Household portfolios and financial preparedness for retirement∗

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Abstract

Using a lifecycle model of consumption, saving and portfolio choice combined with linked survey and administrative data on wealth and lifetime earnings we evaluate measures of retirement preparedness. We estimate heterogeneous discount factors for households and compare the estimates of their patience to their replacement rates – the simple measure often used to evaluate the adequacy of retirement savings. We find first that the specification of the model’s asset structure matters quantitatively for preference parameter estimates – households appear to be much more patient when they are assumed to have access only to a risk-free asset compared to when we account for the fact that much of their wealth is stored in higher-return tax-advantaged private pensions and in housing. Second we find that only the most patient households achieve the replacement rates out of final earnings that are often recommended by policy-makers and industry as sensible benchmarks for retirement preparedness. Notwithstanding this, we find that even quite impatient households in the population we study achieve high replacement rates out of lifetime average income – a more sensible summary measure of preparedness for retirement.

JEL Classification: D91, D31, E21, D14, H55

Keywords: Lifecycle model; Wealth; Savings; Pensions; Patience; Discount Factors

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1 Introduction

Many countries are implementing policies to encourage saving for retirement. Examples include: mandatory private pension saving in Australia (introduced in 1992), a compulsion for all employers to auto-enroll employees into private pension saving in the United Kingdom (introduced from 2012) and auto-enrollment in the US army in 2010, with a number of US states now implementing or piloting similar initiatives. The rationale for such policies is that individuals, left to their own devices, will not save enough for retirement. This paper examines the evidence for the presumption, implicit or explicit, that there is systematic undersaving.

We have two distinct approaches. The first develops a lifecycle model of consumption and saving and combines this with micro-data on the wealth holdings (from household survey data) and lifetime earnings histories (from linked administrative data) for a sample of English households. The model is a rich one: households can save in each of pension and non-pension wealth; they face risks over earnings, pension fund returns and longevity; and they are heterogeneous in their earnings processes, their fertility and their patience. We use this model to estimate the implied level of patience, for each household in our sample, that would rationalize observed wealth holdings.

The wealth data which, together with our lifetime earnings data and model, identifies the discount factor is wealth close to retirement. Our estimates of discount factors, therefore, summarize how households trade-off consumption during working life and saving for retirement and, as such, the estimated discount factor gives a summary measure of the extent to which households have planned for their retirement. We complement this approach by estimating a more conventional summary measure of how households trade-off working life consumption and saving – replacement rates out of lifetime earnings (see, for example, Munnell & Soto (2005) and Banks et al. (2005)). Comparisons of observed or projected replacement rates (out of either career average or of final earnings) to target levels are often used by policy-makers to assess adequacy of savings among cohorts approaching retirement (e.g. Bridges & Choudhury (2005), Pension Commission (2004)) and by financial advisers in recommendations to clients (e.g. TIAA (2018)). While such approach has the advantage of simplicity, any threshold selected as the appropriate one for saving is necessarily ad-hoc. Furthermore, assuming a single replacement rate for all households, regardless of their personal circumstance (trajectory of income, number of children, life expectancy etc.) is ‘conceptually flawed’ (Scholz & Seshadri (2009), building on earlier work by Gokhale et al. (1999)). We show the relationship between our estimated replacement rates
and the estimated discount factors from our first approach and highlight some implications for users of the simple replacement rate approach.

We have four principle findings. First, we can match the distribution of wealth with modest heterogeneity in the degree of patience. The interquartile range for estimates of the discount factor from our preferred model stretches from 0.92 to 0.97. The median discount factor is 0.95, close to conventional values estimated (or assumed) by papers using lifecycle models that assume homogeneous discounting. We validate these estimates of discount factors in a number of ways. In particular, we show that those with higher education are estimated to be more patient (as has been previously found in the literature), the estimates are associated with other known correlates of patience (e.g. smoking behavior) and that our estimated discount factors correlate with self-reported measures of patience elicited from the same survey respondents.

Second, we highlight the importance of capturing the richness of the savings environment available to households for the estimation of discount factors. The median discount factor estimated using a simple one-risk-free-asset model would be 1.04, with an interquartile range that stretches from 1.01 to 1.08. When we take account in the model of the fact that households, in reality, have access to a tax-advantaged pension fund which incentivizes saving for retirement, the median estimate falls to 0.99. Neither of the models underlying these estimates accounts, though, for the particular features of housing as a store of wealth (for example that it returns a flow of services and that it tends to be purchased using leverage). Once we account for these features in the return on non-pension wealth, the median estimated discount rate factor falls to 0.95 – our preferred estimate.

Third, few of the cohort that we study (those born in England in the 1940s) will have substantially lower income in retirement than their average earnings over the course of their working life. The median replacement rate is 84.5%, and only around one third would have pension income at age 65 of less than 70% of their average lifetime earnings. However, replacement rates are lower when pension income is compared to a proxy for final pre-retirement earnings (which is common practice among policy makers and industry). In particular, the median replacement of the average of the best five years of earnings is 47%, and almost nine out of ten households are found to replace less than 70% of this measure of earnings in retirement.

Finally, bringing together our two approaches, we show that these estimated replacement rates out of a measure of final earnings are low – compared to thresholds for ‘adequacy’ commonly used – even when wealth holdings can be justified without appealing to high levels of individual impatience. This suggests that those using replacement rates to assess retirement preparedness
must be careful with their choice of replacement rate and the associated threshold against with adequacy is assessed. While the ease of communicating simple replacement rate benchmarks is clear, those recommending them need to a) be clear that suggesting high replacement rates out of final earnings represents an extremely demanding threshold and b) be cognizant of the importance of individual circumstances and preferences when recommending savings choices.

Our paper contributes to two distinct literatures. The first is that which has used lifecycle models to assess household retirement saving (most notably Engen, Gale & Uccello (1999) and Scholz, Seshadri & Khitatrakun (2006)). Our paper is most similar to Scholz et al. (2006), who showed that levels of wealth accumulated by the vast majority of households in a now-retired cohort in the US were more than an ‘optimal’ level of wealth that the authors derive. ‘Optimal’ wealth in that paper is pinned down using a lifecycle model in which a homogeneous discount rate is assumed, which is set approximately equal to the rate of return on the model’s (single) asset. We add to their paper by a) enriching the asset structure, primarily by including a risky, higher expected return and tax-advantaged pension asset; b) estimating the distribution of discount rates rather than assuming a value; and c) showing the relationship between those discount rates and the instrument central to notions of optimal saving used by policy-makers – replacement rates.

The second literature to which we contribute is that concerning the estimation of discount factors. Lifecycle models have typically assumed homogeneity of discount rates (or homogeneity conditional a set of observable characteristics – e.g. education). Early estimated structural models include Attanasio et al. (1999), Gourinchas & Parker (2002) and Cagetti (2003); more recently (and focusing on those papers most relevant to the current paper), papers studying Medicaid (De Nardi et al. (2016)), old-age means-tested transfers (Braun et al. (2016)) and intergenerational transfers (Lockwood (2018)) have assumed that agents all have the same degree of patience.

This assumption of a homogeneous discount factor is a strong one. Experiments have provided evidence in support of heterogeneous discount rates (Andersen et al. (2008), Andreoni & Sprenger (2012)), and such heterogeneity has been proposed as a candidate explanation for observed wealth inequality (Krusell & Smith (1998), Hendricks (2007), Hubmer et al. (n.d.)). Using a ‘semi-structural approach’, Alan & Browning (2010) and Alan et al. (forthcoming), use, respectively, data on the consumption growth and on the co-movements of income and consumption to estimate distributions of discount rates, finding substantial dispersion in measures of patience.
Samwick (1998) and Gustman & Steinmeier (2005) both solve lifecycle models and identify, as we do, a discount factor for each household by matching wealth levels in the data. Samwick solves a model with income uncertainty and estimates a discount factor for each household in a cross-sectional data set as that which rationalizes wealth to current income ratios; Gustman & Steinmeier solve a lifecycle model with deterministic income and estimate a discount factor for each household as that which rationalizes wealth stocks conditional on life-history of earnings. Computational advances, combined with our rich data, allow us to relax many of the assumptions made in those papers. We broaden the asset structure beyond the ‘single-safe-asset’ workhorse model to make it more reflective of the assets in which households save for retirement, and we incorporate uncertainty in multiple dimensions (earnings, returns and survival).

The rest of this paper proceeds as follows. Section 2 introduces the linked survey and administrative data that we use. Section 3 outlines the model, while Section 4 discusses the estimation and parameterization of features of the model. Section 5 discusses our results. Section 6 concludes.

2 Data

Our data is formed by linking survey and administrative data. The survey data come from the the English Longitudinal Study of Ageing (ELSA) - a biennial longitudinal survey of the household population of England aged 50 and over.\(^1\) ELSA contains detailed data on demographics, labor market circumstances, income and, most importantly for our purposes, the level and composition of wealth holdings.

ELSA respondents were also asked for their National Insurance number (the equivalent of a US Social Security number) and permission to link to their history of National Insurance contributions. Almost 80\% of ELSA respondents agreed to the linking of their survey records with their administrative data. Data on these contributions allow us to calculate individuals’ state pension entitlements, and (subject to some top-coding) to obtain a detailed history of their earnings. Appendix D.1 discusses how we convert our administrative data on National Insurance contributions into a panel of earnings.

We use the 2002/03 ELSA data (the year for which the sample was linked) together with the

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\(^1\)ELSA is part of a family of international aging surveys, including the Health and Retirement Study (HRS) in the US and the Survey of Health, Ageing and Retirement in Europe (SHARE), that collect a similar set data using broadly comparable methodologies.
linked National Insurance data.

2.1 Sample

We restrict our attention to couple households that contain a man born between 1940 and 1949. There are 1,615 couples of this type in the 2002/03 ELSA data. These individuals would be aged between 52 and 63 (and therefore approaching the UK public pension age of 65) when observed in 2002/03. We exclude households where either partner refused permission to link to their administrative data, since for these households we cannot obtain lifetime earnings. We also exclude households where the man is observed in the NI data for fewer than five years and/or where households have more than five years of self-employment activity (since the NI data are less well suited to calculating the earnings histories of the self-employed – as is discussed in more detail in Appendix D.1). After applying these restrictions, 995 couples, or approximately 62% of couples, remain.

Table 1 provides descriptive statistics for our sample of households and for all ELSA couples in the relevant cohort. The average age of men in our sample when they are observed in 2002/03 is just under 57, with women on average being 54. Nearly 70% of men in our sample reported still being in work, and only 15% defined themselves as retired. Home ownership was the norm for this cohort – in our sample, nearly 90% of households own their home (either outright or still mortgaged).

A comparison of the descriptive statistics from the survey data for our sample and the descriptives for all couples in ELSA in the relevant cohort suggests a close match along most observables between our sample and the full sample of couples in the cohort of interest. An exception to this is with regard to self-employed individuals who are under-represented in our sample. This is unsurprising given that we drop those with significant histories of self-employment though this is accentuated by the self-employed being less likely to grant permission to link to their administrative records (see Bozio, Crawford, Emmerson & Tetlow (2010)).

2.2 Wealth measures in ELSA

Our model contains each of private pension wealth and non-pension wealth (which, in turn, comprises each of housing and more liquid wealth). Therefore we must define an empirical analogue for each of these. The sum of these components is referred to as ‘private wealth’.

Private pension wealth includes both Defined Benefit (DB) and Defined Contribution (DC)
Table 1: Descriptive statistics for couples in ELSA of cohort 1940-1949

<table>
<thead>
<tr>
<th>Individual characteristics</th>
<th>Our sample</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean age</td>
<td>Male 56.8</td>
<td>Male 56.9</td>
</tr>
<tr>
<td></td>
<td>Female 54.1</td>
<td>Female 54.0</td>
</tr>
<tr>
<td>Low education</td>
<td>Male 35.3%</td>
<td>Male 36.5%</td>
</tr>
<tr>
<td></td>
<td>Female 43.5%</td>
<td>Female 41.9%</td>
</tr>
<tr>
<td>Mid education</td>
<td>Male 27.3%</td>
<td>Male 26.6%</td>
</tr>
<tr>
<td></td>
<td>Female 32.2%</td>
<td>Female 31.1%</td>
</tr>
<tr>
<td>High education</td>
<td>Male 37.4%</td>
<td>Male 36.8%</td>
</tr>
<tr>
<td></td>
<td>Female 24.3%</td>
<td>Female 22.4%</td>
</tr>
<tr>
<td>Employee</td>
<td>Male 60.5%</td>
<td>Male 55.4%</td>
</tr>
<tr>
<td></td>
<td>Female 59.5%</td>
<td>Female 52.9%</td>
</tr>
<tr>
<td>Self-employed</td>
<td>Male 11.0%</td>
<td>Male 15.2%</td>
</tr>
<tr>
<td></td>
<td>Female 3.6%</td>
<td>Female 4.6%</td>
</tr>
<tr>
<td>Retired</td>
<td>Male 14.9%</td>
<td>Male 14.3%</td>
</tr>
<tr>
<td></td>
<td>Female 11.1%</td>
<td>Female 10.8%</td>
</tr>
<tr>
<td>Other</td>
<td>Male 13.6%</td>
<td>Male 15.0%</td>
</tr>
<tr>
<td></td>
<td>Female 25.8%</td>
<td>Female 27.2%</td>
</tr>
<tr>
<td>Household characteristics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Owner occupier</td>
<td>89.6%</td>
<td>87.6%</td>
</tr>
<tr>
<td>Median total income</td>
<td>£22,544</td>
<td>£22,166</td>
</tr>
<tr>
<td>Median employment income</td>
<td>£17,870</td>
<td>£17,189</td>
</tr>
<tr>
<td>Median asset income</td>
<td>£185</td>
<td>£182</td>
</tr>
<tr>
<td>Median private pension wealth</td>
<td>£95,593</td>
<td>£72,384</td>
</tr>
<tr>
<td>Median housing wealth</td>
<td>£120,000</td>
<td>£121,000</td>
</tr>
<tr>
<td>Median non-pension, non-housing wealth</td>
<td>£32,200</td>
<td>£30,625</td>
</tr>
<tr>
<td>Sample size</td>
<td>995</td>
<td>1,615</td>
</tr>
</tbody>
</table>
pensions. Wealth is calculated differently for each of these. DC pension wealth is the reported value of funds held. We calculate DB pension wealth as the capital sum that would be required in our survey year to purchase the projected stream of DB income to which the household is entitled given their reported accrual of rights to date and the rules of the pension scheme. To calculate this, we first calculate the capital sum required at age 65, given annuity rates at that age.\(^2\) The capital sum required in 2002/03 is then calculated by assuming that households would achieve a real rate of return of equal to the model’s mean return on DC wealth between 2002/03 and the year in which they reach the age of 65.

Housing wealth is the gross value of owner-occupied housing less any mortgage debt. Liquid wealth is the sum of all other non-pension wealth less any outstanding non-mortgage debt. The largest component of this is cash (and we refer to this form of wealth as ‘cash’ below), but it also includes other liquid financial wealth, net non-primary housing wealth and other wealth (business wealth and physical assets such as land, antiques and collectibles).

Table 2 summarizes, for our sample of couples, the distribution of wealth held in each of these components. The mean level of total wealth is £407,400, of which around 36.9% is held in housing, 34.3% is in private pension wealth, and 28.8% is in cash and other liquid assets. This compares to mean lifetime earnings of £985,400. Holdings of wealth and each of its components rise with lifetime earnings, and ratios of mean total wealth to mean lifetime earnings are highest among those at the bottom and top of the lifetime earnings distribution (see also Venti & Wise (1999), Gustman & Steinmeier (1999), Bozio et al. (2017)), although the very large difference between mean and median wealth in the bottom decile highlights the fact that wealth is particularly skewed at that part of the distribution and that many households who have had low lifetime earnings hold little private wealth.

### 3 A model of saving for retirement

We solve and estimate a lifecycle model of consumption, saving and portfolio choice. Details are given in this section. Briefly, its key features include: decisions made (collectively) by households that differ in their patience, uncertainty over employment, earnings, returns on a pension fund and mortality; exogenous heterogeneity over the earnings process and fertility; and a careful specification of the tax and benefit system.

\(^2\)The annuity rate is calculated using the model’s risk free interest rate and survival probabilities (details are given below).
<table>
<thead>
<tr>
<th>Lifetime earnings</th>
<th>Median</th>
<th>Mean</th>
<th>Median</th>
<th>Mean</th>
<th>Median</th>
<th>Mean</th>
<th>Median</th>
<th>Mean</th>
<th>Median</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>888.9</td>
<td>985.4</td>
<td>280.7</td>
<td>407.4</td>
<td>95.6</td>
<td>139.8</td>
<td>32.2</td>
<td>117.2</td>
<td>120.0</td>
<td>150.5</td>
</tr>
<tr>
<td>Lifetime earnings decile:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lowest</td>
<td>299.8</td>
<td>287.6</td>
<td>64.3</td>
<td>251.9</td>
<td>14.0</td>
<td>35.5</td>
<td>2.0</td>
<td>152.5</td>
<td>23.5</td>
<td>63.9</td>
</tr>
<tr>
<td>2</td>
<td>517.4</td>
<td>510.3</td>
<td>146.2</td>
<td>216.7</td>
<td>35.7</td>
<td>65.6</td>
<td>10.2</td>
<td>53.4</td>
<td>75.0</td>
<td>97.7</td>
</tr>
<tr>
<td>3</td>
<td>663.3</td>
<td>658.3</td>
<td>176.7</td>
<td>302.9</td>
<td>52.0</td>
<td>80.5</td>
<td>19.3</td>
<td>109.2</td>
<td>89.5</td>
<td>113.2</td>
</tr>
<tr>
<td>4</td>
<td>768.0</td>
<td>765.4</td>
<td>208.7</td>
<td>274.6</td>
<td>62.5</td>
<td>94.0</td>
<td>19.4</td>
<td>71.9</td>
<td>95.0</td>
<td>108.7</td>
</tr>
<tr>
<td>5</td>
<td>851.7</td>
<td>849.5</td>
<td>242.2</td>
<td>331.0</td>
<td>72.3</td>
<td>109.6</td>
<td>27.2</td>
<td>88.5</td>
<td>100.5</td>
<td>132.9</td>
</tr>
<tr>
<td>6</td>
<td>932.6</td>
<td>935.8</td>
<td>252.5</td>
<td>336.6</td>
<td>98.1</td>
<td>118.8</td>
<td>30.0</td>
<td>82.0</td>
<td>115.0</td>
<td>135.8</td>
</tr>
<tr>
<td>7</td>
<td>1051.7</td>
<td>1049.1</td>
<td>348.1</td>
<td>376.5</td>
<td>109.5</td>
<td>132.3</td>
<td>46.9</td>
<td>94.1</td>
<td>132.5</td>
<td>150.1</td>
</tr>
<tr>
<td>8</td>
<td>1202.2</td>
<td>1201.4</td>
<td>319.7</td>
<td>413.4</td>
<td>125.2</td>
<td>148.7</td>
<td>34.5</td>
<td>76.7</td>
<td>140.0</td>
<td>188.0</td>
</tr>
<tr>
<td>9</td>
<td>1440.9</td>
<td>1447.6</td>
<td>546.6</td>
<td>619.3</td>
<td>196.2</td>
<td>241.3</td>
<td>66.5</td>
<td>144.7</td>
<td>221.3</td>
<td>233.4</td>
</tr>
<tr>
<td>Highest</td>
<td>1894.0</td>
<td>2155.7</td>
<td>683.7</td>
<td>952.7</td>
<td>287.4</td>
<td>372.3</td>
<td>103.7</td>
<td>298.8</td>
<td>250.0</td>
<td>281.6</td>
</tr>
</tbody>
</table>

Table 2: Observed private net wealth (£,000s), by lifetime earnings
3.1 Preferences and the economic environment

3.1.1 Preferences

Household utility in each period (one year in the model) is assumed to exhibit constant relative risk aversion in equivalized consumption, multiplied by the number of equivalized adults in the household:

\[ U(c_t) = n_t \left( \frac{c_t}{n_t} \right)^{1-\gamma} \]

where \( \left( \frac{c_t}{n_t} \right) \) is household equivalized consumption and \( n_t \) is the number of equivalized adults in the household. The subscript \( t \) refers to the age of the household (taken to be the age of the male).

3.1.2 Employment and earnings

Employment starts at age 20 and can continue until age 64, at which point employment is no longer possible. In each period, households get an employment offer with probability \( \pi_j \), where \( j \) represents education group. Households are divided into three education groups: high-school drop outs, high school graduates and those with at least some college.\(^3\)

Earnings \((e_{it})\) are therefore given by:

\[ e_{it} = \tilde{e}_{it} \quad w.p. \quad \pi_j \]
\[ 0 \quad w.p. \quad 1 - \pi_j \]

where \( (\tilde{e}) \) is household productivity, defined below. \( 1-\pi_j \) can therefore be interpreted as the probability of a spell of long-term unemployment (lasting a year) for both members of the couple.

The log productivity of a household (the sum of earnings of both members of the couple) is given by the sum of a fixed effect, a quadratic in age \((t)\) and a stochastic process which contains a first order autoregressive process with normally distributed innovation. Households receive this value unless they receive an unemployment shock:

\(^3\)In the UK context, these are defined as having compulsory education only, compulsory but no post-secondary education and those with some post-secondary education. We use the (better known) US terminology though in this paper.
\[
\begin{align*}
\ln \tilde{c}_{it} &= \alpha_i + \gamma_1 t + \gamma_2 t^2 + u_{it} \\
u_{it} &= \rho^j u_{it-1} + \xi_{it} \\
\xi &\sim N \left(0, \sigma_j^2\right)
\end{align*}
\]

(1)

The coefficients of the earnings process depend on education.

### 3.1.3 Household heterogeneity

Households are ex-ante heterogeneous. They differ by their education. Let \( \tilde{j} \) index the combination of education levels of each member of a couple (and so takes one of nine values).\(^4\) They also differ in the fixed effect in their earnings process (\( \alpha_i \)) which they are assumed to know from the start of life. We also include in the vector of fixed effects the number of children in the household at each age (\( \{k_{it}\}_{t=20}^{100} \)). The implication of this assumption is that couples know with certainty from the age of 20 exactly how many children they will have and when those children will be born.\(^5\) In what follows, we summarize the ‘type’ of household \( i \) by \( \theta_i = (\tilde{j}_i, \alpha_i, \{k_{it}\}_{t=20}^{100}) \). Each household has its own type (\( \theta_i \)) which will be part of the state space, and so each household faces a different optimization problem which must be individually solved.

### 3.1.4 Assets

**Private pensions**  Households can, each period, pay a proportion of their pre-tax income into a Defined Contribution (DC, i.e., 401(k)-style) pension. These funds earn a risky rate of return. The stock of wealth held in DC funds is modeled as illiquid in two senses. First, the wealth cannot be accessed until age 65. Second, there is a compulsion in the model, as there was in reality for decades in the UK (until 2014), to use three quarters of the fund to purchase an annuity. We assume that annuitization happens at the age of 65. While the flow of income from

\(^4\)While we assume that the education of the husband only determines the earnings process, the education of both spouses will be relevant for survival probabilities (and therefore also for annuity pricing).

\(^5\)An alternative assumption – that children arrive probabilistically according to a process that depends on age (see Hong & Ríos-Rull (2012) & Jørgensen (2017)) – would add substantially to the computational cost (as we would have to add the number of children as a state variable and include an additional dimension of integration) and would, we conjecture, be no closer to reality than assuming that households know how many children they will have.
an annuity is taxed, the non-annuitized quarter of the DC fund can be taken in cash tax free (both in the model and in reality), giving this form of saving a degree of tax advantage.\(^6\)

The evolution of the stock of wealth in the DC fund \((DC)\) depends on flows into the fund \((dc)\) and the return on the fund in each year \((\phi)\).

\[
DC_{t+1} = (1 + \phi_{t+1})(DC_t + dc_t)
\]

The return on DC funds is assumed to be normally distributed with mean \(\bar{\phi}\) and variance \(\sigma_{\phi}^2\). Returns are truncated at -100\% and each household in a given year earns the same return.

Annuity rates \((q^\bar{j})\) are assumed to be actuarially fair after a proportion \((z)\) has been deducted to meet the administrative costs (including profits) associated with the provision of annuities. Survival probabilities will vary by education, therefore, the annuity rate depends on the education of each member of the couple \((\bar{j})\). Private pension income at each age post-retirement \((pp_t)\) is the annuity rate at age 64 (which yields an income flow starting from the age of 65) multiplied by the three quarters of the stock of wealth accrued by the end of age 64:

\[
pp_t = q^3_{64}(0.75)(DC_{64} + dc_{64})
\]

This reflects a timing convention that contributions to the pension are made at the end of the period and therefore those in the final year of potential employment (age 64) do not accrue a return and are not subject to risk.

**Non-pension wealth** Household resources that are neither used for consumption nor paid into a private pension are saved in non-pension wealth. The challenge here is to specify an asset that reflects the fact that many households hold a mix of cash and housing, while not taking these to be two separate state variables over the whole of the lifecycle.\(^7\) The return on these two forms of non-pension wealth are very different. Cash has a low rate of return, while the return

---

\(^6\)Pension saving is advantaged by the tax system to a greater extent in the UK than in the US. In both countries contributions are exempt from income tax, but in the US, contributions are subject to Social Security and Medicare taxes, while in the UK, contributions can be made exempt of the equivalent levies (National Insurance Contributions). As a further advantage to pension saving in the UK one quarter of withdrawals are exempt from income tax – while all withdrawals in the US are subject to federal income tax.

\(^7\)Computational constraints preclude us from incorporating housing wealth and cash separately early in life when the annual pension saving decision is being made – solving and estimating the model household-by-household with three different assets: cash, housing and pension wealth would not be possible. Our secure access data can only be used on desktop computers and, cannot, therefore be ported to High Powered Computers.
on housing has tended to be higher for many decades, and those higher returns have typically been magnified by leverage. Failing to take into account that much of wealth accumulation has been due to these returns by assuming that non-pension wealth evolves according to a standard intertemporal budget constraint with a risk-free interest rate would mean that estimates of patience would be biased substantially upwards.

Our treatment of non-pension wealth is different pre- and post-retirement. Taking pre-retirement first, we take non-pension wealth to comprise cash and housing. We do not model the choice of how much households save in each of cash and housing but specify that the shares held in each form (and therefore the overall return on non-pension wealth) is an exogenous function of age and total wealth holdings.\(^8\) We do this by defining two functions. The first is \(s(a)\) – the share of non-pension wealth held in \textit{net} housing – which we assume is a function of non-pension wealth. The second is \(lev(t)\) – the leverage ratio (the ratio of mortgage debt to gross housing wealth) – which we assume is a function of age. These functions will be estimated outside the model – details will be given in Section 4.

Using \(s(a)\), we can decompose non-pension wealth into cash (\(a^c_t\)) and net housing (\(nh_t\)):

\[
a^c_t = (1 - s(a_t))a_t
\]

\[
nh_t = s(a_t)a_t
\]

and using \(lev(t)\) we can further decompose net housing into gross housing (\(gh\)) and mortgage debt (\(mort\)) and therefore calculate the overall return on net housing (\(r^h(t)\)):

\[
r^h(t)nh_t = (r^{hcg} + r^{hr})\underbrace{1}_{gh_t}nh_t - r^{mort}\underbrace{lev(t)}_{mort_t}nh_t
\]

where, following Kaplan & Violante (2014) the return on gross housing wealth comprises a capital gain (\(r^{hcg}\)) and a service flow (\(r^{hr}\)). \(r^{mort}\) is the mortgage interest rate.

Cash assets earn a return \(r^c\) and therefore the overall return on non-pension wealth pre-retirement is:

\[
r(a_t, t) = s(a_t)r^h(t) + (1 - s(a_t))r^c
\]

\(^8\)Kaplan et al. (2016) adopt a similar approach in splitting wealth into equities and housing, although they make the additional assumption that the proportion held in each form of wealth does not vary with wealth.
The implication of these assumptions is that prior to the age of 65, we keep track of just one non-pension wealth state variable \(a_t\). The intertemporal budget constraint for the composite non-housing wealth variable before the age of 65 is given by:

\[
a_{t+1} = \left(1 + r(a_t, t)\right) \left(a_t + y_t - c_t - dc_t\right) \quad \forall \ t \leq 64
\] (4)

Non-pension wealth is assumed to be liquid pre-retirement.

Turning to the post-retirement period, from the age of 65, we treat cash and housing as separate state variables. Here, we first give the initial post-retirement (age 65) values of these state as a function of composite non-pension wealth holdings at age 64 \(a_{64}\), the last period before we split the assets, followed by the intertemporal budget constraints for the rest of retirement. Cash assets at the age of 65 are given by:

\[
a^c_{65} = \left(1 + r^c\right) \left((1 - s(a_{64}))a_{64} + y_{64} - c_{64}\right) + \left(0.25\right) \left(DC_{64} + dc_{64}\right)
\] (5)

where \((1 - s(a_{64}))a_{64}\) is cash at the age of 64, to which we must add savings \((y_{64} - c_{64})\), interest and the tax-free lump sum from the pension saving. In keeping track of housing wealth, we can choose to keep track of gross or net housing wealth as each is a deterministic function of the former and age (through the leverage ratio). We take the state variable here to be gross housing wealth at the age of 65:

\[
gh_{65} = (1 + r^{hc9}) \frac{1}{\left(1 - lev(64)\right)} \underbrace{s(a_{64})a_{64}}_{s(a_{64})a_{64}} \underbrace{gh_{64}}_{gh_{64}}
\] (6)

where \(s(a_{64})a_{64}\) is net housing at the age of 64, which must be converted into gross housing wealth, which in turn accrues a capital gain \(r^{hc9}\).

Through retirement, households continue to accrue a service flow from their owner-occupied housing, and must continue to pay their mortgage (until it is paid off at time \(t\) s.t. \(lev(t) = 0\)). That is they get income from housing of\(^9\):

\[
r^{hr}gh_t - \underbrace{r^{mort}lev(t)gh_t}_{mort_t}
\]

\(^9\)This income on net housing is rolled into the return on net housing pre retirement (see equation (2)). In the model exposition we treat it as income post-retirement – see equation (9).
We add one more assumption – the capital value of housing cannot be consumed.\(^\text{10}\)

Cash and gross housing evolve as follows from the age of 66 onwards:

\[
a_{t+1}^c = (1 + r^c)(a_t^c + y_t - c_t) \quad \forall \ t \geq 65
\]

\[(7)\]

\[
gh_{t+1} = (1 + r^{h_{eq}})gh_t
\]

\[(8)\]

The approach described above, while our preferred approach, makes a number of assumptions which are admittedly strong (for example, the fact that housing wealth accumulates continuously rather than discretely, and the change in the liquidity of housing wealth on retirement). Therefore, in addition to the results from this model, we also show results of two alternative methods for treating housing wealth. First, we make no account for housing wealth in the calculation of rates of return and in the formulation of the inter-temporal budget constraint, but still include housing wealth in the data used for estimation. This approach makes no allowances for the different rates of return on housing and cash, nor of the fact that housing tends to be leveraged; nevertheless the approach is relatively common (Scholz et al. (2006), French (2005), De Nardi et al. (2010)). We show our results under this specification (referring to it as a specification with ‘no housing treatment’). The second method is to deduct an estimate of housing costs (mortgage interest and capital) from income, and then to estimate parameters with non-housing wealth (following Cagetti (2003)). We give more details on this and show results in Appendix B.

### 3.1.5 Public pensions

The UK public pension system is described in Appendix A. Briefly, payments are substantially smaller than Social Security is in the US, and payments are only weakly related to earnings – as most of the payment is proportional only to the number of years in work, with earnings when in work not relevant.\(^\text{11}\)

\(^{10}\)In reality, the vast majority of households do not decumulate their housing wealth in the UK (Blundell, Crawford, French & Tetlow (2016)). In place of our simplifying assumption the observed behavior could be generated by a combination of a bequest motive, transaction costs associated with liquidating housing wealth and the service flow households receive by staying in one’s home.

\(^{11}\)To give a sense of the scale and variation in entitlements, the median public pension entitlement in our sample is approximately £7,800 with an the interquartile range stretching from £6,330 – £9,100 in 2002. Adjusting for prices to 2014 and converting to US dollars using the average exchange rate for that year yields $15,100, $18,600
We model households as receiving a public pension from the age of 65. Its level depends on the fixed effect in the earnings process, final pre-retirement productivity and education. Parameterization and estimation of that function is described in Section 4.1.

3.1.6 Taxes, transfers and net income

Taxes and other transfers are based on the UK tax system in 2002/2003 - the year represented by our main data source. The modeled components of the tax and benefit system are income tax, National Insurance (the UK equivalent of the US Payroll Tax), Jobseekers’ Allowance (a payment to the unemployed), Child Benefit and the Minimum Income Guarantee for pensioners. These are described in more detail in Appendix F. Household net income is a function ($\tau$):

$$y_t = \tau(e_t, a_t, a^c_t, gh_t, pp_t, sp_t, h_t, k_t, dc_t, t)$$ (9)

that depends on household earnings ($e_t$), non-pension wealth pre-retirement ($a_t$), cash and gross housing post-retirement ($a^c_t, gh_t$), private pension payments ($pp_t$), public (or ‘state’) pension payments ($sp_t$), number of adults still alive ($h_t$), number of dependent children ($k_t$), chosen contributions to the pension fund ($dc_t$) (since those attract tax relief) and finally, on the age of the household ($t$) (the UK tax system taxes the elderly to a lesser extent than those of working age).

3.1.7 Uncertainty

Four features of the household’s problem are uncertain. These are mortality, productivity, employment and the return on the pension fund. The probability distribution over mortality is summarized by the sequence of probabilities $\{s^j_{i,m}, s^j_{i,f}\}_{i=20}^{100}$ which gives the probability of surviving to age $t$, conditional on being alive at age $t-1$. Survival probabilities vary by gender and education. We assume no mortality before the age of 65. The joint distribution over earnings (summarizing realizations of both productivity and employment) and pension fund returns is $21,700 for the 25th, 50th and 75th percentiles. This compares to values (calculated using the HRS) of household Social Security Society income for a similarly selected sample in the US of $18,800, $26,360 and $33,400.

12It is worth noting that, given the weak relationship between lifetime average earnings and pension entitlement, we do not include in the state space a measure of lifetime earnings (such as Average Indexed Monthly Earnings) – included in many models that incorporate Social Security.
given by \( F(\phi_{t+1}, e_{t+1}|e_t) \). The stochastic processes underlying the two of these are mutually independent, though the distribution of \( e \) in period \( t + 1 \) depends on its value in period \( t \).

### 3.2 Household maximization problem and value functions

We can now outline the household maximization problem and value functions. The period of life where decisions are modeled starts at age 20, and all individuals are assumed to die by the age of 100 at the latest. We do not model marriage or divorce – couples are assumed to start their productive life already married and stay together until death. We now discuss in turn the optimization problem facing retired households and working age households.

**Retired household’s problem** Households in the model enter retirement at the age of 65. Retirement here is associated with two distinct events. The first is withdrawal from the labor market. The second is the conversion of three quarters of the stock of wealth held in the DC fund into a life annuity, with the remaining quarter taken as a tax-free lump sum. The state variables that summarize the household’s problem in retirement are age \((t)\), cash \((a)\), gross housing \((gh)\), private pension income \((pp)\), final pre-retirement productivity \((\tilde{e}_{64})\) and household composition \((h)\). This last variable takes a value of 1, 2, or 3 indicating, respectively, that both spouses are still alive, only the male is alive and only the female is alive. Households in retirement make a single choice in each period - their level of consumption \((c)\).

The problem facing retired households with both spouses alive at time \( t \) is therefore (Appendix G.1 gives the corresponding problem for households in which one spouse has died.):

\[
V_t(a_t^c, gh_t, pp_t, \tilde{e}_{64}, h_t = 1; \theta_t) = \max_{c_t} \left( u(c_t) + \beta_i s_{t+1}^{j,m} s_{t+1}^{j,f} V_{t+1}(a_{t+1}^c, gh_{t+1}, pp_{t+1}, \tilde{e}_{64}, h_{t+1} = 1; \theta_t) 
\right) 
\]

\[
+ \beta_i s_{t+1}^{j,m} (1 - s_{t+1}^{j,f}) V_{t+1}(a_{t+1}^c, gh_{t+1}, pp_{t+1}, \tilde{e}_{64}, h_{t+1} = 2; \theta_t)
\]

\[
+ \beta_i (1 - s_{t+1}^{j,m}) s_{t+1}^{j,f} V_{t+1}(a_{t+1}^c, gh_{t+1}, pp_{t+1}, \tilde{e}_{64}, h_{t+1} = 3; \theta_t)
\]

s.t. \( y_t = \tau(e_t, a_t^c, gh_t, pp_t, sp_t, h_t, k_t, dc_t, t) \)

and intertemporal budget constraints (5) to (8)
the maximization problem and decision rules, faced by each household differ is indicated by the inclusion of $\theta_i$ as an argument.

**Working age household’s problem** The state variables that summarize the household’s problem during working life are age ($t$), non-pension wealth ($a$), pension wealth ($DC$) and productivity ($\tilde{e}$). At each age during working life households make two choices: their level of consumption ($c_t$) and their payments into a DC fund ($dc_t$). The balance of their resources is saved in a risk-free asset. The problem facing working households (we assume no mortality during working life, so $h$ – household composition is equal to 1 for all periods) is:

$$V_t(a_t, DC_t, \tilde{e}_t; \theta_i) = \max_{c_t, dc_t} \left( u(c_t) + \beta_i \int V_{t+1}(a_{t+1}, DC_{t+1}, \tilde{e}_{t+1}; \theta_i) dF(\phi_{t+1}, e_{t+1}|e_t) \right)$$

subject to:

$$y_t = \tau(e_t, a_t, a^c_t, gh_t, pp_t, sp_t, h_t, k_t, dc_t, t)$$

$$a_{t+1} = (1 + r(a_t, t))(a_t + y_t - c_t - dc_t)$$

$$DC_{t+1} = (1 + \phi_{t+1})(DC_t + dc_t)$$

### 3.3 Model solution and the simulation of consumption and savings behavior

The model outlined contains two choice variables (consumption and pension saving), six state variables post-retirement (age, cash, housing wealth, pension income, final productivity and household composition) and four state variables pre-retirement (age, non-pension wealth, pension wealth and productivity). There is no analytical solution to the maximization problem. Full details on the methods used in solving the households’ problem and simulating their behavior are given in Appendix G. In short, we obtain decision rules numerically by solving the households’ problem, first for the final period of life, storing the value functions for this period and then iterating backwards. After having obtained decision rules (the pension saving function and consumption function), we simulate behavior (consumption and saving) using these and realizations for the stochastic variables. Following Scholz et al. (2006), instead of using draws from a pseudo-random number generator, we use actual realizations for the stochastic components drawn from the data on these households. That is, in each year in which we simulate behavior using the calculated decision rules, we use households’ actual employment status, realized earnings and the average return observed on DC assets in that year.
4 Estimation and parameterization

Estimation follows a two step process. In the first step, we set features of the economic environment and a sub-set of preference parameters with reference to the institutional structure or existing literature. In the second step, we estimate the earnings process and the distribution of discount rates. We discuss these steps in turn.

4.1 Step 1: Parameterization

This section discusses the model parameterization – first, the parameters that set the economic conditions (rates of return, policy environment) faced by our cohort of interest, and second, demographic and preference parameters.

4.1.1 Economic environment

Return on cash  The rate of return on cash is set at the average real return on cash balances, which was 1.6% between 1952 and 2012 (see Table 1 of Barclays Capital (2012)).

Share of non-pension wealth in housing  To calculate \( s(a_t) \) – the share of non-pension wealth held in housing – we use data from the Wealth and Assets Survey which is a dedicated wealth survey of the British population that started in 2006. We regress the proportion of non-pension wealth held in housing wealth on a quadratic in total wealth. We also include cohort dummies and a set of time dummies constrained to sum to zero (a normalization suggested by Deaton & Paxson (1994) given the identification problem implied by the collinearity of age, period and cohort). We do this only for those who own a house (recall that 90% of the households in our sample are home-owners). The modeled relationship is shown in Figure 16 in Appendix E – the share of wealth held in housing is high for those with the lowest wealth levels (approximately 0.9), falling to 0.65 for those with £1m of non-pension wealth.

Return on housing  To specify the return on housing (given in equation (2)) we need to specify two rates of return \((r_{hc} \text{ – the housing capital gain and } r_{hr} \text{ – the rental income from owner occupied housing})\), the mortgage interest rate \((r_{mort})\) and the leverage ratio \((lev(t))\).

In setting rates of return we follow the broad approach outlined by Kaplan & Violante (2014). The average real capital gain over the period 1976 to 2002 – the first year that we have data to the year in which our sample of households are observed – was 3.2% (Nationwide Building
Society (2014)). Our model abstracts from housing risk, so we follow Kaplan & Violante (2014) by subtracting the variance of returns, giving a real capital gain of 2.28%. The service flow from housing is calculated using the ratio of the estimate of aggregate housing consumption to the estimate of the value of the housing stock in the National Income and Product Accounts. This yields an average of 8.4%. We risk-adjust by subtracting the variance of returns and also subtract depreciation (1%, using data from the National Income and Product Accounts) and insurance costs (0.35%, taken from Kaplan & Violante (2014)). This yields a return of 7.02%.\(^{13}\)

The mortgage rate is set at 5.9% and is calculated using data from the Bank of England (Bank of England (2017)). This is the average rate over time and over all products where historical data is available. Finally, the leverage ratio \(\text{lev}(t)\) is set in a manner similar to the housing share. Using the Wealth and Asset Survey we regress the observed leverage ratio on a quadratic in age, as well as cohort and time dummies. The estimated function is given in Figure 17 in Appendix E.

**Pension fund returns** We base the mean and standard deviation of pension fund returns on an index known as the “DCisions index”. This is an index of total fund returns that reflects the asset allocation decisions made by leading DC pension plans in their default investment strategies. This index provides information on returns stretching back to 1994. For years prior to 1994 when the DCisions index is not available, we estimate \(\phi_t\) using the FTSE all-share index (on which data is available back to the early 1960s) and the ratio between the FTSE all-share index and the DCisions index over the period where both are available (1994 - 2010). We discuss how this is estimated in Appendix H. We use the mean and standard deviation of this time series in our model. These parameters are, respectively, \(\bar{\phi} = 3.97\%\) and \(\sigma_\phi = 13.8\%\).

**Unemployment rate** The period in our model is a year. We consider a household in the data to be unemployed for a year if they have total earnings of less than £4,402 - the level of unemployment benefit payable to an unemployed couple in 2002/03. In our data, the incidence of unemployment, so defined, between the ages of 25 and 50 is 8.4%, 6.5% and 6.4% for our low-, medium- and high-education groups respectively. These are the unemployment probabilities

\(^{13}\)This is the consumption value from having access to owner-occupied housing. We assume that this value does not increase as the value of a given quantity of housing increases. Therefore, households earn this 7.02% return on the value of their current stock of gross housing in 2002 housing prices (all other quantities in the model are also expressed in prices of this year). We convert housing wealth in other years to these prices by deflating (inflating) values in years after (before) that year by \(r^{h,c}\) per year.
used in the model.

**Public Pension**  We model public pension entitlements as a quadratic in decile of fixed effect and decile of ‘final earnings’ where the coefficients are allowed to vary by education. Final earnings in the data are measured as the average decile of the last five years of observed earnings. For estimating this process we only use data on those aged over 60 (for whom public pension entitlements are largely determined).

Figure 1 illustrates the relationship between modeled public pension entitlements and each of final earnings and earnings fixed effect. In the left hand panel we give predicted public pension as final earnings decile varies (where for each earnings decile we set the fixed effect decile equal to its average for that group). In the right hand panel we give public pension as fixed effect decile increases (with final earnings similarly set at its group-specific mean). In the case of both state variables, there is only a slight gradient with respect to decile.

![Figure 1: Public pension process](image)

**Annuity rates**  The actuarially fair annuity rates are calculated using survival probabilities (the estimation of which are described below) and the risk-free interest rate. The administrative load is assumed to be 10% of the value of the DC fund to be annuitized. This is taken from Murthi et al. (1999) who apply the methodology of Mitchell et al. (1999) to the UK. We give the annuity rates in Appendix I (with the formula given in Section 3 of the online supplemental material accompanying this paper).
4.1.2 Demographic and preference parameters

Survival probabilities  Our objective is to have survival curves for each gender and education group for our cohort of interest. The UK’s Office for National Statistics (ONS) produces \textit{period} and \textit{cohort} survival curves by gender but not by education. Period survival curves give survival probabilities by age given the age-specific mortality rates at a particular point in time – they therefore make no allowance for any later actual or projected changes in mortality. Cohort survival curves give survival probabilities by age for a given cohort and allow age-specific mortality rates to vary for known or projected changes in mortality over time. Cohort survival curves are thus more appropriate for our purposes.

We use observed deaths in the ELSA panel between 2002/03 and 2012/13 to estimate a set of \textit{period} survival curves by gender and education. Using these we calculate the difference between these education-age-gender period survival probabilities and the ONS’ age-gender period survival probabilities. We then adjust the ONS’ age-gender cohort survival curves (for the 1945 cohort) using these differences to obtain education-age-gender cohort survival probabilities.

Equivalence scale  The number of equivalent adults in a household $n$ is set using the ‘modified OECD equivalence scale’ (see Anyaegbu (2010) for a discussion). The first adult in a household counts for one equivalent adult, subsequent adults and children aged 14 and over count for half an equivalent adult, while children aged 13 or younger account for 30\% of an equivalent adult.

Coefficient of relative risk aversion  The coefficient of relative risk aversion has proved a difficult coefficient to identify.\textsuperscript{14} Our approach is to set the coefficient of relative risk aversion to 3, following the most closely-related work (Scholz et al. (2006)).

Summary  These parameters of the model and values assigned are summarized in Table 3.

4.2 Step 2: Estimation

4.2.1 Estimation of earnings processes

To estimate the parameters of the earnings process we aggregate individual earnings histories into household earnings histories. We then divide households into three groups according to

\textsuperscript{14}See Chiappori & Paiella (2011) for a review and recent empirical contribution.
### Table 3: Parameterization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate</td>
<td>{\pi}_{j=1}^3</td>
<td>8.6%, 6.5%, 6.4%</td>
</tr>
<tr>
<td>Return on cash</td>
<td>r^c</td>
<td>1.6%</td>
</tr>
<tr>
<td>Housing capital gain</td>
<td>r^{h_{cg}}</td>
<td>2.3%</td>
</tr>
<tr>
<td>Housing rental yield</td>
<td>r^{h_{hr}}</td>
<td>7.0%</td>
</tr>
<tr>
<td>Mortgage interest rate</td>
<td>r^{mort}</td>
<td>5.9%</td>
</tr>
<tr>
<td>Mean return on DC fund</td>
<td>\bar{\phi}</td>
<td>4.0%</td>
</tr>
<tr>
<td>Variance of return on DC fund</td>
<td>\sigma_2^\phi</td>
<td>13.8%</td>
</tr>
<tr>
<td>Survival probabilities</td>
<td>\delta_j^{j,m}, \delta_j^{j,f}</td>
<td>ONS Life Tables adjusted for education using survival differences by education observed in ELSA</td>
</tr>
<tr>
<td>Administrative load on annuities</td>
<td>z</td>
<td>10%</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion</td>
<td>\gamma</td>
<td>3</td>
</tr>
<tr>
<td>Equivalence scale</td>
<td>n</td>
<td>Modified OECD scale</td>
</tr>
</tbody>
</table>

the education of the man in the couple (indexed by \(j\)). The parameters to be estimated are \(\{\alpha_i\}_{i=1}^N\) and \(\{\delta_1^j, \delta_2^j, \rho^j, \sigma_2^j\}_{j=1}^3\).

To allow for measurement error in earnings, we augment the earnings process given in equation (1) with an \textit{iid} measurement error term \(m_{it}\). The assumed data generating process for our earnings data is given in equations (11) to (13):

\[
\begin{align*}
\ln \tilde{e}_{it}^{\text{data}} &= \alpha_i + \delta_1^j t_i + \delta_2^j t_i^2 + v_{it} \\
v_{it} &= u_{it} + m_{it} \\
u_{it} &= \rho^j u_{it-1} + \xi_{it}
\end{align*}
\]

(11) (12) (13)

The approach to estimation is a standard one (see, for example, Low et al. (2010)). It involves first running a fixed effects regression and estimating the household fixed effect and quadratic in age. Residuals \(r\) are then obtained:

\[r_{it} = \ln \tilde{e}_{it}^{\text{data}} - \hat{\alpha}_i - \hat{\delta}_1^j t_i - \hat{\delta}_2^j t_i^2\]

The parameters of the wage process are obtained by choosing those that minimize the distance between the empirical covariance matrix of differences in these residuals and the theoretical
covariance matrix implied by equations (11) to (13). Estimates of the parameters of the earnings process for each of three education groups are given in Table 15 in Appendix I.

4.3 Estimation of discount factor

Conditional on a discount factor, the model solution gives decision rules for each household. We can use these rules, combined with realized earnings and investment returns that they receive to simulate behavior (consumption, savings in each asset and therefore total wealth \( A = a + DC \)) at each age:

\[
A^\text{sim}_{it} = f_i \left( \beta_i, \theta_i, \{e_{i,\tau}\}_{\tau=1}^{t} \right) \tag{14}
\]

where \( \beta_i \) is patience, \( \theta_i \) is household type and \( \{e_{i,\tau}\}_{\tau=1}^{t} \) contains earnings shocks up to \( t \). The discount factor estimate is that for which simulated wealth and observed wealth coincide in 2002 (when wealth is observed).

\[
A^\text{data}_{i,2002} = A^\text{sim}_{i,2002} = f_i \left( \hat{\beta}_i, \theta_i, \{e_{i,\tau}\}_{\tau=1}^{2002} \right) \tag{15}
\]

We calculate a discount factor for all households whose wealth can be rationalized with a discount factor in the range 0.5 to 1.5. 97.3% of households in our sample have wealth levels that can be rationalized with patience in this range (and 96.0% in the narrower range of 0.8 to 1.2).

5 Results

The model laid out in the previous section has three purposes. First, it will be used to estimate the distribution of discount factors which would rationalize observed household wealth holdings. Second, it will be used to understand the importance of accounting for household portfolios in models of saving for retirement. Third, it will be used to document a link between measures of preparedness for retirement used by policy-makers (replacement rates) and discount factors, the preference parameter which bears most heavily on the preparedness for retirement of agents in lifecycle models.
5.1 Estimates of discount factors

Table 4 summarizes the distribution of discount factors for each of three versions of our model. The first row, which we give as a benchmark, gives estimates derived from a simple one-asset lifecycle model where all wealth accrues the risk-free rate (1.6%) as a return. The median discount factor is 1.042 with an interquartile range stretching from 1.011 to 1.080. These rates are extremely high relative to those which have been found in the literature. The second row adds the capacity for agents to save in an illiquid tax-advantaged pension asset, which earns an expected return that is higher than the safe return but is risky. All of non-pension wealth still earns the safe rate of return. The median estimated discount factor falls to 0.999. Additionally the distribution shows less variation, with an interquartile range of 0.976 to 1.021, as part of heterogeneity in wealth is explained by differential (across households of different income and with different fertility profiles) incentives to avail of the tax breaks associated with pension saving.

Neither model takes into account that much of non-pension wealth is held in housing. The third row gives estimates from our preferred version of our model that, in addition to including the pension asset, also calibrates the return on non-pension wealth to take into account housing (as discussed in Section 3.1.4). The median discount factor is 0.950, with an interquartile range of 0.923 to 0.975. Figure 2 illustrates the three distributions summarized in Table 4.

5.1.1 Validation of discount factor estimates

The discount factors we estimate are identified by fitting total private wealth simulated by our model to observed total private wealth holdings. One test of the model is therefore how holdings of individual components of wealth fit the data. Figure 3 illustrates how modeled non-pension wealth (left-hand panel) and modeled pension wealth (right-hand panel) compare to that observed in the data for each household. Non-pension wealth (which includes housing and non-housing assets) is strongly clustered around the 45-degree line - the correlation coefficient
between modeled and observed non-pension wealth is 0.8577. Pension wealth is more clustered around low levels, but is also strongly correlated, with a correlation coefficient of 0.7282. Table 5 shows the mean holdings of each of pension and non-pension wealth in the data and the mean holdings implied by the model by decile of lifetime earnings. Figure 9c in Appendix B uses the quantities in this table to graph the ratio of mean pension wealth to mean total wealth – showing that, in addition to matching the level, the model also replicates the increase in the pension share by lifetime wealth. The model therefore performs well in terms of fitting household portfolio choices.

An additional test of the validity of our estimates is that we can use the breadth of data
Table 5: Observed and modeled wealth, Means (£,000s)

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Modeled</td>
<td>Pension</td>
<td>Non-pension</td>
</tr>
<tr>
<td></td>
<td>Data</td>
<td>Modeled</td>
<td>Data</td>
<td>Modeled</td>
</tr>
<tr>
<td>All</td>
<td>387.4</td>
<td>387.3</td>
<td>142.9</td>
<td>120.8</td>
</tr>
<tr>
<td>Lifetime earnings decile:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lowest</td>
<td>172.8</td>
<td>172.9</td>
<td>41.2</td>
<td>22.6</td>
</tr>
<tr>
<td>2</td>
<td>218.9</td>
<td>218.6</td>
<td>66.3</td>
<td>45.3</td>
</tr>
<tr>
<td>3</td>
<td>240.1</td>
<td>240.0</td>
<td>78.5</td>
<td>50.3</td>
</tr>
<tr>
<td>4</td>
<td>277.4</td>
<td>277.0</td>
<td>94.9</td>
<td>72.7</td>
</tr>
<tr>
<td>5</td>
<td>296.5</td>
<td>296.2</td>
<td>110.6</td>
<td>75.2</td>
</tr>
<tr>
<td>6</td>
<td>343.7</td>
<td>343.3</td>
<td>121.2</td>
<td>95.1</td>
</tr>
<tr>
<td>7</td>
<td>376.5</td>
<td>376.6</td>
<td>132.3</td>
<td>104.5</td>
</tr>
<tr>
<td>8</td>
<td>413.4</td>
<td>413.3</td>
<td>148.7</td>
<td>126.3</td>
</tr>
<tr>
<td>9</td>
<td>619.3</td>
<td>619.2</td>
<td>241.3</td>
<td>216.8</td>
</tr>
<tr>
<td>Highest</td>
<td>881.7</td>
<td>882.6</td>
<td>378.3</td>
<td>384.6</td>
</tr>
</tbody>
</table>

The totals in the data columns differ slightly from those Table 2 as the sample used here are only those for whom we can rationalize observed wealth holdings with a discount factor between 0.5 and 1.5.
collected in ELSA to validate the estimated discount factors using information that is not used at all in the modeling process. More specifically, we can examine how the estimated discount factors vary with individual characteristics thought to correlate with, or be indicative of, individuals’ time preference. We examine three particular characteristics: education, self-reported financial planning horizon, and current and former smoking behavior.

The left-hand panel of Figure 4 illustrates how the distribution of estimated discount factors varies for our three education groups. The distribution of those with lower education lies to the left of those with more education. This is consistent with the bulk of existing literature on education and discount factors (for example, Lawrance (1991), Cagetti (2003), Dohmen et al. (2010)).

The right-hand panel of Figure 4 illustrates how the distribution of estimated discount factors varies according to individuals' answers to the question “In deciding how much of your income to spend or save, people are likely to think about different financial planning periods. In planning your family’s saving and spending, which of the following time periods is more important to you?” The options given to respondents are: the next few weeks, the next few months, the next year, the next few years, the next 5-10 years, longer than 10 years. For ease of illustration we group individuals into those who respond with a period up to ‘the next year’ and those who respond with a period ‘the next few years’ or longer (a finer categorization is used later in Table 6. Those reporting longer planning horizons are estimated to have higher discount factors. This is strong validation that at least some of the heterogeneity in discount factors estimated through the modeling process is indeed reflective of heterogeneity in individuals’ preferences. Samwick (1998) shows the relationship between a similar question and his estimates of discount rates and also finds that the estimated and stated measures of patience correlate.

In Table 6, we report the results of median regression analysis examining the correlation between estimated discount factors and individuals’ self-reported planning horizon, education level, smoking behavior and these three characteristics simultaneously. These confirm statistically significant differences in median discount rates between the groups illustrated in Figure 4 and also show that estimated discount factors are significantly negatively correlated with current smoking behavior (see Khwaja et al. (2007) which documents, in a different setting, a correlation between some measures of time preference and smoking behavior). We also find that the correlation between discount factors and each characteristic holds up even when the others are being controlled for. We take this as evidence that the distribution of discount factors we have estimated is indeed reflective of heterogeneity in time preferences.
Table 6: Median regressions of estimated discount factor on household characteristics

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planning (weeks)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Planning (up to a year)</td>
<td>0.017***</td>
<td>0.011**</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Planning (a few years)</td>
<td>0.033***</td>
<td>0.019***</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Planning (5+ years)</td>
<td>0.036***</td>
<td>0.023***</td>
<td>(0.005)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Low educ.</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Mid educ.</td>
<td>0.030***</td>
<td>0.027***</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>High educ.</td>
<td>0.040***</td>
<td>0.033***</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Never smoked</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Former smoker</td>
<td>-0.007</td>
<td>-0.004</td>
<td>(0.004)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Smoker</td>
<td>-0.018***</td>
<td>-0.007*</td>
<td>(0.005)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.923***</td>
<td>0.932***</td>
<td>0.956***</td>
<td>0.921***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Observations</td>
<td>954</td>
<td>943</td>
<td>964</td>
<td>930</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1
5.2 Household portfolios and saving for retirement

5.2.1 The role of pensions

The previous section illustrated the importance of the asset structure of our lifecycle model for the estimated distribution of discount factors. To examine the role of household portfolios in more detail, we can use our model to evaluate what would happen, at our estimated discount factors, if some of the features of the asset structure we have added were removed. Table 7 shows how the distribution of each of total private wealth and its two components, pension wealth and non-pension wealth, vary with the characteristics of the model’s asset structure. We show four scenarios:

1. Our baseline

2. Removing the pension tax advantage. Pension saving is now done out of net income, and yields a non-taxable annuity in retirement (rather than being done out of gross income and yielding a taxable annuity). The pension remains risky and earns a higher expected return than cash.

3. Removing the pension asset entirely (and so retirement funds are all saved in the composite non-pension wealth asset).

4. Additionally, removing access to housing (and so all wealth is held in cash)

Comparing the baseline with the third row of the top panel shows that removal of the pension would, holding the distribution of discount factors constant, reduce mean accumulated
wealth from approximately £451,000 to approximately £353,000. The proportionate reduction is greater at the top of the wealth distribution (where the fall was approximately 20% at the 90th percentile) than at the middle and bottom (where the fall was just over 10% at the median and just under 10% at the 10th percentile). The result that, conditional on a set of discount factors, wealth accumulation would be less in the absence of access to the pension is isomorphic to the result presented in Section 5.1 that estimated discount factors would be higher if no access to the pension was assumed.

Pension wealth differs from non-pension wealth in two respects that are quantitatively important for increasing saving. First it is treated advantageously by the tax system. Second, it has a higher expected return than the cash portion of non-pension wealth. The second row of each panel in Table 7 allows the effects of these two aspects to be disentangled. Of the approximately 22% fall in mean wealth with the removal of the pension asset (from £450,900 to £353,600), two-fifths is due to the favorable tax treatment (removal of which would lead to a fall from in mean wealth from £450,900 to £411,200); with the remaining three-fifth due to the returns available from the risky asset.

Finally, removing access to housing (which forces all of non-pension wealth to be held in cash and so substantially reduces the rate of return on offer) dramatically reduces wealth further. Mean wealth would fall to £120,700 and private wealth in the bottom 10 percent is very close to zero. Those at the bottom of the lifetime earnings distribution have reasonable earnings replacement from the public pension as well as an asset-tested transfer that essentially taxes wealth accumulation over a certain range. Both of these factors mean that they have limited incentives to save for retirement; these incentives are further reduced in the counterfactuals studied here.

We can use these counterfactual estimates to place our results in the context of the literature which estimates the extent to which household responses to pension tax incentives represent new saving or whether households respond to them by saving more in pension wealth at the expense of other forms of wealth. Poterba et al. (1995) and, more recently, Gelber (2011) argued that 401(k) saving is largely ‘new’ saving that would not have been saved in the absence of the savings vehicle being offered; while Engen et al. (1994) and, more recently, Chetty et al. (2014) argued that subsidies largely increase pension wealth at the expense of other forms of wealth. Our results are intermediate to these sets of studies – the approximately £75,000 fall in

\[15\] In our setting the pension asset also gives access to an annuity though the effect of this on wealth accumulation is small.
pension wealth as a result of removing tax advantages (from £192,100 to £117,200) is offset by an increase in non-pension wealth of approximately £35,000). Additionally, the fall in pension income will be (proportionately) less than the fall in pension wealth due to the fact that the figure after the removal of the tax advantages represents already-taxed income. Therefore, our estimated model implies that tax incentives toward pension saving incentivize households to switch from non-pension wealth to pension wealth to a greater extent than incentivizing them to do new saving (Engen et al. (1994), Chetty et al. (2014)).

Table 7: Distribution of modeled wealth: pension tax advantage counterfactual analysis at age 64

<table>
<thead>
<tr>
<th></th>
<th>p10</th>
<th>p25</th>
<th>Median</th>
<th>p75</th>
<th>p90</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total wealth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline</td>
<td>85.0</td>
<td>185.6</td>
<td>334.6</td>
<td>578.5</td>
<td>883.6</td>
<td>450.9</td>
</tr>
<tr>
<td>No pension tax adv.</td>
<td>81.2</td>
<td>172.9</td>
<td>315.8</td>
<td>522.7</td>
<td>797.9</td>
<td>411.2</td>
</tr>
<tr>
<td>No pension asset</td>
<td>77.8</td>
<td>164.5</td>
<td>295.3</td>
<td>480.1</td>
<td>711.3</td>
<td>353.6</td>
</tr>
<tr>
<td>No pension and no housing</td>
<td>0.5</td>
<td>25.4</td>
<td>80.2</td>
<td>168.0</td>
<td>279.1</td>
<td>120.7</td>
</tr>
<tr>
<td><strong>Pension wealth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline</td>
<td>14.7</td>
<td>49.9</td>
<td>118.0</td>
<td>240.2</td>
<td>394.7</td>
<td>192.1</td>
</tr>
<tr>
<td>No pension tax adv.</td>
<td>0.0</td>
<td>19.6</td>
<td>60.7</td>
<td>137.0</td>
<td>237.4</td>
<td>117.2</td>
</tr>
<tr>
<td>No pension asset</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>No pension and no housing</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>Non-pension wealth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline</td>
<td>59.1</td>
<td>126.8</td>
<td>223.8</td>
<td>350.7</td>
<td>508.2</td>
<td>258.8</td>
</tr>
<tr>
<td>No pension tax adv.</td>
<td>78.9</td>
<td>148.1</td>
<td>243.6</td>
<td>394.7</td>
<td>568.6</td>
<td>294.0</td>
</tr>
<tr>
<td>No pension asset</td>
<td>77.8</td>
<td>164.5</td>
<td>295.3</td>
<td>480.1</td>
<td>711.3</td>
<td>353.6</td>
</tr>
<tr>
<td>No pension and no housing</td>
<td>0.5</td>
<td>25.4</td>
<td>80.2</td>
<td>168.0</td>
<td>279.1</td>
<td>120.7</td>
</tr>
</tbody>
</table>

5.2.2 The potential role of portfolios in previous estimates of ‘oversaving’

Scholz et al. (2006) used similar microdata to that which we use to compare the retirement saving of a now retired cohort to a benchmark (which they refer to as ‘optimal’ wealth) which is pinned down by a single-asset lifecycle model with a homogeneous discount rate assumed to be approximately equal to the interest rate.\footnote{The interest rate assumed was 4\%, with a discount factor of 0.96.} They find that that a large majority of households
(almost 85%) had accumulated more than this benchmark – that is that there was little evidence of households having undersaved for retirement.

In this section, we implement a similar exercise to theirs. Rather than allow an estimated discount factor to explain the heterogeneity in wealth accumulation as we do in the rest of this paper, we set the discount rate for all households equal to the risk-free interest rate (1.6%, and so $\beta = 1.016^{-1}$), and evaluate observed wealth relative to a benchmark level produced by the resulting parameterized model.

We first do this using a one-asset model in which households can only save in cash. Notwithstanding the fact that our cohort is a different one and our setting is the UK (where the public pension system differs – especially for higher earners) we get a very similar result: 87.8% of households have ‘oversaved’ relative to a benchmark where the discount rate is set equal to interest rate. Additionally, as Scholz et al. found, the deficits among the undersavers are much smaller than the surpluses among the oversavers – among the 12% of households who have undersaved, the median deficit is £31,500, while the median surplus among the oversavers is about £185,000.

In generating this result, we have followed Scholz et al. in setting the discount rate equal to the (risk-free) interest rate in determining the benchmark against which accumulated wealth will be compared. Adding any feature to the model which increases the return on saving (either by introducing an asset that tends to have a higher return, or by introducing an asset which has tax breaks associated with it) will, all else equal, increase the benchmark required to have greater than ‘optimal’ saving, and so reduce the proportion of households found to be oversaving. Scholz et al. anticipate that portfolio structure is a contributory factor in generating their result – when they increase the rate of return to 7%, the rate of oversaving falls to approximately 64.1%, and when they (separately) exclude half of property wealth from their data, the rate of oversaving falls to 61.2%. We showed in the previous section the effect that adding our additional assets would have on saving, conditional on estimated discount factors; here, we compare here how adding our additional features to the portfolio structure affects adjudications of over-saving relative to a model that is parameterized with a fixed homogeneous discount factor.

There are some other differences between our model and that of Scholz et al. (2006). First they assume Defined Benefit wealth is exogenous – we treat it as if it has accumulated as DC wealth – though we show in Appendix C that our conclusions are essentially unchanged if we treated it as exogenous. Second, reflecting institutional differences between the US and the UK, they additionally model the risk of out-of-pocket medical expenses in retirement.
Figure 5 and Table 8 summarize saving, relative to the benchmark, for three models: i) the one-asset model, in which all wealth accrues a return equal to the discount rate, ii) a two-asset model, which adds the tax-advantaged private pension, and iii) our preferred model, in which the return on non-pension wealth is set to take into account the high (and typically leveraged) returns that housing has accrued. Figure 5 shows a scatter of observed and ‘optimal’ wealth for each model. Table 8 shows median wealth (data and optimal), the percentage oversaving, conditional deficits and surpluses and the $R^2$ from a regression of observed wealth on predicted wealth and a constant.\(^{18}\) The proportion oversaving falls from 87.8% to 65.4% when the private pension is added (as each of higher expected returns and the tax incentives increase the benchmark against which we compare optimal wealth). Taking account of potential housing returns reduces this to 18.1%, with the median deficit now larger than the median surplus.

<table>
<thead>
<tr>
<th>Spec</th>
<th>Median wealth</th>
<th>Percent oversaving</th>
<th>Median surplus</th>
<th>Median deficit</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Optimal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One asset</td>
<td>280.7</td>
<td>117.9</td>
<td>87.8</td>
<td>185.0</td>
<td>31.5</td>
</tr>
<tr>
<td>Simple two asset</td>
<td>280.7</td>
<td>203.8</td>
<td>65.4</td>
<td>133.2</td>
<td>65.6</td>
</tr>
<tr>
<td>Modeled housing</td>
<td>280.7</td>
<td>482.9</td>
<td>18.1</td>
<td>198.0</td>
<td>238.2</td>
</tr>
</tbody>
</table>

From this we take that the modeled portfolio structure matters substantially in exercises that use lifecycle models to derive benchmarks against which to evaluate household saving. For two reasons, we do not, however, take these results to challenge the central contention of Scholz et al. (2006) – that their cohort of interest have, for the most part, prepared well for retirement.

\(^{18}\)Our models explain less of the variation compared to Scholz et al. (2006), who report an $R^2$ of 86%.
First, housing wealth has appreciated to a much greater extent in the UK than in the US and so the effect of adding housing might be more modest in their context. Second, while enriching the model’s asset structure to account for incentives to save more for retirement raises the bar against which realized wealth will be assessed, the fact that they set the discount rate equal to the model’s interest rate (reflected in our setting by setting the discount rate equal to the risk-free interest rate) makes the bar a very demanding one – households are evaluated against a benchmark in which they are no more impatient than they are compensated for by risk free returns. We show in the next section that most households who are more impatient that this still replace a reasonable share of their lifetime average earnings in retirement.

The fact that most of our households are found, in our baseline, to have undersaved relative to the modeled benchmark is isomorphic to the fact that we estimate that most households have an estimated discount factor of less than 0.984 (that used to derive the benchmark). In the next section we compare those estimated discount factors to another benchmark used to evaluate whether households have saved appropriately for retirement – replacement rates.

5.3 Household preparedness for retirement: comparing replacement rates and patience estimates

The rate of replacement of lifetime average or final earnings by pension income in retirement is a metric often used by policy-makers and private pension providers to make assessments of individuals’ preparedness for retirement. There are strong theoretical arguments why optimal saving behavior would typically lead to replacement rates of less than 100%: one no longer needs to save for retirement, children have become more self-sufficient and average tax rates on retirement income tend to be lower than on income during working life. However, the actual replacement rate desired varies across individuals, depending on their circumstances and preferences. This means that the benchmark target thresholds used to assess savings adequacy by policy makers and the industry are at best necessarily ad hoc and at worst ‘conceptually flawed’ (Scholz & Seshadri (2009)). However, because of their simplicity and a lack of viable alternatives, replacement rates continue to be extensively used as a basis for which to organize retirement planning advice (see TIAA (2018) and online calculators by many other providers of retirement saving products) and as an official basis for evaluating the retirement preparedness of those approaching retirement (Bridges, Benjamin and Choudhury, Sharmila (2005), Pension Commission (2004)).
The data we use in this paper, on lifetime earnings and late life wealth holdings, is ideally suited to examining the likely retirement income replacement rates of our cohort of couples. To do so we project forward current public and private pension entitlements, assuming that individuals maintain their current labor market activity and private pension savings behavior from when they are observed in 2002/03 to when they reach age 65. We can then compare gross pension income (we consider broader measures of retirement income below) at age 65 with measures of average gross working life earnings.

The distribution of two definitions of replacement rates is shown in Figure 6. The solid line shows the distribution of pension income at age 65 relative to average (real) working life earnings between ages 20 and 64. Replacement rates, even on this narrow measure (which only include public and private pension income in the numerator), tend to be high, with a median of 84.5% and only about a third of households having pension income in retirement of less than 70% of their average lifetime earnings. This indicates that households in our sample appear well prepared for retirement, in the sense that they will be little worse off in retirement compared to the average of their working life.

How can we rationalize this with the popular and policy concern about systematic undersaving? At least in part this is likely to be caused by industry focusing on households being able to have similar levels of net income in retirement to what they had towards the end of working life. The dashed line in Figure 6 shows the distribution of total pension income at age 65 relative to average earnings in the best five years of working life. We use this denominator to proxy for ‘final earnings’ since we do not observe hours worked in our data, and we do not want a measure of final earnings that is biased downwards by individuals reducing their hours as they move into retirement. Since this denominator is obviously higher than average working life earnings, the resulting replacement rates are lower, with a median of just below 50%. On this definition over four-fifths of households would have a replacement rate of less than 70%, more consistent with the general consensus of concern.

It is questionable though, whether encouraging households to aim to replace such proportions out of their final earnings is sensible – this is essentially encouraging all households to act in the manner that only those who are the most patient act. Table 9 illustrates this. It shows median replacement rates, by decile of estimated discount factor on three definitions of retirement income: pension income only, pension income plus the annuitized value of liquid wealth (that measure of wealth that was described as ‘cash’ in the model), and pension income plus the annuitized value of all wealth (‘cash’ plus housing). The panel on the left shows median
replacement rates out of average lifetime earnings; that on the right shows replacement rates out of the best five years earnings.

Three points are apparent from the table. The first, which is simply an implication of the manner in which the discount factors are estimated, is that replacement rate rise with decile of estimated discount factor. Second, the replacement rates out of lifetime earnings are reasonable for even for those who are estimated as impatient. Even those in the lowest decile of patience ($\beta < 0.89$) have a median replacement rates of 61%, 63% and 73% on, respectively, the narrowest, middle and broadest measure of retirement income. On the middle definition, by the second decile the median replacement rate is above 70%; by the sixth decile it is above 100% – emphasizing once again that this is a cohort that will not have systematically lower living standards in retirement than over the course of their working life.

Third, most of the distribution of replacement rates out of the best five years earnings would not meet the thresholds suggested by policy-makers and industry even among households who are very patient. On the measure of pension income which includes annuitized liquid wealth but not housing, the median replacement rate is less than 50% for the bottom three deciles, and it does not reach 70% until the ninth decile. This suggests that policymakers and industry should be cautious of their common use of standard replacement rates applied to measures of later-life or final earnings. Revealed behavior indicates that households, across most of the distribution of patience, aim for much lower replacement rates. Attempting to shift most households towards providing a replacement of the order of 70% or so applied to measures or later-life or final earnings, essentially involves attempting to make them act as if they are much more patient that they actually are and even if successful, risks imposing welfare costs early in life.
Table 9: Median income replacement rates by estimated discount factors

<table>
<thead>
<tr>
<th>Decile of estimated discount factor:</th>
<th>Replacement of average lifetime earnings</th>
<th>Replacement of average best 5yrs earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2) (3)</td>
<td>(1) (2) (3)</td>
</tr>
<tr>
<td>All</td>
<td>0.845 0.978 1.373</td>
<td>0.498 0.582 0.786</td>
</tr>
<tr>
<td>1 ([\beta \leq 0.89])</td>
<td>0.610 0.630 0.732</td>
<td>0.367 0.385 0.424</td>
</tr>
<tr>
<td>2 ([0.89 &lt; \beta \leq 0.92])</td>
<td>0.701 0.730 0.924</td>
<td>0.422 0.446 0.572</td>
</tr>
<tr>
<td>3 ([0.92 &lt; \beta \leq 0.93])</td>
<td>0.720 0.767 1.042</td>
<td>0.451 0.483 0.667</td>
</tr>
<tr>
<td>4 ([0.93 &lt; \beta \leq 0.94])</td>
<td>0.776 0.882 1.172</td>
<td>0.485 0.548 0.711</td>
</tr>
<tr>
<td>5 ([0.94 &lt; \beta \leq 0.95])</td>
<td>0.811 0.915 1.293</td>
<td>0.488 0.539 0.791</td>
</tr>
<tr>
<td>6 ([0.95 &lt; \beta \leq 0.96])</td>
<td>0.910 1.020 1.422</td>
<td>0.542 0.602 0.822</td>
</tr>
<tr>
<td>7 ([0.96 &lt; \beta \leq 0.97])</td>
<td>0.945 1.167 1.585</td>
<td>0.567 0.656 0.965</td>
</tr>
<tr>
<td>8 ([0.97 &lt; \beta \leq 0.98])</td>
<td>0.977 1.264 1.714</td>
<td>0.551 0.677 0.962</td>
</tr>
<tr>
<td>9 ([0.98 &lt; \beta \leq 1.00])</td>
<td>1.121 1.528 2.075</td>
<td>0.579 0.821 1.188</td>
</tr>
<tr>
<td>10 ([\beta &gt; 1.00])</td>
<td>1.307 2.452 3.712</td>
<td>0.602 1.153 1.675</td>
</tr>
</tbody>
</table>

6 Conclusions

In this paper, we have examined the presumption that households are ‘undersaving’ for retirement that has, implicitly or explicitly, motivated a significant quantity of international policy reforms to encourage private saving.

Using a lifecycle model of consumption, non-pension and pension saving, combined with rich micro-data from survey and linked administrative sources, we have estimated the distribution of household discount factors that would be required to rationalize observed late-life wealth holdings, given lifetime earnings histories, for a sample of English households. Using our preferred specification of the model we find a median discount factor of 0.95 and an interquartile range of 0.92-0.97 – close to conventional values estimated or assumed in the literature. We show how heterogeneity in discount factors relates to heterogeneity in replacement rates and find that even those with low estimated patience have modest replacement rates out of lifetime income,
while even among those who we estimate to be the most patient many fail to have accumulated sufficient funds to replace much of their peak (or final) earnings. This calls into question the rationale for the focus by the pension industry and by some policymakers on the ability of households to replace the level of income enjoyed towards the end of working life, rather than over the whole of working life.

While we interpret our results as suggesting that concerns about undersaving relative to conventional benchmarks might be exaggerated, we do not mean to suggest that there is no one for whom undersaving for retirement is a concern. We conclude with three notes of caution. First, for reasons to do with data availability, our sample only includes couples and we do not model divorce risk. There may be particular at-risk groups outside of this population, such as those who find themselves single late in life (see Munnell et al. (2018) for a recent discussion of the association of divorce and retirement security).

Second, we do not account for the risk of needing to pay for long-term care in retirement in our model (while there is universal care of hospital and out-patient health costs in the UK, nursing home care is only paid for those with the lowest assets). De Nardi et al. (2010) have shown that the risk of large expenses of nursing home stays explains the fact that households do not decumulate wealth in retirement. However, Kopecky & Koreshkova (2014) show that the effect of nursing home care risks on the wealth distribution is modest – explaining about 6.7% of aggregate wealth holdings – with approximately half of that accounted for by wealth accumulated pre-retirement. This latter fact reassures us that our abstraction from these risks will have only minor impacts on our estimated measures of patience. However, it remains an open question whether those risks should bear more heavily on saving given that medical costs and the time spent in care might both be higher for future cohorts.

Finally, our sample represents those born in 1940s. Hood & Joyce (2013) document lower levels of wealth accumulation by younger cohorts in the UK relative to the cohort we study. Younger cohorts might differ in the extent to which they discount the future, the rates of return that they will expect and experience and the government policies that support incomes in retirement (public pension benefits, tax incentives for private pension saving). Any of these could lead to those cohorts having lower living standards in retirement than those which our

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19 Supporting this evidence of a modest impact of this risk on wealth accumulation, we note that ELSA respondents in 2016 were asked “Have you thought about how to pay for your care and support needs?” Of those aged 52 to 62 (the ages of those in our sample), 63% say that they have not thought about it at all, and 31% say that they have thought about it a little. Fewer than 6% say that they have thought about it a lot.
cohort of interest will have.
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