.000	0.073	0.21	94.1	1.62	0.000	0.089	0.14	93.8	1.61
Conventional	$\hat{h}^*_{\mathrm{PT,ROT}}$	0.025	0.038	0.53	85.6	0.84	0.038	0.045	0.29	83.8	0.82
Conventional	$\hat{h}_{\mathrm{CE}}$	0.019	0.040	0.38	90.3	0.90	0.038	0.045	0.29	82.4	0.81
FLCI, $M=2$	$\hat{h}^*_{\mathrm{RMSE},2}$	0.019	0.041	0.36	94.7	1.00	0.058	0.041	0.37	77.0	0.81
FLCI, $M=6$	$\hat{h}^*_{\mathrm{RMSE},6}$	0.009	0.050	0.23	96.5	1.23	0.025	0.050	0.23	94.7	1.00
FLCI, $M=\hat{M}_{\mathrm{ROT}}$	$\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.007	0.053	0.21	96.1	1.31	0.009	0.064	0.14	96.3	1.29
Design 2											
RBC	$h=\hat{h}^*_{\mathrm{PT}}$, $b=\hat{b}^*_{\mathrm{PT}}$	0.027	0.037	0.57	88.0	0.82	0.073	0.038	0.53	49.0	0.68
RBC	$h=b=\hat{h}^*_{\mathrm{PT}}$	0.013	0.043	0.57	93.2	0.97	0.032	0.045	0.53	84.3	0.82
RBC	$h=\hat{h}_{\mathrm{CE}}$, $b=\hat{b}_{\mathrm{CE}}$	0.014	0.043	0.40	92.7	0.96	0.032	0.045	0.36	84.8	0.83
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},2}$	0.003	0.054	0.36	94.6	1.21	0.007	0.054	0.37	94.5	0.98
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},6}$	0.000	0.068	0.23	94.3	1.53	0.000	0.068	0.23	94.4	1.24
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.001	0.068	0.25	94.2	1.51	0.001	0.075	0.20	94.0	1.35
Conventional	$\hat{h}^*_{\mathrm{PT,ROT}}$	0.026	0.035	0.70	85.1	0.79	0.060	0.039	0.43	62.8	0.71
Conventional	$\hat{h}_{\mathrm{CE}}$	0.019	0.039	0.40	90.8	0.88	0.050	0.041	0.36	72.2	0.75
FLCI, $M=2$	$\hat{h}^*_{\mathrm{RMSE},2}$	0.018	0.041	0.36	94.9	1.00	0.055	0.041	0.37	79.1	0.81
FLCI, $M=6$	$\hat{h}^*_{\mathrm{RMSE},6}$	0.009	0.050	0.23	96.5	1.23	0.025	0.050	0.23	94.7	1.00
FLCI, $M=\hat{M}_{\mathrm{ROT}}$	$\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.009	0.049	0.25	95.7	1.22	0.019	0.054	0.20	94.2	1.09

Monte Carlo simulation: beta distribution for x_i (continued)

		$M=2$					$M=6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 3											
RBC	$h=\hat{h}^*_{\text{PT}}$, $b=\hat{b}^*_{\text{PT}}$	-0.031	0.037	0.55	86.2	0.83	-0.070	0.039	0.49	52.9	0.71
RBC	$h=b=\hat{h}^*_{\text{PT}}$	-0.012	0.044	0.55	93.9	0.98	-0.024	0.047	0.49	89.4	0.85
RBC	$h=\hat{h}_{\text{CE}}$, $b=\hat{b}_{\text{CE}}$	-0.011	0.044	0.39	92.9	0.99	-0.018	0.049	0.31	91.3	0.89
RBC	$h=b=\hat{h}^*_{\text{RMSE},2}$	-0.002	0.054	0.36	94.6	1.21	-0.007	0.054	0.37	94.5	0.98
RBC	$h=b=\hat{h}^*_{\text{RMSE},6}$	0.000	0.068	0.23	94.3	1.53	0.000	0.068	0.23	94.3	1.24
RBC	$h=b=\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.000	0.072	0.22	94.2	1.60	0.000	0.085	0.15	93.9	1.54
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	-0.025	0.036	0.65	85.3	0.80	-0.051	0.041	0.37	72.1	0.75
Conventional	$\hat{h}_{\text{CE}}$	-0.018	0.040	0.39	91.3	0.89	-0.040	0.044	0.31	81.6	0.79
FLCI, $M=2$	$\hat{h}^*_{\text{RMSE},2}$	-0.018	0.041	0.36	95.1	1.00	-0.054	0.041	0.37	79.6	0.81
FLCI, $M=6$	$\hat{h}^*_{\text{RMSE},6}$	-0.008	0.050	0.23	96.6	1.23	-0.024	0.050	0.23	94.8	1.00
FLCI, $M=\hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	-0.007	0.052	0.22	96.1	1.29	-0.011	0.061	0.15	96.3	1.23

Legend: SE—average standard error; $E[h]$—average (over Monte Carlo draws) bandwidth; Cov—coverage of CIs (in %); RL—relative (to optimal FLCI) length.

Bandwidth descriptions: $\hat{h}^*_{\text{PT}}$—plugin estimate of pointwise MSE optimal bandwidth (bw); $\hat{b}^*_{\text{PT}}$—analog for estimate of the bias; $\hat{h}_{\text{CE}}$—plugin estimate of coverage error optimal bw; $\hat{b}_{\text{CE}}$—analog for estimate of the bias; The implementation of Calonico et al. (2018) is used for all four bws. $\hat{h}^*_{\text{RMSE},2}$, $\hat{h}^*_{\text{RMSE},6}$—RMSE optimal bw, assuming $M=2$, and $M=6$, respectively. $\hat{h}^*_{\text{PT,ROT}}$—Fan and Gijbels (1996) rule of thumb; $\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$—RMSE optimal bw, using rule-of-thumb for M. 50,000 Monte Carlo draws.

Table S3: Monte Carlo simulation: heteroskedastic errors

		$M=2$					$M=6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 1											
RBC	$h=\hat{h}^*_{\mathrm{PT}}$, $b=\hat{b}^*_{\mathrm{PT}}$	0.058	0.049	0.69	78.8	0.83	0.160	0.050	0.63	6.7	0.70
RBC	$h=b=\hat{h}^*_{\mathrm{PT}}$	0.019	0.058	0.69	94.3	0.97	0.044	0.060	0.63	91.0	0.84
RBC	$h=\hat{h}_{\mathrm{CE}}$, $b=\hat{b}_{\mathrm{CE}}$	0.029	0.054	0.45	90.4	0.91	0.065	0.057	0.37	76.3	0.80
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},2}$	0.003	0.070	0.43	94.5	1.17	0.006	0.070	0.42	94.5	0.99
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},6}$	0.000	0.083	0.28	94.4	1.40	0.000	0.084	0.27	94.4	1.18
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.001	0.087	0.27	94.2	1.46	0.000	0.105	0.16	93.8	1.47
Conventional	$\hat{h}^*_{\mathrm{PT,ROT}}$	0.038	0.045	0.73	81.1	0.76	0.074	0.053	0.40	67.6	0.75
Conventional	$\hat{h}_{\mathrm{CE}}$	0.028	0.051	0.45	89.8	0.86	0.064	0.055	0.37	75.1	0.77
FLCI, $M=2$	$\hat{h}^*_{\mathrm{RMSE},2}$	0.029	0.052	0.43	95.1	1.00	0.083	0.052	0.42	73.6	0.83
FLCI, $M=6$	$\hat{h}^*_{\mathrm{RMSE},6}$	0.013	0.061	0.28	97.3	1.20	0.036	0.062	0.27	94.8	1.00
FLCI, $M=\hat{M}_{\mathrm{ROT}}$	$\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.012	0.064	0.27	96.4	1.25	0.013	0.077	0.16	96.9	1.25
Design 2											
RBC	$h=\hat{h}^*_{\mathrm{PT}}$, $b=\hat{b}^*_{\mathrm{PT}}$	0.040	0.049	0.69	87.2	0.83	0.121	0.049	0.69	29.9	0.69
RBC	$h=b=\hat{h}^*_{\mathrm{PT}}$	0.022	0.058	0.69	93.5	0.97	0.064	0.058	0.69	79.9	0.81
RBC	$h=\hat{h}_{\mathrm{CE}}$, $b=\hat{b}_{\mathrm{CE}}$	0.026	0.054	0.46	91.4	0.90	0.074	0.054	0.44	69.6	0.76
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},2}$	0.005	0.069	0.43	94.5	1.16	0.014	0.070	0.43	94.0	0.98
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},6}$	0.000	0.083	0.28	94.4	1.39	0.001	0.083	0.27	94.4	1.17
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.003	0.081	0.31	94.5	1.36	0.003	0.090	0.24	93.5	1.26
Conventional	$\hat{h}^*_{\mathrm{PT,ROT}}$	0.034	0.043	0.85	84.1	0.73	0.091	0.047	0.61	49.6	0.66
Conventional	$\hat{h}_{\mathrm{CE}}$	0.027	0.050	0.46	90.7	0.85	0.076	0.051	0.44	65.0	0.72
FLCI, $M=2$	$\hat{h}^*_{\mathrm{RMSE},2}$	0.026	0.051	0.43	95.6	1.00	0.076	0.052	0.43	77.7	0.83
FLCI, $M=6$	$\hat{h}^*_{\mathrm{RMSE},6}$	0.013	0.061	0.28	97.3	1.20	0.037	0.061	0.27	94.8	1.00
FLCI, $M=\hat{M}_{\mathrm{ROT}}$	$\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.015	0.060	0.31	96.5	1.18	0.029	0.066	0.24	92.8	1.08

Monte Carlo simulation: heteroskedastic errors (continued)

		$M=2$					$M=6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 3											
RBC	$h=\hat{h}^*_{\text{PT}}$, $b=\hat{b}^*_{\text{PT}}$	-0.040	0.049	0.69	87.2	0.83	-0.118	0.049	0.69	33.2	0.69
RBC	$h=b=\hat{h}^*_{\text{PT}}$	-0.020	0.058	0.69	93.4	0.97	-0.058	0.058	0.69	81.1	0.81
RBC	$h=\hat{h}_{\text{CE}}$, $b=\hat{b}_{\text{CE}}$	-0.024	0.054	0.46	91.4	0.90	-0.066	0.055	0.44	74.3	0.77
RBC	$h=b=\hat{h}^*_{\text{RMSE},2}$	-0.005	0.069	0.43	94.5	1.17	-0.014	0.070	0.42	93.9	0.98
RBC	$h=b=\hat{h}^*_{\text{RMSE},6}$	0.000	0.083	0.28	94.4	1.39	-0.001	0.084	0.27	94.3	1.17
RBC	$h=b=\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	-0.002	0.084	0.29	94.4	1.41	-0.001	0.099	0.19	93.9	1.38
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	-0.035	0.044	0.82	83.2	0.73	-0.085	0.049	0.53	56.2	0.69
Conventional	$\hat{h}_{\text{CE}}$	-0.026	0.050	0.46	90.4	0.85	-0.075	0.052	0.44	65.1	0.72
FLCI, $M=2$	$\hat{h}^*_{\text{RMSE},2}$	-0.026	0.051	0.43	95.6	1.00	-0.075	0.052	0.42	78.2	0.83
FLCI, $M=6$	$\hat{h}^*_{\text{RMSE},6}$	-0.013	0.061	0.28	97.2	1.20	-0.037	0.062	0.27	94.7	1.00
FLCI, $M=\hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	-0.013	0.062	0.29	96.8	1.22	-0.018	0.072	0.19	96.5	1.18

Legend: SE—average standard error; $E[h]$—average (over Monte Carlo draws) bandwidth; Cov—coverage of CIs (in %); RL—relative (to optimal FLCI) length.

Bandwidth descriptions: $\hat{h}^*_{\text{PT}}$—plugin estimate of pointwise MSE optimal bandwidth (bw); $\hat{b}^*_{\text{PT}}$—analog for estimate of the bias; $\hat{h}_{\text{CE}}$—plugin estimate of coverage error optimal bw; $\hat{b}_{\text{CE}}$—analog for estimate of the bias; The implementation of Calonico et al. (2018) is used for all four bws. $\hat{h}^*_{\text{RMSE},2}$, $\hat{h}^*_{\text{RMSE},6}$—RMSE optimal bw, assuming $M=2$, and $M=6$, respectively. $\hat{h}^*_{\text{PT,ROT}}$—Fan and Gijbels (1996) rule of thumb; $\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$—RMSE optimal bw, using rule-of-thumb for M. 50,000 Monte Carlo draws.

Table S4: Monte Carlo simulation: heteroskedastic errors and beta distribution for x_i

		$M=2$					$M=6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 1											
RBC	$h=\hat{h}^*_{\text{PT}}$, $b=\hat{b}^*_{\text{PT}}$	0.027	0.050	0.50	90.8	0.90	0.062	0.052	0.44	72.1	0.79
RBC	$h=b=\hat{h}^*_{\text{PT}}$	0.006	0.059	0.50	94.5	1.05	0.011	0.062	0.44	93.0	0.94
RBC	$h=\hat{h}_{\text{CE}}$, $b=\hat{b}_{\text{CE}}$	0.009	0.057	0.37	94.0	1.03	0.015	0.060	0.31	92.4	0.92
RBC	$h=b=\hat{h}^*_{\text{RMSE},2}$	0.003	0.062	0.44	94.6	1.11	0.008	0.062	0.43	94.4	0.95
RBC	$h=b=\hat{h}^*_{\text{RMSE},6}$	0.000	0.075	0.27	94.5	1.35	0.000	0.076	0.26	94.5	1.16
RBC	$h=b=\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.001	0.083	0.23	94.3	1.50	0.001	0.096	0.16	94.0	1.46
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	0.029	0.048	0.64	88.3	0.86	0.054	0.052	0.36	78.5	0.80
Conventional	$\hat{h}_{\text{CE}}$	0.018	0.052	0.37	92.4	0.94	0.043	0.055	0.31	84.0	0.83
FLCI, $M=2$	$\hat{h}^*_{\text{RMSE},2}$	0.025	0.049	0.44	94.9	1.00	0.073	0.050	0.43	76.0	0.85
FLCI, $M=6$	$\hat{h}^*_{\text{RMSE},6}$	0.012	0.057	0.27	97.1	1.18	0.033	0.058	0.26	94.7	1.00
FLCI, $M=\hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.008	0.063	0.23	97.0	1.31	0.013	0.070	0.16	96.8	1.24
Design 2											
RBC	$h=\hat{h}^*_{\text{PT}}$, $b=\hat{b}^*_{\text{PT}}$	0.024	0.050	0.50	91.8	0.89	0.069	0.050	0.49	69.8	0.76
RBC	$h=b=\hat{h}^*_{\text{PT}}$	0.010	0.058	0.50	94.3	1.05	0.025	0.059	0.49	91.3	0.90
RBC	$h=\hat{h}_{\text{CE}}$, $b=\hat{b}_{\text{CE}}$	0.012	0.057	0.37	93.8	1.02	0.032	0.058	0.36	89.0	0.88
RBC	$h=b=\hat{h}^*_{\text{RMSE},2}$	0.006	0.061	0.44	94.5	1.10	0.017	0.062	0.44	93.6	0.94
RBC	$h=b=\hat{h}^*_{\text{RMSE},6}$	0.001	0.075	0.27	94.5	1.35	0.001	0.075	0.27	94.5	1.15
RBC	$h=b=\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.001	0.081	0.25	94.3	1.46	0.002	0.085	0.22	94.0	1.29
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	0.028	0.047	0.73	89.6	0.84	0.072	0.048	0.57	64.1	0.74
Conventional	$\hat{h}_{\text{CE}}$	0.018	0.052	0.37	92.7	0.93	0.050	0.053	0.36	80.8	0.80
FLCI, $M=2$	$\hat{h}^*_{\text{RMSE},2}$	0.023	0.049	0.44	95.4	1.00	0.068	0.049	0.44	79.3	0.85
FLCI, $M=6$	$\hat{h}^*_{\text{RMSE},6}$	0.012	0.057	0.27	97.1	1.18	0.034	0.057	0.27	94.8	1.00

Monte Carlo simulation: heteroskedastic errors and beta distribution for x_i (continued)

		$M=2$					$M=6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
FLCI, $M=\hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.009	0.061	0.25	97.1	1.27	0.023	0.063	0.22	94.8	1.12
Design 3											
RBC	$h=\hat{h}^*_{\text{PT}}$, $b=\hat{b}^*_{\text{PT}}$	-0.027	0.050	0.50	91.0	0.90	-0.071	0.051	0.47	69.0	0.78
RBC	$h=b=\hat{h}^*_{\text{PT}}$	-0.008	0.059	0.50	94.5	1.05	-0.020	0.060	0.47	92.8	0.92
RBC	$h=\hat{h}_{\text{CE}}$, $b=\hat{b}_{\text{CE}}$	-0.011	0.057	0.37	93.8	1.03	-0.021	0.060	0.32	91.6	0.92
RBC	$h=b=\hat{h}^*_{\text{RMSE},2}$	-0.005	0.061	0.44	94.6	1.10	-0.016	0.062	0.44	93.7	0.94
RBC	$h=b=\hat{h}^*_{\text{RMSE},6}$	0.000	0.075	0.27	94.5	1.35	0.000	0.076	0.27	94.5	1.15
RBC	$h=b=\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.000	0.083	0.23	94.3	1.49	0.000	0.092	0.17	94.2	1.40
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	-0.027	0.047	0.70	89.4	0.85	-0.066	0.050	0.49	69.4	0.76
Conventional	$\hat{h}_{\text{CE}}$	-0.017	0.052	0.37	93.0	0.93	-0.043	0.054	0.32	84.5	0.82
FLCI, $M=2$	$\hat{h}^*_{\text{RMSE},2}$	-0.022	0.049	0.44	95.5	1.00	-0.067	0.049	0.44	79.9	0.85
FLCI, $M=6$	$\hat{h}^*_{\text{RMSE},6}$	-0.011	0.057	0.27	97.2	1.18	-0.032	0.057	0.27	94.9	1.00
FLCI, $M=\hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	-0.008	0.062	0.23	97.1	1.30	-0.014	0.068	0.17	96.8	1.21

Legend: SE—average standard error; $E[h]$—average (over Monte Carlo draws) bandwidth; Cov—coverage of CIs (in %); RL—relative (to optimal FLCI) length.

Bandwidth descriptions: $\hat{h}^*_{\text{PT}}$—plugin estimate of pointwise MSE optimal bandwidth (bw); $\hat{b}^*_{\text{PT}}$—analog for estimate of the bias; $\hat{h}_{\text{CE}}$—plugin estimate of coverage error optimal bw; $\hat{b}_{\text{CE}}$—analog for estimate of the bias; The implementation of Calonico et al. (2018) is used for all four bws. $\hat{h}^*_{\text{RMSE},2}$, $\hat{h}^*_{\text{RMSE},6}$—RMSE optimal bw, assuming $M=2$, and $M=6$, respectively. $\hat{h}^*_{\text{PT,ROT}}$—Fan and Gijbels (1996) rule of thumb; $\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$—RMSE optimal bw, using rule-of-thumb for M. 50,000 Monte Carlo draws.

Table S5: Monte Carlo simulation: log-normal errors

		$M=2$					$M=6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 1											
RBC	$h=\hat{h}^*_{\mathrm{PT}}$, $b=\hat{b}^*_{\mathrm{PT}}$	0.062	0.034	0.73	57.3	0.73	0.151	0.035	0.60	0.2	0.62
RBC	$h=b=\hat{h}^*_{\mathrm{PT}}$	0.022	0.041	0.73	94.5	0.88	0.036	0.045	0.60	91.5	0.78
RBC	$h=\hat{h}_{\mathrm{CE}}$, $b=\hat{b}_{\mathrm{CE}}$	0.042	0.037	0.55	83.0	0.79	0.111	0.037	0.50	18.8	0.66
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},2}$	0.001	0.058	0.35	91.1	1.24	0.003	0.057	0.35	91.5	1.01
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},6}$	0.000	0.070	0.23	89.7	1.52	0.000	0.070	0.23	89.6	1.23
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.000	0.072	0.22	89.3	1.56	0.000	0.087	0.14	87.6	1.54
Conventional	$\hat{h}^*_{\mathrm{PT,ROT}}$	0.032	0.034	0.55	78.7	0.74	0.048	0.044	0.31	82.2	0.77
Conventional	$\hat{h}_{\mathrm{CE}}$	0.041	0.034	0.55	81.2	0.73	0.107	0.035	0.50	16.4	0.61
FLCI, $M=2$	$\hat{h}^*_{\mathrm{RMSE},2}$	0.021	0.041	0.35	96.2	1.00	0.062	0.041	0.35	79.1	0.81
FLCI, $M=6$	$\hat{h}^*_{\mathrm{RMSE},6}$	0.009	0.051	0.23	95.5	1.23	0.027	0.050	0.23	96.2	1.00
FLCI, $M=\hat{M}_{\mathrm{ROT}}$	$\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.007	0.052	0.22	94.6	1.27	0.010	0.064	0.14	94.8	1.27
Design 2											
RBC	$h=\hat{h}^*_{\mathrm{PT}}$, $b=\hat{b}^*_{\mathrm{PT}}$	0.042	0.033	0.76	80.5	0.72	0.127	0.033	0.76	2.5	0.59
RBC	$h=b=\hat{h}^*_{\mathrm{PT}}$	0.024	0.040	0.76	93.3	0.86	0.073	0.040	0.76	52.2	0.70
RBC	$h=\hat{h}_{\mathrm{CE}}$, $b=\hat{b}_{\mathrm{CE}}$	0.033	0.036	0.56	89.8	0.79	0.097	0.037	0.55	19.0	0.64
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},2}$	0.002	0.057	0.35	91.3	1.24	0.006	0.057	0.35	91.9	1.01
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},6}$	0.000	0.070	0.23	89.6	1.52	0.000	0.070	0.23	89.7	1.23
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.002	0.063	0.29	90.3	1.37	0.000	0.075	0.19	89.0	1.32
Conventional	$\hat{h}^*_{\mathrm{PT,ROT}}$	0.032	0.030	0.76	77.4	0.66	0.072	0.037	0.43	53.3	0.66
Conventional	$\hat{h}_{\mathrm{CE}}$	0.034	0.033	0.56	87.8	0.72	0.099	0.034	0.55	11.4	0.59
FLCI, $M=2$	$\hat{h}^*_{\mathrm{RMSE},2}$	0.019	0.041	0.35	96.4	1.00	0.059	0.041	0.35	83.5	0.81
FLCI, $M=6$	$\hat{h}^*_{\mathrm{RMSE},6}$	0.009	0.051	0.23	95.6	1.23	0.027	0.051	0.23	96.5	1.00
FLCI, $M=\hat{M}_{\mathrm{ROT}}$	$\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.013	0.046	0.29	94.7	1.11	0.019	0.055	0.19	94.9	1.08

Monte Carlo simulation: log-normal errors (continued)

Method	Bandwidth	$M=2$ Bias	SE	$E[h]$	Cov	RL	$M=6$ Bias	SE	$E_m[h]$	Cov	RL
Design 3											
RBC	$h=\hat{h}^*_{\mathrm{PT}}$, $b=\hat{b}^*_{\mathrm{PT}}$	-0.043	0.034	0.76	67.1	0.73	-0.121	0.034	0.72	12.3	0.60
RBC	$h=b=\hat{h}^*_{\mathrm{PT}}$	-0.024	0.040	0.76	83.9	0.86	-0.065	0.041	0.72	56.1	0.72
RBC	$h=\hat{h}_{\mathrm{CE}}$, $b=\hat{b}_{\mathrm{CE}}$	-0.030	0.037	0.55	78.7	0.80	-0.077	0.039	0.51	45.8	0.69
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},2}$	-0.002	0.057	0.36	90.7	1.23	-0.006	0.057	0.36	90.1	1.00
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},6}$	0.000	0.069	0.23	89.7	1.50	0.000	0.069	0.23	89.7	1.22
RBC	$h=b=\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.000	0.068	0.25	89.6	1.48	0.000	0.083	0.16	88.1	1.46
Conventional	$\hat{h}^*_{\mathrm{PT,ROT}}$	-0.031	0.033	0.71	69.3	0.72	-0.063	0.040	0.39	54.9	0.71
Conventional	$\hat{h}_{\mathrm{CE}}$	-0.033	0.034	0.55	74.2	0.73	-0.093	0.035	0.51	27.8	0.61
FLCI, $M=2$	$\hat{h}^*_{\mathrm{RMSE},2}$	-0.020	0.041	0.36	89.9	1.00	-0.059	0.041	0.36	70.1	0.81
FLCI, $M=6$	$\hat{h}^*_{\mathrm{RMSE},6}$	-0.009	0.050	0.23	93.0	1.23	-0.027	0.050	0.23	88.4	1.00
FLCI, $M=\hat{M}_{\mathrm{ROT}}$	$\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	-0.010	0.050	0.25	91.8	1.22	-0.012	0.060	0.16	91.6	1.21

Legend: SE—average standard error; $E[h]$—average (over Monte Carlo draws) bandwidth; Cov—coverage of CIs (in %); RL—relative (to optimal FLCI) length.

Bandwidth descriptions: $\hat{h}^*_{\mathrm{PT}}$—plugin estimate of pointwise MSE optimal bandwidth (bw); $\hat{b}^*_{\mathrm{PT}}$—analog for estimate of the bias; $\hat{h}_{\mathrm{CE}}$—plugin estimate of coverage error optimal bw; $\hat{b}_{\mathrm{CE}}$—analog for estimate of the bias; The implementation of Calonico et al. (2018) is used for all four bws. $\hat{h}^*_{\mathrm{RMSE},2}$, $\hat{h}^*_{\mathrm{RMSE},6}$—RMSE optimal bw, assuming $M=2$, and $M=6$, respectively. $\hat{h}^*_{\mathrm{PT,ROT}}$—Fan and Gijbels (1996) rule of thumb; $\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$—RMSE optimal bw, using rule-of-thumb for M. 50,000 Monte Carlo draws.

Table S6: Monte Carlo simulation: log-normal errors and beta distribution for x_i

		$M=2$					$M=6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 1											
RBC	$h=\hat{h}^*_{\text{PT}}$, $b=\hat{b}^*_{\text{PT}}$	0.027	0.035	0.55	88.4	0.82	0.049	0.039	0.41	65.1	0.73
RBC	$h=b=\hat{h}^*_{\text{PT}}$	0.006	0.041	0.55	91.7	0.96	0.007	0.047	0.41	87.0	0.90
RBC	$h=\hat{h}_{\text{CE}}$, $b=\hat{b}_{\text{CE}}$	0.011	0.041	0.46	92.2	0.94	0.015	0.045	0.41	88.9	0.85
RBC	$h=b=\hat{h}^*_{\text{RMSE},2}$	0.001	0.052	0.36	91.7	1.19	0.004	0.051	0.36	91.9	0.98
RBC	$h=b=\hat{h}^*_{\text{RMSE},6}$	0.000	0.064	0.22	90.1	1.48	0.000	0.064	0.22	90.2	1.21
RBC	$h=b=\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.000	0.067	0.21	89.8	1.56	0.000	0.081	0.13	88.5	1.54
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	0.024	0.035	0.52	87.9	0.82	0.037	0.043	0.28	87.9	0.81
Conventional	$\hat{h}_{\text{CE}}$	0.026	0.036	0.46	91.3	0.82	0.064	0.037	0.41	51.5	0.70
FLCI, $M=2$	$\hat{h}^*_{\text{RMSE},2}$	0.019	0.039	0.36	96.2	1.00	0.055	0.039	0.36	81.0	0.82
FLCI, $M=6$	$\hat{h}^*_{\text{RMSE},6}$	0.008	0.047	0.22	95.7	1.22	0.024	0.047	0.22	96.1	1.00
FLCI, $M=\hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.006	0.049	0.21	95.0	1.28	0.009	0.059	0.13	94.9	1.27
Design 2											
RBC	$h=\hat{h}^*_{\text{PT}}$, $b=\hat{b}^*_{\text{PT}}$	0.024	0.035	0.56	91.4	0.81	0.067	0.036	0.52	49.0	0.68
RBC	$h=b=\hat{h}^*_{\text{PT}}$	0.010	0.041	0.56	92.2	0.95	0.028	0.042	0.52	81.6	0.81
RBC	$h=\hat{h}_{\text{CE}}$, $b=\hat{b}_{\text{CE}}$	0.015	0.040	0.47	92.9	0.93	0.037	0.042	0.45	81.9	0.79
RBC	$h=b=\hat{h}^*_{\text{RMSE},2}$	0.002	0.052	0.36	91.8	1.19	0.007	0.051	0.36	92.4	0.98
RBC	$h=b=\hat{h}^*_{\text{RMSE},6}$	0.000	0.064	0.22	90.0	1.48	0.000	0.064	0.22	90.1	1.21
RBC	$h=b=\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.001	0.062	0.25	90.4	1.44	0.000	0.069	0.19	89.5	1.31
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	0.026	0.033	0.69	88.9	0.77	0.059	0.037	0.43	64.9	0.70
Conventional	$\hat{h}_{\text{CE}}$	0.023	0.036	0.47	92.5	0.82	0.068	0.036	0.45	48.7	0.68
FLCI, $M=2$	$\hat{h}^*_{\text{RMSE},2}$	0.018	0.039	0.36	96.4	1.00	0.053	0.039	0.36	84.5	0.82
FLCI, $M=6$	$\hat{h}^*_{\text{RMSE},6}$	0.008	0.047	0.22	95.8	1.22	0.024	0.047	0.22	96.4	1.00

Monte Carlo simulation: log-normal errors and beta distribution for x_i (continued)

		$M=2$					$M=6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
FLCI, $M=\hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.009	0.046	0.25	95.3	1.19	0.018	0.050	0.19	95.0	1.08
Design 3											
RBC	$h=\hat{h}^*_{\text{PT}}$, $b=\hat{b}^*_{\text{PT}}$	-0.032	0.035	0.55	77.2	0.82	-0.068	0.038	0.47	50.6	0.72
RBC	$h=b=\hat{h}^*_{\text{PT}}$	-0.015	0.041	0.55	88.9	0.95	-0.025	0.045	0.47	85.4	0.85
RBC	$h=\hat{h}_{\text{CE}}$, $b=\hat{b}_{\text{CE}}$	-0.014	0.041	0.44	87.8	0.96	-0.022	0.045	0.37	85.4	0.86
RBC	$h=b=\hat{h}^*_{\text{RMSE},2}$	-0.002	0.051	0.36	91.0	1.18	-0.006	0.051	0.36	90.5	0.97
RBC	$h=b=\hat{h}^*_{\text{RMSE},6}$	0.000	0.064	0.23	90.1	1.47	0.000	0.064	0.23	90.1	1.21
RBC	$h=b=\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.000	0.066	0.22	89.7	1.53	0.000	0.077	0.15	88.7	1.47
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	-0.025	0.035	0.64	77.5	0.81	-0.050	0.040	0.36	63.6	0.76
Conventional	$\hat{h}_{\text{CE}}$	-0.023	0.036	0.44	82.3	0.84	-0.055	0.038	0.37	62.4	0.73
FLCI, $M=2$	$\hat{h}^*_{\text{RMSE},2}$	-0.018	0.039	0.36	90.1	1.00	-0.053	0.039	0.36	71.1	0.82
FLCI, $M=6$	$\hat{h}^*_{\text{RMSE},6}$	-0.008	0.047	0.23	93.1	1.22	-0.024	0.047	0.23	88.6	1.00
FLCI, $M=\hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	-0.007	0.049	0.22	92.6	1.27	-0.011	0.056	0.15	91.9	1.22

Legend: SE—average standard error; $E[h]$—average (over Monte Carlo draws) bandwidth; Cov—coverage of CIs (in %); RL—relative (to optimal FLCI) length.

Bandwidth descriptions: $\hat{h}^*_{\text{PT}}$—plugin estimate of pointwise MSE optimal bandwidth (bw); $\hat{b}^*_{\text{PT}}$—analog for estimate of the bias; $\hat{h}_{\text{CE}}$—plugin estimate of coverage error optimal bw; $\hat{b}_{\text{CE}}$—analog for estimate of the bias; The implementation of Calonico et al. (2018) is used for all four bws. $\hat{h}^*_{\text{RMSE},2}$, $\hat{h}^*_{\text{RMSE},6}$—RMSE optimal bw, assuming $M=2$, and $M=6$, respectively. $\hat{h}^*_{\text{PT,ROT}}$—Fan and Gijbels (1996) rule of thumb; $\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$—RMSE optimal bw, using rule-of-thumb for M. 50,000 Monte Carlo draws.

Table S7: Monte Carlo simulation: $\mathrm{sd}(u_i) = 1/4$

		$M = 2$					$M = 6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 1											
RBC	$h = \hat{h}^*_{\mathrm{PT}}$, $b = \hat{b}^*_{\mathrm{PT}}$	0.058	0.018	0.68	4.5	0.64	0.116	0.020	0.49	0.0	0.57
RBC	$h = b = \hat{h}^*_{\mathrm{PT}}$	0.019	0.022	0.68	90.2	0.80	0.017	0.026	0.49	91.2	0.76
RBC	$h = \hat{h}_{\mathrm{CE}}$, $b = \hat{b}_{\mathrm{CE}}$	0.024	0.022	0.39	77.4	0.78	0.041	0.025	0.28	60.8	0.72
RBC	$h = b = \hat{h}^*_{\mathrm{RMSE},2}$	0.000	0.035	0.27	94.3	1.26	0.000	0.035	0.28	94.3	1.01
RBC	$h = b = \hat{h}^*_{\mathrm{RMSE},6}$	0.000	0.043	0.18	93.8	1.57	0.000	0.043	0.18	93.8	1.26
RBC	$h = b = \hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.000	0.045	0.16	93.7	1.64	0.000	0.056	0.11	93.0	1.62
Conventional	$\hat{h}^*_{\mathrm{PT,ROT}}$	0.022	0.021	0.38	76.3	0.77	0.028	0.026	0.24	78.9	0.77
Conventional	$\hat{h}_{\mathrm{CE}}$	0.023	0.021	0.39	76.9	0.76	0.040	0.024	0.28	61.0	0.71
FLCI, $M = 2$	$\hat{h}^*_{\mathrm{RMSE},2}$	0.013	0.025	0.27	94.7	1.00	0.038	0.025	0.28	73.9	0.80
FLCI, $M = 6$	$\hat{h}^*_{\mathrm{RMSE},6}$	0.005	0.031	0.18	96.5	1.25	0.016	0.031	0.18	94.5	1.00
FLCI, $M = \hat{M}_{\mathrm{ROT}}$	$\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.004	0.032	0.16	96.2	1.30	0.006	0.040	0.11	96.2	1.29
Design 2											
RBC	$h = \hat{h}^*_{\mathrm{PT}}$, $b = \hat{b}^*_{\mathrm{PT}}$	0.043	0.017	0.77	28.8	0.63	0.128	0.017	0.76	0.0	0.51
RBC	$h = b = \hat{h}^*_{\mathrm{PT}}$	0.026	0.021	0.77	76.5	0.75	0.075	0.021	0.76	5.5	0.61
RBC	$h = \hat{h}_{\mathrm{CE}}$, $b = \hat{b}_{\mathrm{CE}}$	0.026	0.020	0.47	70.2	0.73	0.061	0.022	0.37	24.8	0.64
RBC	$h = b = \hat{h}^*_{\mathrm{RMSE},2}$	0.000	0.035	0.27	94.3	1.26	0.001	0.035	0.28	94.3	1.01
RBC	$h = b = \hat{h}^*_{\mathrm{RMSE},6}$	0.000	0.043	0.18	93.8	1.57	0.000	0.043	0.18	93.8	1.25
RBC	$h = b = \hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.000	0.038	0.24	93.5	1.38	0.000	0.047	0.15	93.6	1.37
Conventional	$\hat{h}^*_{\mathrm{PT,ROT}}$	0.029	0.018	0.57	58.5	0.66	0.048	0.023	0.32	46.0	0.67
Conventional	$\hat{h}_{\mathrm{CE}}$	0.027	0.019	0.47	66.1	0.69	0.062	0.021	0.37	21.8	0.62
FLCI, $M = 2$	$\hat{h}^*_{\mathrm{RMSE},2}$	0.012	0.025	0.27	94.8	1.00	0.039	0.024	0.28	73.6	0.80
FLCI, $M = 6$	$\hat{h}^*_{\mathrm{RMSE},6}$	0.005	0.031	0.18	96.5	1.25	0.016	0.030	0.18	94.6	1.00
FLCI, $M = \hat{M}_{\mathrm{ROT}}$	$\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.009	0.027	0.24	92.9	1.09	0.011	0.033	0.15	95.4	1.10

Monte Carlo simulation: $\mathrm{sd}(u_i) = 1/4$ (continued)

		$M = 2$					$M = 6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 3											
RBC	$h = \hat{h}^*_{\mathrm{PT}}$, $b = \hat{b}^*_{\mathrm{PT}}$	-0.042	0.017	0.76	32.3	0.63	-0.107	0.018	0.63	1.7	0.54
RBC	$h = b = \hat{h}^*_{\mathrm{PT}}$	-0.023	0.021	0.76	77.5	0.76	-0.048	0.023	0.63	45.4	0.67
RBC	$h = \hat{h}_{\mathrm{CE}}$, $b = \hat{b}_{\mathrm{CE}}$	-0.024	0.021	0.46	75.0	0.75	-0.046	0.023	0.35	49.5	0.68
RBC	$h = b = \hat{h}^*_{\mathrm{RMSE},2}$	0.000	0.035	0.27	94.3	1.26	-0.001	0.035	0.28	94.4	1.01
RBC	$h = b = \hat{h}^*_{\mathrm{RMSE},6}$	0.000	0.043	0.18	93.8	1.57	0.000	0.043	0.18	93.8	1.25
RBC	$h = b = \hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	0.000	0.042	0.19	93.8	1.54	0.000	0.053	0.12	93.3	1.53
Conventional	$\hat{h}^*_{\mathrm{PT,ROT}}$	-0.026	0.019	0.49	65.2	0.69	-0.041	0.024	0.29	58.7	0.70
Conventional	$\hat{h}_{\mathrm{CE}}$	-0.026	0.019	0.46	66.5	0.69	-0.057	0.022	0.35	30.4	0.63
FLCI, $M = 2$	$\hat{h}^*_{\mathrm{RMSE},2}$	-0.012	0.025	0.27	94.7	1.00	-0.038	0.025	0.28	73.9	0.80
FLCI, $M = 6$	$\hat{h}^*_{\mathrm{RMSE},6}$	-0.005	0.031	0.18	96.4	1.25	-0.016	0.031	0.18	94.5	1.00
FLCI, $M = \hat{M}_{\mathrm{ROT}}$	$\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$	-0.006	0.030	0.19	95.8	1.22	-0.007	0.037	0.12	96.2	1.23

Legend: SE—average standard error; $E[h]$—average (over Monte Carlo draws) bandwidth; Cov—coverage of CIs (in %); RL—relative (to optimal FLCI) length.

Bandwidth descriptions: $\hat{h}^*_{\mathrm{PT}}$—plugin estimate of pointwise MSE optimal bandwidth (bw); $\hat{b}^*_{\mathrm{PT}}$—analog for estimate of the bias; $\hat{h}_{\mathrm{CE}}$—plugin estimate of coverage error optimal bw; $\hat{b}_{\mathrm{CE}}$—analog for estimate of the bias; The implementation of Calonico et al. (2018) is used for all four bws. $\hat{h}^*_{\mathrm{RMSE},2}$, $\hat{h}^*_{\mathrm{RMSE},6}$—RMSE optimal bw, assuming $M = 2$, and $M = 6$, respectively. $\hat{h}^*_{\mathrm{PT,ROT}}$—Fan and Gijbels (1996) rule of thumb; $\hat{h}^*_{\mathrm{RMSE},\hat{M}_{\mathrm{ROT}}}$—RMSE optimal bw, using rule-of-thumb for M. 50,000 Monte Carlo draws.

Table S8: Monte Carlo simulation: smooth DGP with $\lambda = 40$

		$M = 2$					$M = 6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 1											
RBC	$h = \hat{h}^*_{\text{PT}}$, $b = \hat{b}^*_{\text{PT}}$	0.062	0.035	0.74	57.7	0.73	0.151	0.036	0.61	0.2	0.61
RBC	$h = b = \hat{h}^*_{\text{PT}}$	0.024	0.042	0.74	93.3	0.88	0.039	0.047	0.61	90.1	0.78
RBC	$h = \hat{h}_{\text{CE}}$, $b = \hat{b}_{\text{CE}}$	0.029	0.041	0.46	86.1	0.85	0.059	0.045	0.34	72.6	0.76
RBC	$h = b = \hat{h}^*_{\text{RMSE},2}$	0.001	0.061	0.36	94.5	1.27	0.003	0.061	0.36	94.5	1.01
RBC	$h = b = \hat{h}^*_{\text{RMSE},6}$	0.000	0.076	0.23	94.2	1.58	0.000	0.075	0.23	94.2	1.26
RBC	$h = b = \hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.000	0.078	0.22	93.9	1.63	0.000	0.097	0.14	93.4	1.63
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	0.032	0.036	0.57	77.0	0.75	0.050	0.046	0.32	76.9	0.77
Conventional	$\hat{h}_{\text{CE}}$	0.028	0.039	0.46	85.7	0.80	0.057	0.044	0.34	72.8	0.74
FLCI, $M = 2$	$\hat{h}^*_{\text{RMSE},2}$	0.021	0.043	0.36	95.0	1.00	0.063	0.043	0.36	76.2	0.80
FLCI, $M = 6$	$\hat{h}^*_{\text{RMSE},6}$	0.009	0.054	0.23	96.6	1.25	0.027	0.053	0.23	94.7	1.00
FLCI, $M = \hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.008	0.055	0.22	95.6	1.29	0.010	0.069	0.14	96.3	1.29
Design 2											
RBC	$h = \hat{h}^*_{\text{PT}}$, $b = \hat{b}^*_{\text{PT}}$	0.041	0.035	0.77	77.4	0.72	0.124	0.035	0.77	5.4	0.58
RBC	$h = b = \hat{h}^*_{\text{PT}}$	0.024	0.042	0.77	91.4	0.87	0.072	0.042	0.77	58.0	0.70
RBC	$h = \hat{h}_{\text{CE}}$, $b = \hat{b}_{\text{CE}}$	0.026	0.040	0.49	88.1	0.83	0.071	0.041	0.44	56.4	0.69
RBC	$h = b = \hat{h}^*_{\text{RMSE},2}$	0.002	0.061	0.36	94.5	1.27	0.007	0.061	0.36	94.4	1.01
RBC	$h = b = \hat{h}^*_{\text{RMSE},6}$	0.000	0.076	0.23	94.2	1.58	0.000	0.075	0.23	94.2	1.26
RBC	$h = b = \hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.002	0.068	0.30	94.0	1.43	0.000	0.083	0.20	93.8	1.38
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	0.030	0.032	0.78	76.0	0.67	0.071	0.040	0.44	54.7	0.66
Conventional	$\hat{h}_{\text{CE}}$	0.027	0.037	0.49	86.7	0.77	0.072	0.039	0.44	52.5	0.66
FLCI, $M = 2$	$\hat{h}^*_{\text{RMSE},2}$	0.019	0.043	0.36	95.3	1.00	0.058	0.043	0.36	80.0	0.80
FLCI, $M = 6$	$\hat{h}^*_{\text{RMSE},6}$	0.009	0.054	0.23	96.6	1.25	0.027	0.053	0.23	94.8	1.00
FLCI, $M = \hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	0.013	0.048	0.30	94.5	1.13	0.019	0.059	0.20	94.4	1.10

Monte Carlo simulation: smooth DGP, with $\lambda = 40$ (continued)

		$M = 2$					$M = 6$				
Method	Bandwidth	Bias	SE	$E[h]$	Cov	RL	Bias	SE	$E_m[h]$	Cov	RL
Design 3											
RBC	$h = \hat{h}^*_{\text{PT}}$, $b = \hat{b}^*_{\text{PT}}$	-0.041	0.035	0.77	77.0	0.72	-0.119	0.035	0.74	11.0	0.59
RBC	$h = b = \hat{h}^*_{\text{PT}}$	-0.023	0.042	0.77	91.3	0.87	-0.064	0.042	0.74	62.4	0.71
RBC	$h = \hat{h}_{\text{CE}}$, $b = \hat{b}_{\text{CE}}$	-0.025	0.040	0.49	88.6	0.83	-0.061	0.043	0.43	66.1	0.71
RBC	$h = b = \hat{h}^*_{\text{RMSE},2}$	-0.002	0.061	0.36	94.5	1.27	-0.007	0.061	0.36	94.3	1.01
RBC	$h = b = \hat{h}^*_{\text{RMSE},6}$	0.000	0.076	0.23	94.2	1.58	-0.001	0.075	0.23	94.2	1.26
RBC	$h = b = \hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	-0.001	0.074	0.25	94.2	1.54	0.000	0.092	0.16	93.6	1.54
Conventional	$\hat{h}^*_{\text{PT,ROT}}$	-0.030	0.033	0.72	75.9	0.69	-0.062	0.042	0.39	63.4	0.70
Conventional	$\hat{h}_{\text{CE}}$	-0.027	0.037	0.49	86.4	0.78	-0.071	0.040	0.43	53.9	0.66
FLCI, $M = 2$	$\hat{h}^*_{\text{RMSE},2}$	-0.019	0.043	0.36	95.1	1.00	-0.058	0.043	0.36	79.8	0.80
FLCI, $M = 6$	$\hat{h}^*_{\text{RMSE},6}$	-0.009	0.054	0.23	96.5	1.25	-0.027	0.053	0.23	94.7	1.00
FLCI, $M = \hat{M}_{\text{ROT}}$	$\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$	-0.010	0.052	0.25	95.7	1.22	-0.013	0.065	0.16	96.1	1.22

Legend: SE—average standard error; $E[h]$—average (over Monte Carlo draws) bandwidth; Cov—coverage of CIs (in %); RL—relative (to optimal FLCI) length.

Bandwidth descriptions: $\hat{h}^*_{\text{PT}}$—plugin estimate of pointwise MSE optimal bandwidth (bw); $\hat{b}^*_{\text{PT}}$—analog for estimate of the bias; $\hat{h}_{\text{CE}}$—plugin estimate of coverage error optimal bw; $\hat{b}_{\text{CE}}$—analog for estimate of the bias; The implementation of Calonico et al. (2018) is used for all four bws. $\hat{h}^*_{\text{RMSE},2}$, $\hat{h}^*_{\text{RMSE},6}$—RMSE optimal bw, assuming $M = 2$, and $M = 6$, respectively. $\hat{h}^*_{\text{PT,ROT}}$—Fan and Gijbels (1996) rule of thumb; $\hat{h}^*_{\text{RMSE},\hat{M}_{\text{ROT}}}$—RMSE optimal bw, using rule-of-thumb for M. 50,000 Monte Carlo draws.