Estimation of Dynastic Life-Cycle Discrete Choice Models [1]

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Abstract

This paper explores the estimation of a class of life-cycle discrete choice dynastic models. It provides a new representation of the value function for these class of models. It compare a multi stage conditional choice probability (CCP) estimator based on the new value function representation with a modified version of the full solution maximum likelihood estimator (MLE) in a Monte Carlo study. The modified CCP estimator performs comparably to the MLE in a finite sample but greatly reduces the computational cost. Using the proposed estimator, we estimate a dynastic model and use the estimated model to conduct counterfactual simulations to investigate the role Nature versus Nurture in intergenerational mobility. We find that Nature accounts for 20 percent of the observed intergenerational immobility at the bottom of income distribution. That means that 80 percent of mobility at the bottom of the income distribution is explained by economic decision and economic/institutional constraints. (**JEL classification**: C13, J13, J22, J62)

1 Introduction

The importance of parents' altruism toward their children and children's altruism toward their parents has long been recognized as an important factor underlying the economic behavior of individuals. Economic models that incorporate these intergenerational links are normally referred to as dynastic models. Many important economic behaviors – and hence the welfare effect of many public policies – critically depend on whether these dynastic links are explicitly modeled. For example, several papers have documented that (i) the distribution of wealth is more concentrated than that of labor earnings and (ii) it is characterized by a smaller of fraction of households owning a larger fraction of total wealth over time.

There are different models of dynastic transfers explaining the persistence in wealth and income across generations (for example, the Loury, 1981, model of transmission of human capital and the Laitner, 1992, model of bequests); however, in these models fertility is exogenous. Barro and Becker (1988, 1989) develop dynastic models with endogenous fertility; however, in their models endogenizing fertility leads to a lack of persistence in earnings and wealth because wealthier households have more children and therefore dynastic transfers do not depend on wealth and income. The data clearly show persistence in income across generations. Subsequently, dynastic models with endogenous fertility that capture the dynastic persistence of income and wealth have been analyzed extensively, but such models have not been estimated mainly because of computational feasibility considerations. This paper develops an estimator for dynastic models of dynastic transfers and estimates a model quantifying the different factors generating the persistence of income.

Alvarez (1999) combines the main features of the above-mentioned models by incorporating the fertility decision into the Laitner (1981) and Loury (1992) dynastic transfer models. While some models, as Laitner (1981), incorporate an elaborate finite life-cycle model for adults in each generation, in other models there is one period of childhood and one period of adulthood. The framework we study incorporates all these elements and develops a model in which altruistic parents make discrete choices of birth, labor supply, and discrete and continuous investment choices in children. In particular, in order to accommodate many models in the literature, parents choose time with children and a continuous monetary investment in their children every year over their life-cycle. The model can also be extended to include bequests. The model is a partial equilibrium model, and as in most dynastic models and in the basic setup, there is one decision- maker in a household; however, we show that it can be easily extended to a

unitary household.[1]

While the study of dynastic models has been widespread in the economic literature, these studies have been largely theoretical or quantitative theory. However, the estimation of these models and the use of these estimated models to conduct counterfactual policy analysis are nonexistent. There are two main reasons for this gap; the first is data limitation and the second is computational feasibility. Ideally, one would need data on the choices and characteristics of multiple generations linked across time to estimate these dynastic models. The number of generations needed for estimation can be reduced to two by analyzing the stationary equilibrium properties of these model. Recently data on the choices and characteristics of at least two generations have become available in the National Longitudinal Survey of Youth (NLSY79), Panel Study of Income Dynamics (PSID), and a number of European administrative datasets.

There are two main estimators used in the literature to estimate dynamic discrete choice models: full solution method using the "nested fixed point" algorithm (NFXP) (see Wolpin, 1984; Miller, 1984; Pakes, 1986; and Rust, 1987, for early examples) and "conditional choice probability" (CCP) (see Hotz and Miller, 1993; Altug and Miller, 1998; and Aguirregabiria, 1999) estimators that do not require the solution to the fixed points. More recently Aguirregabiria and Mira (2002) showed that an appropriately formed CCP-based estimator, "nested pseudo likelihood" (NPL), is asymptotically equivalent to an NFXP estimator. The major limitation of the NFXP estimation procedure is that it suffers from the curse of dimensionality (i.e., as the number of states in the state space increases, the number of computations increases at a rate faster than linear). Dynastic models add an additional loop to this estimation procedure: a nested fixed point squared. Therefore, this estimation procedure suffers from the curse of dimensionality squared. However, even with a CCP estimator or an NPL estimator, estimation of the dynastic model requires dealing with further complications that are not present in single-agent dynamic discrete choice models.

The main difficulty is deriving the representation of the value functions of the problem. This difficulty is associated with the non-standard nature of the problem. A dynastic model has finite number of periods in the life-cycle in each generation and infinitely many generations are linked by the altruistic preferences. This framework does not fit into a finite horizon dynamic discrete choice model since in the last period,

[1] In a companion paper, we extend the current framework to incorporate non-unitary households (Gayle, Golan, and Soytas , 2014).

there is a continuation value associated with the next generation's problem that is linked to the current generation by the transfers and the discount factor. Therefore, we need to find a representation for the next generation's continuation value if we want to treat the problem as a standard finite-period problem and solve it by backward induction[2]. In this paper, we propose a new estimation procedure based on a representation of the period value functions in terms of period primitives. In particular, we show that an appropriately defined alternative representation of the continuation value enables us to apply a CCP estimator to the dynastic model. The general principles used in the estimation technique are well known in the literature[3] and hence the main contribution of this paper is showing how these principles can be combined to estimate dynastic models. In a Monte Carlo study, we demonstrate that a multi stage CCP estimator based on the new value function representation have good small-sample properties that compare favorably to a full solution NFXP estimator. For this comparison we use a pseudo maximum likelihood estimator (PML) so that our results would be more comparable to those of the NFXP maximum likelihood estimator.

We use the GMM version of the estimator developed in this paper to estimate a dynastic model of intergenerational transmission of human capital with unitary households. The estimated model captures well the labor supply, time with children, and fertility decisions of households. We then demonstrates the usefulness of our framework for policy analysis. This is done by conducting counterfactual simulations to investigate the role of the automatic transmission of education across generation (Nature) on integenerational mobility at bottom of the income distribution. We find that without the Nature on the intergenerational education production function mobility at the bottom of the income distribution would have been 20 percent higher. That means that 80 percent of mobility at the bottom of the income distribution is explained by economic decision and economic/institutional constraints. Lastly, not accounting for the re-optimization of subsequent generations in the model, as is done in the approach outlined in this paper, will overstate the effect of Nature on mobility by between 20 and 90 percent.

Dynastic models have been used to study numerous topics in economics. These topics include explaining the cross-sectional correlation between parental wages and fertility (see Jones, Schoonbroodt, and

[2] Obviously, we can always solve the problem by NFXP if we assume that the problem is stationary in the generations. In this case, the solution to the dynamic programming problem requires solving the fixed point problem for the period value functions. However, as one can easily anticipate, we encounter the same computational burden of full solution. Therefore our specific interest is CCP-type estimators.

[3] See Hotz and Miller (1993), Hotz, Miller, Sanders and Smith (1994), Altug and Miller (1998) and Aguirregabiiria and Miria (2002) for the seminal contributions from which these general principles are derived.

Tertilt, 2010, for a detailed overview of this literature), the relationship between inequality and growth (see, e.g., De la Croix and Doepke, 2003), the relationship between human capital formation and social mobility (see Heckman and Mosso, 2014, for a survey of this literature), the relation among bequests, saving, and the distribution of wealth and earnings (see De Nardi, 2004; Cagetti and De Nardi, 2008, among others),[4] and the optimality of different ways of funding social security. These models have been used to shed light on the effect of education, child care subsidies, child labor regulations, and wealth and income redistribution policies on individual welfare. Reviewing this vast and diverse literature is beyond the scope of this paper; however, a short review of two of the literature segments will suffice to illustrate the need to estimate these models and hence the wide applicability of our estimation technique.

The first segment explains the widespread negative cross-sectional correlation between parental wage and fertility. The basic dynastic model as formulated by Barro and Becker (1989) cannot explain this negative correlation because wealthier parents increase the number of offspring, keeping transfer levels the same as less wealthy parents. Attempts in the literature to account for this negative correlation range from appropriately calibrating the model parameters so that the substitution effects are larger than the income effects, introducing the quality of children as a choice variable with an appropriate assumption about the cost of child-rearing (Becker and Lewis, 1973; Becker and Tomes,1976; Moav 2005),[5] to introducing non-homotheticity in preferences (see, e.g., Galor and Weil, 2000; Greenwood and Seshadri, 2002; or Fernandez, Guner, and Knowles, 2005). As summarized in Alvarez (1999), depending on the functional form assumptions of the primitives and values of the structural parameters, dynastic models could generate the negative correlation between parental wages and fertility.[6] Therefore, whether the basic dynastic model can explain this negative cross-sectional correlation between parental wages and fertility is an empirical question requiring careful exploration of the source of identification and estimation (see Gayle, Golan, and Soytas, 2014; 2015, for examples of these types of analysis).

The effects of the social security system on both capital accumulation and wealth distribution have been of great interest to economists and policy- makers for decades (see, for instance, Kotlikoff and

[4] For example, De Nardi (2004) model explicitly focuses on the transmission of physical and human capital from parents to children and intergenerational links. She shows that such a model can can induce savings behavior that generates a distribution of wealth that (i) is much more concentrated than that of labor earnings and (ii) also makes the rich keep large amounts of assets in old age to leave bequests to their descendants.

[5] See Jones, Schconbroodt, and Tertilt (2010, section 5.2).

[6] Recently Mookherjie, Prina, and Ray (2012) demonstrated that incorporating dynamic analysis of return to human capital can help explain the negative cross-sectional correlation between parental wages and fertility.

Summers, 1981; Caballé and Luisa, 2003; among others). However, the optimal form of funding social security may depend on whether or not these intergenerational links are explicitly modelled. For example, Fuster, Imrohoroglu, and Imrohoroglu (2007) argue that when households insure members in the same family line, privatizing social security without compensation is favored by 52% of the population. If social security participants are fully compensated for their contributions and the transition to privatization is financed by a combination of debt and a consumption tax, 58% experience a welfare gain. These gains and the resulting public support for social security reform depend critically on a flexible labor market. If the elasticity of the labor supply is low, then support for privatization disappears. Therefore, it is important to estimate these models because policy implications often depend on the value of key structural parameters. In Fuster, Imrohoroglu and Imrohoroglu (2007) the key structural parameter was the elasticity of labor supply, but in other models it may be the altruism parameters themselves.

The rest of the paper is organized as follows. Section 2 presents the basic gender-less life-cycle dynastic model with only discrete choices. Section 3 presents the generic estimator of the life-cycle model and presents the Monte Carlo study. Section 4 extends the framework to include continuous choices and transfers, intra-household behaviors, and gender. Section 5 presents the basic framework of our empirical application. Section 6 presents our empirical results. Section 7 concludes, and all proofs are provided in an appendix.

2 Theoretical Framework

The theoretical framework is developed to allow for estimation of a rich group of dynastic models and allows us to address many relevant policy questions. This section develops a model of altruistic parents who make transfers to their children. The transfers are discrete and can allow for (i) discrete time investment in children and (ii) monetary investment with discrete levels. Section 4 extends this basic framework to allow for continuous choices and transfers. This allows us to use the framework to analyze bequests or any continuous monetary transfers by parents to their children. We incorporate two important aspects of the problem. First, fertility is endogenous. Endogenous fertility has important implications for intergenerational transfers and the quantity-quality trade-offs made by parents when they choose transfers as the well as number of offspring. Second, we include a life-cycle for each generation. The life-cycle is important to understanding fertility behavior, spacing of children, and the timing of different

types of investments. This section analyzes a model with one gender-less decision-maker. We later extend this framework to a unitary household.[7]

We build on previous dynastic models that analyze transfers and intergenerational transmission of human capital. In some models, such as Loury (1981) and Becker and Tomes (1986), fertility is exogenous, whereas in others, such as Becker and Barro (1988) and Barro and Becker (1989), fertility is endogenous. The Barro-Becker framework is extended in our model by incorporating a life-cycle behavior model, based on previous work, such as Heckman, Hotz and Walker (1985) and Hotz and Miller (1988), into an infinite-horizon model of dynasties. Our life-cycle model includes individuals choices about time allocation decisions, investments in children, and fertility. We formulate a partial equilibrium discrete choice model that incorporates life-cycle considerations of individuals from each generation into the larger framework. Adults in each generation derive utility from their own consumption, leisure, and the utility of their adult offspring. The utility of adult offspring is determined probabilistically by the educational outcome of childhood, which in turn is determined by parental time and monetary inputs during early childhood, parental characteristics (such as education), and luck. Parents make decisions in each period about fertility, labor supply, time spent with children, and monetary transfers. For simplicity, the only intergenerational transfers are transfers of human capital, as in Loury (1981). However, the framework can include any other choice of transfer that is discrete. We assume no borrowing or savings for simplicity. The model assumes that the educational outcome of children is revealed at the last period of parent's life-cycle regardless of the birth date of the children. This assumption is similar to the Barro-Becker assumptions. In the parents' life-cycle, adult children's behavior and choices do not affect the choices of parents. As in Barro-Becker, the choices can only be made by the children in their own life-cycle which starts immediately after the parents' life-cycle ends[8].

In the model adults live for T periods. Each adult from generation $g \in \{0, ...\infty\}$ makes discrete choices about labor supply (h_t), time spent with children (d_t), and birth (b_t), in every period $t = 1...T$. For labor time individuals choose no work, part-time, or full-time ($h_t \in (0, 1, 2)$); for time spent with children individuals choose none, low, or high ($d_t \in (0, 1, 2)$). The birth decision is binary ($b_t \in (0, 1)$). The individual does not make any choices during childhood, when $t = 0$. All the discrete choices can be

[7] Treatment of households, with two decision-makers (with separate utility functions), marriage, and divorce, is involved and is beyond the scope of this paper. See Gayle, Golan, and Soytas (2014) for more details on one such model.

[8] In a model where adult children's behavior and choices do affect investment in children and fertility of the parents, solutions to the problems are significantly more complicated and it is not clear whether a solution exists.

combined into one set of mutually exclusive discrete choice, represented as k, such that $k \in (0, 1...17)$. Let I_{kt} be an indicator for a particular choice k at age t; I_{kt} takes the value 1 if the kth choice is chosen at age t and 0 otherwise. These indicators are defined as follows:

$$\begin{aligned} I_{0t} &= I\{h_t = 0\}I\{d_t = 0\}I\{b_t = 0\},\ I_{1t} = I\{h_t = 0\}I\{d_t = 0\}I\{b_t = 1\},\ ..., \\ I_{16t} &= I\{h_t = 1\}I\{d_t = 2\}I\{b_t = 1\},\ I_{17t} = I\{h_t = 2\}I\{d_t = 2\}I\{b_t = 1\}. \end{aligned} \tag{1}$$

Since these indicators are mutually exclusive, then $\sum_{k=0}^{17} I_{kt} = 1$. We define a vector, x, to include the time-invariant characteristics of the individual's education, skill, and race. Incorporating this vector, we further define the vector z to include all past discrete choices as well as time-invariant characteristics, such that $z_t = (\{I_{k1}\}_{k=0}^{17}, ..., \{I_{kt-1}\}_{k=0}^{17}, x)$.

We assume the utility function is the same for adults in all generations. An individual receives utility from discrete choice and from consumption of a composite good, c_t. The utility from consumption and leisure is assumed to be additively separable because the discrete choice, I_{kt}, is a proxy for leisure and is additively separable from consumption. The utility from I_{kt} is further decomposed into two additive components: a systematic component, denoted by $u_{1kt}(z_t)$, and an idiosyncratic component, denoted by ε_{kt}. The systematic component associated with each discrete choice k represents an individual's net instantaneous utility associated with the disutility from market work, the disutility/utility from parental time investment, and the disutility/utility from birth. The idiosyncratic component represents a preference shock associated with each discrete choice k that is transitory in nature. To capture this feature of ε_{kt}, we assume that the vector $(\varepsilon_{0t}, .., \varepsilon_{17t})$ is independent and identically distributed across the population and time and is drawn from a population with a common distribution function, $F_\varepsilon(\varepsilon_{0t}, .., \varepsilon_{17t})$. The distribution function is assumed to be absolutely continuous with respect to the Lebesgue measure and has a continuously differentiable density.

Per-period utility from the composite consumption good is denoted $u_{2t}(c_t, z_t)$. We assume that $u_{2t}(c_t, z_t)$ is concave in c; that is, $\partial u_{2t}(c_t, z_t)/\partial c_t > 0$ and $\partial^2 u_{2t}(c_t, z_t)/\partial c_t^2 < 0$. Implicit in this specification is the intertemporally separable utility from the consumption good, but not necessarily for the discrete choices, since u_{2t} is a function of z_t, which is itself a function of past discrete choices but is not a function of the lagged values of c_t.

Altruistic preferences are introduced under the same assumption as the Barro-Becker model: Parents obtain utility from their adult offspring's expected lifetime utility. Two separable discount factors capture the altruistic component of the model. The first, β, is the standard rate of time preference parameter, and the second, $\lambda N^{-\nu}$, is the intergenerational discount factor, where N is the number of offspring an individual has over her lifetime. Here λ $(0 < \lambda < 1)$ should be understood as the individual's weighting of her offsprings' utility relative to her own utility. The individual discounts the utility of each additional child by a factor of $-\nu$, where $0 < \nu < 1$.[9]

We let earnings (w_t) be given by the earnings function $w_t(z_t, h_t)$, which depends on the individual's time-invariant characteristics, choices that affect human capital accumulated with work experience, and the current level of labor supply (h_t). The choices and characteristics of parents are mapped onto their offspring's characteristics (x') via a stochastic production function of several variables. The offspring's characteristics are affected by their parents' time-invariant characteristics, their parents' monetary and time investments, and the presence and timing of siblings. These variables are mapped into the child's skill and educational outcome by the function $M(x'|z_{_{T+1}})$ where $z_{_{T+1}}$ includes all parental choices and characteristics and contains information on the choices of time inputs and monetary inputs. Because $z_{_{T+1}}$ also contains information on all birth decisions, it captures the number of siblings and their ages. We assume there are four mutually exclusive educational outcomes for offspring: less than high school (LH), high school (HS), some college (SC), and college (Coll). Therefore, $M(x'|z_{_{T+1}})$ is a mapping of parental inputs and characteristics into a probability distribution over these four outcomes.

We normalize the price of consumption to 1. Raising children requires parental time (d_t) and market expenditure. The per-period cost of expenditures for raising a child is denoted pc_{nt}. Therefore, the per-period budget constraint is given by

$$w_t \geq c_t + pc_{nt}. \tag{2}$$

The sequence of optimal choice for both discrete choice and consumption is denoted as I^o_{kt} and c^o_t, respectively. We can thus denote the expected lifetime utility at time $t = 0$ of a person with characteristics x in generation g, excluding the dynastic component, as

$$U_{gT}(x) = E_0 \left[\textstyle\sum_{t=0}^{T} \beta^t [\sum_{k=0}^{17} I^o_{kt} \{u_{1kt}(z_t) + \varepsilon_{kt}\} + u_{2t}(c^o_t, z_t)] | x \right]. \tag{3}$$

[9] Note that this formulation can be written as an infinite discounted sum (over generations) of per-period utilities as in the Barro-Becker formulation.

The total discounted expected lifetime utility of an adult in generation g including the dynastic component is

$$U_g(x) = U_{gT}(x) + \beta^T \lambda E_0 \left[N^{-\nu} \sum_{n=1}^{N} U_{g+1,n}(x'_n) | x \right], \tag{4}$$

where $U_{g+1,n}(x'_n)$ is the expected utility of child n $(n = 1, .., N)$ with characteristics x'_n. In this model, individuals are altruistic and derive utility from their offspring's utility, subject to discount factors β and $\lambda N^{-\nu}$.[10] This formulation is similar to the one in Barro-Becker, but it is extended to allow for differences in gender and "types."

To simplify presentation of the model, we assume that pc_{nt} is proportional to an individual's current earnings and the number of children, but we allow this proportion to depend on the state variables. This assumption allows us to capture the differential expenditures on children made by individuals with different incomes and characteristics. Practically, this allows us to proxy for differences in social norms of child-rearing among different socioeconomic classes.[11] Explicitly, we assume that

$$pc_{nt} = \alpha_{Nc}(z_t)(N_t + b_t) w_t(x, h_t) \tag{5}$$

and, incorporating the assumption that individuals cannot borrow or save[12] and equation (5), the budget constraint becomes

$$w_t(x, h_t) = c_t + \alpha_{Nc}(z_t)(N_t + b_t) w_t(x, h_t). \tag{6}$$

Solving for consumption from equation (6) and substituting for consumption in the utility equation, we can rewrite the third component of the per-period utility function, specified as $u_{2kt}(z_t)$, as a function of just z_t as follows;

$$u_{2kt}(z_t) = u_t[w_t(x, h_t) - \alpha_{Nc}(z_t)(N_t + b_t) w_t(x, h_t), z_t]. \tag{7}$$

Note that the discrete choices now map into different levels of utility from consumption. Therefore, we can eliminate the consumption decision as choice and write the systematic contemporary utility associated

[10] Note that since we add life-cycle, the regularity condition that implies that the discount factor of the children's utilities, $\beta^T \lambda N^{-\nu}$ is between zero and 1 is satisfied for any N, as β is also between zero and one.

[11] In general, individuals can choose expenditures on children, but we do not observe spending in our data used for estimation in empirical section.

[12] This assumption is not important for any of the results obtained in this paper. However, it simplifies the presentation by allowing all choices to be discrete. See Section 4 for a relaxation of the assumption at all the choices are discrete.

with each discrete choice k as[13]

$$u_{kt}(z_t) = u_{1kt}(z_t) + u_{2kt}(z_t). \tag{8}$$

Incorporating the budget constraint manipulation, we can rewrite equation (3) as

$$U_{gT}(x) = E_0 \left[\sum_{t=0}^{T} \beta^t \sum_{k=0}^{17} I_{kt}^o [u_{kt}(z_t) + \varepsilon_{kt}] | x \right]. \tag{9}$$

Alvarez (1999) theoretically analyzes and generalizes the conditions under which dynastic models with endogenous fertility lead to intergenerational persistence in income and wealth. Following his analysis, we show empirically which assumptions are relaxed in our model and lead to persistence in income. The first is constant cost per child. In our model, the per-period costs of raising a child and transferring human capital is the cost described in equations (5) and (6), as well as the opportunity cost of time investment in children. Time investment in children and labor market time are modeled as discrete choice with three levels. This introduces nonlinearity. Even if we were able to capture the proportional increase in time with children as the number of children increases, the nonlinearity in labor supply decisions implies that the opportunity cost of time investment in children is not linear. Thus, the cost of transfer of human capital per child is not constant. Furthermore, in contrast to standard dynastic models and those analyzed in Alvarez (1999), we incorporate dynamic elements of the life-cycle that involve age and experience effect. The opportunity cost of time with children therefore incorporates returns to experience, which are nonlinear. Therefore, estimating a dynastic model with accounting for the individual demographics and heterogeneity can be important in understanding the extent of the life-cycle dynamics and help us sort out the importance of different mechanisms leading to persistence in outcomes across generations. The nonlinearity involved in labor supply is realistic; parents labor market time is often not proportional to the number of children they have, and hours in the labor market for a given wage rate are not always flexible and depend on occupation. Furthermore, fertility decisions are made sequentially, and due to age effects, the cost of a child varies over the life-cycle. The second condition is non-separability in preferences, aggregation of the utilities from children, and the feasible set. In our model, the latter is relaxed; that

[13] In our formulation utility from consumption and leisure is assumed to be additively separable, and hence $u_{1kt}(z_t) + \varepsilon_{kt}$ captures the utility of leisure corresponding to the choices of labor supply , time spent with children (d_t) and the birth decision (b_t) associated with choice k in period t. In the empirical implementation we account by setting the levels of high level of labor supply and time with children to be feasible and satisfy a time budget constraint.

is, the separability of the feasible set across generations. This is because the opportunity costs of the children depend on their education and labor market skills. However, education and labor market skills of children are linked with their parents' skills and education through the production function of education.

2.1 Optimal Discrete Choice

The individual then chooses the sequence of alternatives yielding the highest utility by following the decision rule $I(z_t, \varepsilon_t)$, where ε_t is the vector $(\varepsilon_{0t}, ..., \varepsilon_{17t})$. The optimal decision rules are given by

$$I^o(z_0, \varepsilon_0) = \arg\max_I E_I \left[\sum_{t=0}^{T} \beta^t \{ \sum_{k=0}^{17} I_{kt}[u_{kt}(z_t) + \varepsilon_{kt}] \} + \beta^T \lambda N^{-\nu} \sum_{n=1}^{N} U_{g+1,n}(x'_n) | x \right] \tag{10}$$

where the expectations are taken over the future realizations of z and ε induced by I^o. In any period $t < T$, the individual's maximization problem can be decomposed into two parts: the utility received at t plus the discounted future utility from behaving optimally in the future. Therefore, we can write the value function of the problem, which represents the expected present discounted value of lifetime utility from following I^o, given z_t and ε_t, as

$$V(z_{t+1}, \varepsilon_{t+1}) = \max_I E_I \left(\sum_{t'=t+1}^{T} \beta^{t'-t} \sum_{k=0}^{17} I_{kt'}[u_{kt'}(z_{t'}) + \varepsilon_{kt'}] + \beta^{T-t} \lambda N^{-\nu} \sum_{n=1}^{N} U_{g+1,n}(x'_n) | z_{t+1}, \varepsilon_{t+1} \right). \tag{11}$$

By Bellman's principle of optimality, the value function can be defined recursively as

$$\begin{aligned} V(z_t, \varepsilon_t) &= \max_I \left[\sum_{k=0}^{17} I_{kt} \{ u_{kt}(z_t) + \varepsilon_{kt} + \beta E(V(z_{t+1}, \varepsilon_{t+1}) | z_t, I_{kt} = 1) \} \right] \\ &= \sum_{k=0}^{17} I^o_{kt}(z_t, \varepsilon_t)[u_{kt}(z_t) + \varepsilon_{kt} + \beta \sum_{z'} \int V(z', \varepsilon) f(\varepsilon) d\varepsilon F(z' | z_t, I^o_{kt} = 1)], \end{aligned} \tag{12}$$

where $f(\varepsilon)$ is the continuously differentiable density of $F_\varepsilon(\varepsilon_{0t}, .., \varepsilon_{17t})$, and $F(z'|z_t, I_{kt} = 1)$ is a transition function for state variables, which is conditional on choice k. In this simple version, the transitions of the state variables are deterministic given the choices of labor market experience, time spent with children, and number of children.

Since ε_t is unobserved, we further define the ex ante (or integrated) value function, $V(z_t)$, as the continuation value of being in state z_t before ε_t is observed by the individual. Therefore, $V(z_t)$ is given by integrating $V(z_t, \varepsilon_t)$ with respect to the density of ε_t. Defining the probability of choice k at age t by

$p_k(z_t) = E[I^o_{kt} = 1|z_t]$, the ex ante value function can be written as

$$V(z_t) = \sum_{k=0}^{17} p_k(z_t) \left[u_{kt}(z_t) + E_\varepsilon[\varepsilon_{kt}|I_{kt} = 1, z_t] + \beta \sum_{z'} V(z')F(z'|z_t, I_{kt} = 1) \right] \tag{13}$$

This representation of the problem is a not new or is it unique to dynastic models[14], but pedagogically it shows that $V(z_t)$ is a function of the CCPs, the expected value of the preference shock, the per-period utility, the transition function, and the ex ante continuation value. All components except the conditional probability and the ex ante value function are primitives of the initial decision problem. By writing the CCPs as a function of just the primitives and the ex ante value function, we can characterize the optimal solution of the problem (i.e., the ex ante value function) as implicitly dependent on just the primitives of the original problem.

As is standard in the dynamic discrete choice literature, we define the conditional value function, $\upsilon_k(z_t)$, as the present discounted value (net of ε_t) of choosing k and behaving optimally from period t onward:

$$\upsilon_k(z_t) = u_{kt}(z_t) + \beta \sum_{z'} V(z')F(z'|z_t, I_{kt} = 1). \tag{14}$$

The conditional value function is the key component to the CCPs. Equation (10) can now be rewritten using the individual's optimal decision rule at t to solve

$$I^o(z_t, \varepsilon_t) = \arg\max_I \sum_{k=0}^{17} I_{kt}[\upsilon_k(z_t) + \varepsilon_{kt}]. \tag{15}$$

Therefore, the probability of observing choice k, conditional on z_t, is $p_k(z_t)$ and is found by integrating overt ε_t in the decision rule in equation (15):

$$p_k(z_t) = \int I^o(z_t, \varepsilon_t) f_\varepsilon(\varepsilon_t) d\varepsilon_t = \int \left[\prod_{k \neq k'} 1\{\upsilon_k(z_t) - \upsilon_{k'}(z_t) \geq \varepsilon_{tk'} - \varepsilon_{kt}\} \right] f_\varepsilon(\varepsilon_t) d\varepsilon_t. \tag{16}$$

Therefore, $p_k(z_t)$ is now entirely a function the primitives of the model (i.e., $u_{kt}(z_t), \beta, F(z_{t+1}|z_t, I_{kt} = 1)$, and $f_\varepsilon(\varepsilon_t)$) and the ex ante value function. Hence substituting equation (16) into equation (13) gives an implicit equation defining the ex ante value function as a function of only the primitives of the model.

[14] See for example Aguirregabiria and Mira (2002).

3 A Generic Estimator of the Life-Cycle Dynastic Discrete Choice Model

We use a partial solution, multi stage estimation procedure to accommodate the non-standard features of the model. By assuming stationarity across generations and discrete state space in the dynamic programming problem, we obtain an analytical representation of the value function. The alternative value function depends on the CCPs, the transition function of the state variable, and the structural parameters of the model. In the first stage, we estimate the CCPs and the transition function. The second stage forms either moment conditions or likelihood functions to estimate the remaining structural parameters using a PML or a GMM, respectively. For each iteration in the estimation procedure the CCP is used to generate a value representation to form the terminal value in the life-cycle problem, which can then be solved by backward induction to obtain the life-cycle valuation functions.

3.1 An Alternative Representation of the Problem

The alternative representation of the continuation value of the intergenerational problem is developed below. The Hotz and Miller estimation technique for standard single-agent problems is adapted to the dynastic problem using the following representation. Define $N_T(z_T)$ to be the number of total number of children at the end of the life-cycle given state variable z_T. In addition, we recursively define a transition function $F_k^o(z_{t'}|z_t)$ for the one-period-ahead $t'-t$:

$$F_k^o(z_{t'}|z_t) = \begin{cases} F(z_{t'}|z_t, I_{kt}=1) & \text{for } t'-t=1 \\ \sum_{r=0}^{17}\sum_{z_{t'-1}} p_r(z_{t'-1})F(z_{t'}|z_{t'-1}, I_{rt'-1}=1)F_k^o(z_{t'-1}|z_t) & \text{for } t'-t>1, \end{cases}$$

This function is a recursive formulation that determines the future probability of a future state $z_{t'}$ conditional on current state z_t and a current choice k.

Proposition 1 *There exists an alternative representation for the ex ante conditional value function at time t that is a function of just the primitives of the problem and the CCPs:*

$$\begin{aligned} v_k(z_t) &= u_{kt}(z_t) + \sum_{t'=t+1}^{T}\beta^{t'-t}\sum_{s=0}^{17}\sum_{z_{t'}} p_s(z_{t'})[u_{st'}(z_{t'}) + E_\varepsilon(\varepsilon_{st'}|I_{st'}=1, z_{t'})]F_k^o(z_{t'}|z_t) \quad (17) \\ &\quad +\lambda\beta^{T-t}\sum_{z_T} N_T(z_T)^{-\nu}\sum_{n=1}^{N_T}\sum_x V(x')\sum_{s=0}^{K_T} M_k^n(x'|z_T)p_s(z_T)F_k^o(z_T|z_t), \end{aligned}$$

Where the intergenerational transition function for the nth child born in a parent's life-cycle, $M_k^n(x'|z_T) = M(x'|z_T)$*, is conditional on a choice* $I_{kT} = 1$*.*

The representation of Proposition 1 highlights the main contribution of this paper. There are three components in equation (17). The first two are normally found in the finite horizon discrete choice dynamic programing model, and are standard in stationary dynamic discrete choice models. The last components is the dynastic component that is non-standard. There are two points worth noting. The first is that without further restrictions, the third component of equation (17), does not have the finite-state-dependence property, which normally simplifies the estimation life-cycle discrete choice models. See Altug and Miller (1998), Gayle and Miller (2004), Arcidiacono and Miller (2011, 2015), Gayle and Golan (2012), and Gayle (2015) for discussion and use of the finite-state-dependence property. Second, an alternative used in literature which estimates or calibrates dynastic models[15] is to replace the dynastic component in equation (17) with a reduced form approximation, and then treat the model as a finite horizon model with a reduced form terminal value function. This reduced form approximation, however, is not in general policy invariant. Therefore, we pursue an alternative strategy which builds on the ideas in Aguirregabira and Mira (2002, 2007) and Pesendorfer and Schmidt-Dengler (2008).

Let $e_k(p, z)$ represent the expected preference shocks conditional on choice k being optimal in state z. The expected preference shocks are written in this notation to convey the shock as a function of the CCPs (see Hotz and Miller, 1993). For example, in the type 1 extreme value case, $e_k(p, z)$ is given by $\gamma - \ln[p_k(z)]$, where γ is Euler's constant. From the representation in Proposition 1 we can define the ex ante conditional lifetime utility at period t, excluding the dynastic component as

$$U_k(z_t) = u_{kt}(z_t) + \sum_{t'=t+1}^{T} \beta^{t'-t} \sum_{s=0}^{17} \sum_{z_{t'}} p_s(z_{t'})[u_{st'}(z_{t'}) + e_s(p, z_{t'})]F_k^o(z_{t'}|z_t).$$

Because $U_k(z_t)$ is a function of just the primitives of the problem and the CCPs, we can write an alternative representation for the ex ante value function at time t:

$$V(z_t) = \sum_{k=0}^{17} p_k(z_t)[U_k(z_t) + e_k(p, z_t) + \lambda\beta^{T-t} \sum_{z_T} N_T(z_T)^{-\nu} \sum_{n=1}^{N_T} \sum_{x} V(x') \sum_{s=0}^{K_T} M_k^n(x'|z_T)p_s(z_T)F_k^o(z_T|z_t)]. \tag{18}$$

Equation (18) is satisfied at every state vector z_t. The problem is stationary over generations, so

[15] See for example Rios-Rull and Sanchez-Marcos (2002).

$z_t = x$ at period $t = 0$ because there is no history of decisions in the state space, and hence the initial state space has finite support on the integers $\{1, ..., X\}$. We define the optimal lifetime intergenerational transition function as $M_k^o(x'|x) = \sum_{z_T} \sum_{n=1}^{N_T(z_T)} \sum_{s=0}^{K_T} p_s(z_T) M_k^n(x'|z_T) F_k^o(z_T|x)$. The matrix M_k^o can be interpreted as the probability that an average descendant of the individual with characteristic x', given that his parents have characteristics x, chooses decision k in the first period and behaves optimally from period 1 to T of the parent's life-cycle. Now, we can express the components of equation (18) in vector or matrix form:

$$V_0 = \begin{bmatrix} V(1) \\ . \\ . \\ . \\ V(X) \end{bmatrix}, \quad U(k) = \begin{bmatrix} U_k(1) \\ . \\ . \\ . \\ U_k(X) \end{bmatrix}, \quad E(k) = \begin{bmatrix} e_k(p,1) \\ . \\ . \\ . \\ e_k(p,X) \end{bmatrix}, P(k) = \begin{bmatrix} p_k(1) \\ . \\ . \\ . \\ p_k(X) \end{bmatrix},$$

$$\iota_X = \begin{bmatrix} 1 \\ . \\ . \\ . \\ 1 \end{bmatrix}_{Xx1}, \text{ and } \quad M^o(k) = \begin{bmatrix} M_k^o(1|1) & ... & M_k^o(X|1) \\ . & & \\ . & & \\ . & & \\ M_k^o(1|X) & ... & M_k^o(X|X) \end{bmatrix}$$

Using these components the vector of the ex ante value function can be expressed as

$$V_0 = \sum_{k=0}^{K} P(k) \otimes [U(k) + E(k) + \lambda \beta^T N_k^{-\nu} \otimes M^o(k) V_0] \tag{19}$$

where $\otimes$ refers to element-by-element multiplication and $N_k(x) = \sum_{z_T} N_T(z_T) F_k^o(z_T|x)$, and the vector $N_k = (N_{kT}(1), , .., N_{kT}(X))'$. Rearranging the terms and solving for V_0, we obtain

$$V_0 = [I_X - \lambda \beta^T \sum_{k=0}^{17} \{P(k) \iota_X'\} \otimes (N_k^{-\nu} \otimes M^o(k))]^{-1} \sum_{k=0}^{17} P(k)[U(k) + E(k)], \tag{20}$$

where I_X denotes the $X \times X$ identity matrix. Equation (20) is based on the dominant diagonal property, which implies that the matrix $I_X - \lambda \beta^T \sum_{k=0}^{17} \{P(k) \iota_X'\} \otimes (N_k^{-\nu} \otimes M^o(k))$ is invertible. The representation

is obtained by combining known results[16] from discrete choice estimation of stationary infinite-horizon problems with the finite horizon properties of the dynastic life-cycle model.

3.2 Estimation

We parameterized the period utility by a vector θ_2, $u_{kt}(z_t, \theta_2)$; the period transition on the observed states is parameterized by a vector θ_3, $F(z_t|z_{t-1}, I_{kT} = 1, \theta_3)$; the intergenerational transitions on permanent characteristics is parameterized by a vector θ_4, $M^n(x'|z_{T+1}, \theta_4)$; and the earnings function is characterized by a vector $\theta_5, w_t(x, h_t, \theta_5)$. Therefore, the conditional value functions, decision rules, and choice probabilities now also depend on $\theta \equiv (\theta_2, \theta_3, \theta_4, \theta_5, \beta, \lambda, \nu)$. Standard estimates of dynamic discrete choice models involve forming the likelihood functions from the CCPs derived in equation (16). This involves solving the value function for each iteration of the likelihood function. The method used to solve the value function depends on the nature of the optimization problems and normally falls into one of two cases

(i) Finite-horizon problems: The problem has an end date (as in a standard life-cycle problem); hence future value function is obtained by backwards induction.

(ii) Stationary infinite-horizon problem: The valuation is obtained by a contraction mapping.

A dynastic discrete choice model in unusual because it involves both a finite-horizon problem and an infinite-horizon problem. Solving both problems for each iteration of the likelihood function is computationally infeasible for all but the simplest of models. We avoid solving the stationary infinite-horizon problem in estimation by replacing the terminal value in the life-cycle problem with equation (20). This converts the problem into a finite-horizon problem that can be solved by backward recursion since the flow utility function is

$$\upsilon_k(z_T) = u_{kT}(z_T) + \lambda N_T(z_T)^{-\nu} \sum_{x'} V(x') \sum_{n=1}^{N_T} M_k^n(x'|z_T). \tag{21}$$

[16] See Aguirregabira and Mira (2002, 2007) and Pesendorfer and Schmidt-Dengler (2008) for the use and derivation of this inversion in context of stationary infinite horizon problems.

Since $u_{kT}(z_T)$ is parameterized by θ_2, the transition $M_k^n(x'|z_T)$ is known since it can be estimated from the data. Observing $F_\varepsilon(\varepsilon_{0t},..,\varepsilon_{17t})$ and calculating $V(x')$ via equation (20),[17] we can calculate the ex ante value function at T using $V(z_T) = \sum_{k=0}^{17} \int I_{kI}^0(z_T,\varepsilon_T)[\upsilon_k(z_T) + \varepsilon_{kT}]f_\varepsilon(\varepsilon_T)d\varepsilon_T$. The conditional value function for $T-1$ is given by $\upsilon_k(z_{T-1}) = u_{kT-1}(z_{T-1}) + \beta \sum_{z_T} V(z_T)F(z_T|z_{T-1}, I_{kT} = 1)$. This is continued backward given $\upsilon_k(z_{T-1})$ to form value function at $T-2$, and so on.

The backward induction procedure outlined above shows that only $M_k^n(x'|z_T)$ in equations (21) and (20) depends on the next generation's outcome. Thus, we can estimate the intergenerational problem with only two generations of data, as is the case in the standard stationary discrete choice models (see Rust, 1987, for example). To estimate the intergenerational problem we let I_{dtg}, z_{dtg}, and ε_{dtg}, respectively, indicate the choice, observed state, and unobserved state at age t in the generation g of dynasty d. Forming the CCPs for each individual in the first observed generation of dynasty d at all ages t yields the components necessary for estimation. Estimation proceeds in two steps.

Step 1: In the first step we estimate the CCP, transition, and earnings functions necessary to compute the inversion in equation (20). The expectation of observed choices conditional on the observed state variable gives an empirical analog to the CCPs at the true parameter values of the problem, θ_1^o, allowing us to estimate the CCPs; we denote this estimate by $\widehat{p_k(z_{dt1})}$. We also estimate θ_3, θ_4, and θ_5, which parameterize the transition and earnings functions $F(z_t|z_{t-1}, I_{kT} = 1, \theta_3)$, $M^n(x'|z_{T+1}, \theta_4)$, and $w_t(x, h_t, \theta_5)$ respectively in this step.

Step 2: The second step can be estimated two ways, the first is a PML (as used in Aguirregabira and Mira (2002)) and the second is a GMM (as used in the original Hotz and Miller (1993)). We can use a PML method and not a pure maximum likelihood estimator because part of the likelihood function is concentrated out using the data. With D dynasties, the PML estimates of $\theta_0 = (\theta_2, \beta, \lambda, \nu)$ are obtained via

$$\widehat{\theta}_{0PML} = \underset{\theta_0}{\arg\max} \left(\sum_{dt1=1}^{D} \sum_{t=0}^{T} \sum_{k}^{17} I_{dt1} \ln[p_k(z_{dt1}; \theta_0, \widehat{\theta}_3, \widehat{\theta}_4, \widehat{\theta}_5)] \right), \tag{22}$$

[17] This manipulation is possible because the alternative value function in equation (20) is a function of only the parameters of the model and the CCP. Since the CCP can be estimated directly from the data, backward recursion becomes possible because the decision in the last period, T, is similar to a static problem when the value of children is replaced with equation (20).

where $p_k(z_{dt1};\theta_0,\widehat{\theta}_3,\widehat{\theta}_4,\widehat{\theta}_5)$ is the CCP defined in equation (16) with the conditional value function replaced with $\upsilon_k(z_{dt1},\theta_0,\widehat{\theta}_3,\widehat{\theta}_4,\widehat{\theta}_5)$, which is calculated by backward recursion using the estimated choice probabilities and the transition functions outlined in Step 1.

An alternative second-step GMM estimator is formed using the inversion found in Hotz and Miller (1993). Under the assumption that ε is distributed independently and identically as type I extreme values, then the Hotz and Miller inversion implies that

$$\log\Big(p_k(z_{dt1};\theta_0,\widehat{\theta}_3,\widehat{\theta}_4,\widehat{\theta}_5)/p_K(z_{dt1};\theta_0,\widehat{\theta}_3,\widehat{\theta}_4,\widehat{\theta}_{53})\Big) = \upsilon_k(z_{dt1},\theta_0,\widehat{\theta}_3,\widehat{\theta}_4,\widehat{\theta}_5) - \upsilon_K(z_{dt1},\theta_0,\widehat{\theta}_3,\widehat{\theta}_4,\widehat{\theta}_5) \quad (23)$$

for any normalized choice K. We can use $\widehat{p_k(z_{dt1})}$, estimated from Step 1, to form an empirical counterpart to equation (23) and estimate the parameters of our model. The moment conditions can be obtained from the difference in the conditional valuation functions calculated for choice k and the base choice 0. The following moment conditions are produced for an individual at age $t \in \{17,....,55\}$:

$$\xi_{jdt}(\theta_0) \equiv \upsilon_k(z_{dt1},\theta_0,\widehat{\theta}_3,\widehat{\theta}_4,\widehat{\theta}_5) - \upsilon_0(z_{dt1},\theta_0,\widehat{\theta}_3,\widehat{\theta}_4,\widehat{\theta}_5) - \ln\left[\widehat{p_k(z_{dt1})}/\widehat{p_0(z_{dt1})}\right]. \quad (24)$$

Therefore, there are 17 orthogonality conditions and thus $j = 1,...,17$. Letting $\xi_{dt}(\theta_0)$ be the vector of moment conditions at t, these vectors are defined as $\xi_{dt}(\theta_0) = (\xi_{1dt}(\theta_0),\xi_{2dt}(\theta_0),...\xi_{17dt}(\theta_0))'$. Therefore, $E[\xi_{dt}(\theta_0^o)|z_{dt}]$ converges to 0 for every consistent estimator of true CCPs, $p_k(z_{dt1};;\theta_0,\widehat{\theta}_3,\widehat{\theta}_4,\widehat{\theta}_5)$, for $t \in \{17,...,55\}$, and where θ_0^o is the true parameter of the model. Define $\xi_d(\theta_0) \equiv (\xi_{d1}(\theta_0)',...,\xi_{dT}(\theta_0)')'$ as the vector of moment restrictions for a given individual over time and define a weight matrix as $\Phi(\theta_0) \equiv E_t[\xi_d(\theta_0)\xi_d(\theta_0)']$. Then the GMM estimate of θ_0 is obtained via

$$\widehat{\theta}_{02SGMM} = \arg\min_{\theta_0}[1/D\textstyle\sum_{d=1}^{D}\xi_d(\theta_0)]'\widehat{\Phi}[1/D\textstyle\sum_{d=1}^{D}\xi_d(\theta_0)]. \quad (25)$$

where $\widehat{\Phi}$ is a consistent estimator of $\Phi(\theta^o)$.

3.3 Monte Carlo Study

To compare the dynamics of the model in a numerical example and to examine the performance of the estimation, we use a simple human capital investment model with intergenerational transfers that has

the structure of the two-period model in Section 1. We generate simulated data from the model for given parameter values, compare the dynamics, and estimate the model parameters for the generated dataset. We estimate the parameters using the NFXP and PML estimators described above. The estimations are repeated for both algorithms for different specifications of the model in terms of sample size (i.e., for 1000, 10000, 20000, 40000). The number of the structural parameters estimated, including the discount factors, is 3.

NFXP estimation of life-cycle dynastic models is possible only in the simplest dynastic structure. Hence for the Monte Carlo study we choose a simple model which can be estimated by both NFXP and PML. To the best of our knowledge, there is no empirical application of life-cycle dynastic model which is estimated by NFXP. Instead, all the empirical application of life-cycle dynastic model specify the terminal value at the end of an individual's life-cycle as a reduced-form function of the state variable. Dynastic models estimated in this fashion are not suitable for conducting counterfactual policy analysis.

For illustration purpose, we start with the model in which the per-period utility function, $u_k(z_t)$, has a linear form. In each period, $t \in \{0, 1\}$, the individual chooses whether to invest or not, $I_k \in \{0, 1\}$. We assume that individuals can have at most one child, $N \leq 1$. The receive utilities associated with each choice are given by

$$u_k(z_t) = \left\{ \begin{array}{cc} z_t & \text{if } k = 0 \\ (1-\theta)z_t & \text{if } k = 1 \end{array} \right\}$$

where $F_\varepsilon(\varepsilon_t)$ is the choice-specific, unobservable part of the utility; it is assumed to be independently distributed type 1 extreme value.

In the environment in this example the individual begins the life-cycle with a particular set of character traits denoted by $z_t \in (0.5, 0.6, 0.7, 0.8, 0.9)$. Note that at $t = 0$ the individual has not made any choices yet, so the vector z_0 depends fully on initial characteristics x. The value of z_1 is given by the

transformation function $F_k(z_t|z_{t-1})$ that given by the transition matrix:

$$F_0(z_t|z_{t-1}) = \begin{pmatrix} 0.85 & 0.13 & 0.02 & 0 & 0 \\ 0.04 & 0.85 & 0.09 & 0.02 & 0 \\ 0.01 & 0.04 & 0.85 & 0.09 & 0.01 \\ 0 & 0.01 & 0.05 & 0.85 & 0.09 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

and

$$F_1(z_t|z_{t-1}) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.1 & 0.9 & 0 & 0 & 0 \\ 0.13 & 0.27 & 0.6 & 0 & 0 \\ 0.01 & 0.11 & 0.28 & 0.6 & 0 \\ 0 & 0.04 & 0.13 & 0.23 & 0.6 \end{pmatrix}.$$

The individual's traits in the next period are determined by the probabilities in the corresponding row, where each row corresponds to one of the initial values $z_0 \in$ (0.5,0.6,0.7,0.8,0.9), and each column represents character traits in the next period, $z_1 \in$ (0.5,0.6,0.7,0.8,0.9). The transition is such that an individual with character traits $z_0 = 0.5$ who chooses not to have a child such that the choice vector $I_0 = 0$ will have characteristics $z_1 = 0.5$ with a probability of 0.85. In this simplified model, the next generation's initial characteristics z_0' depend only on the sum of the financial investment decisions in the life-cycle.

The educational outcome of the offspring is determined by the intergenerational transition function:

$$M(z_0' \mid z_{T+1}) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.4 & 0.4 & 0.1 \\ 0 & 0 & 0.04 & 0.06 & 0.9 \end{pmatrix},$$

where z_{T+1} can take values in {0,1,2}. The next generation's starting character traits are determined by the probabilities given in the row, where each row corresponds to one of the values of $z_{T+1} \in (0, 1, 2)$ and the first row represents investment level $z_{T+1} = 0$. If the individual invests nothing, then the next generation will have the lowest consumption value with complete certainty. The transition is such that

an individual who opts to invest twice in the life-cycle has a probability of 0.9 that the next generation will start his life-cycle with the characteristics $z_0' = 0.9$.

We simulated the model for a given value of the parameters of the model, $(\theta_2, \beta, \lambda) = (0.25, 0.8, 0.95)$, where θ is the structural parameter of interest that gives the marginal cost of investment, and λ and β are the generational and time discount factors, respectively. We solve the dynamic problem for datasets of 1000, 10000, 20000, 40000 individual dynasties and repeat the simulation 100 times. For the CCP estimation, the initial consistent estimates are estimated nonparametrically using the generated sample. Next, we estimate the model by NFXP and PML.[18] Table 1 presents the results of the estimation for each specification. Not surprisingly, we find that the finite-sample properties of the estimators improve monotonically with sample size. In the NFXP estimation, the mean square error (MSE) of θ drops quickly as the sample size increases. The results for the discount factors are similar: MSEs fall as the sample size increases. In the PML estimation, we observe a similar pattern for all estimators. We obtain similar results from the NFXP and PML estimations. For the sample size of 1000, the PML estimate of the MSE of θ_0 is 0.00249 compared with 0.00288 from the NFXP. The PML estimate of the MSE of λ is 0.01253 compared with 0.00901, and the PML estimate of the MSE of β is 0.00396 compared with 0.00305. For the sample sizes of 10000, 20000 and 40000, the MSEs obtained from PML estimation is lower than the MSEs obtained from the NFXP, but the magnitudes are still very close. In terms of biases, the two estimation algorithms are also quite similar. The major difference between the two estimation algorithms is computational time, which varies greatly between the NFXP and PML even though we simulate a very simple model. The average computational time for the NFXP for a sample of 1000 is 347.6 seconds, but it is only 0.65 seconds for the PML estimation, meaning the PML was 530 times faster. For the sample size of 40000, computation times are 509.8 and 12.6 seconds for the NFXP and PML, respectively, a ratio of 40.4.

4 Extensions

The dynastic framework developed so far in this paper has three major drawbacks. First, parts of the parental investment and transfers from parents to children are monetary in nature. Additionally for

[18] As illustrated in the estimation section, intergenerational models at the final step can be estimated either by the PML or GMM method. For this simulation study we used the PML because it is more comparable to the full solution maximium likelihood.

exposition purpose we assume that there were not borrowing or saving. Monetary investment and/or parental transfers, such as paying for college or purchasing a house for their children, are most naturally characterized as a continuous choice. Also it is natural to introduce borrowing and saving as a continuous choice. Second, the framework assumes that gender does not matter. However, there are significant differences in the cost, choices, and opportunities over an individual's lifetime that are gender specific. Third, which is related to gender but not specific to it, is that individuals normally form households and it take a man and a woman to reproduce, and fertility is central to the model. In this section, we consider extensions to the basic framework that account for these three shortcomings.

4.1 Continuous Choice and Transfer

For the estimation technique developed above to be applicable to a dynastic framework two features must be present. First, all choices must be discrete and second, all systematic state variables, at the initial stage and in every period during the life-cycle, must have a discrete support. We replace these assumptions with two weaker assumptions. The first is that there must be at least one discrete choice variable. This requirement is easily satisfied as birth decision is naturally discrete. The second is that the initial systematic state variable (i.e., endowment that an individual starts adult life with) must belong to a finite set with discrete support. This is weaker than the original assumption and is a less restrictive requirement; it is satisfied in a non trivial number of economic dynastic models – for example, in models where human capital is the major intergenerational transfers and even in models of bequests once the amount transferred is discretized. In practice, in most dynamic programing models the state space is normally discretized. This requirement, however, relaxes the assumption that state space is discrete for the entire lifetime and that all choice variables are discrete. While bequests and initial wealth still must be discrete, the framework allows for any transfers and investments the parents make during their lifetime and map into discrete initial conditions of the child, such as education, houses, or other assets that are discrete in nature. If these assumptions are satisfied then we can modify the representation and then use the estimation strategy for mixed discrete and continuous choice model[19].

For illustration purposes, we extend our framework to include continuos choice of assets and bequests, assuming that we observed data on the per-period assets, A_t, which is continuous. We assume that

[19] See Altug and Miller (1998), Gayle and Miller (2004), Gayle and Golan (2012), and Gayle (2015) for application of CCPs estimators with mixed discrete and continuous choices.

individuals beginning their life as adults with asset level A_0. This level is a bequest from the parents. The initial level of assets, j,is discrete with: $A_0 = [A_0^1, .., A_0^J]$. The budget constraint is given by

$$A_{t+1} - (1 + r)A_t = w_t(x, h_t) - pc_{nt} - c_t, \tag{26}$$

where r is the interest rate for borrowing and savings, and the right hand side is the household income net of expenditures on children and consumption. A few remarks are in order. First, in order to map the assets at age T, A_T, to a discrete bequest level that individuals start their life with, A_0', we define a transition function $\Pr(A_0' = A_0^j | A_T)$. Second, there are different ways to model markets completeness or incompleteness that will translate into different restrictions on savings and assets levels. For illustration purposes, we will assume interior solution for all assets choices and will ignore such restrictions in this presentation. We do not restrict the initial and terminal asset levels to be non-negative. However, the framework can be adjusted to include all these different types of constraints.

Let us further assume that the parents' asset levels can potentially affect educational outcomes of children: higher savings of parents increases the probability of a higher level of educational attainment of the child[20]. We redefine the vector of state variable z_t to capture these new assumptions, $z_t = (\{I_{k1}\}_{k=0}^{17}, ..., \{I_{kt-1}\}_{k=0}^{17}, A_1, ..., A_{t-1}, x)$ with $x \in \{A_0, x_1, ..., x_{|X|}\}$, a discrete set with finite support. Thus x includes all the characteristics that a person in endowed with at the beginning of life. In this application it included the initial (discrete) levels of assets inherited from the parents. As before, $M(x' \mid z_{T+1})$ is the intergenerational transition probability of x conditional on a parent's endowment, x, and the parent's choices over his/her lifetime. It includes the education, inherited assets and potentially skills for example (as well as traits such as gender and race). As before, it is derived from an education production function, $M(x'|z_T)$, and is augmented to incorporate $\Pr(A_0' = A_0^j | A_T)$, the assets transition functions.

Let I_{kt}^o and A_t^o be the sequence of optimal choice over the parent's lifetime. Also, plugging the budget constraint in the utility from consumption, we redefine the systematic part of current utility in equation(8) as

$$u_{kt}(z_t, A_t) = u_{1kt}(z_t) + u_t[w_t(x, h_t) - pc_{nt} - A_{t+1} + (1 + r)A_t, z_t]. \tag{27}$$

[20] Assets can be a proxy of the ability to pay for college for example. However, we allow for assets to impact educational outcomes in order to illustrate the general nature of the extension. One can think of the continous variable as expenditure on children if observed in the data.

Then the lifetime expected utility excluding the dynastic component at the start of an adult's life becomes

$$U_{gT}(x) = E_0\left[\sum_{t=0}^{T}\beta^t[\sum_{k=0}^{17} I_{kt}^o\{u_{1kt}(z_t, A_t^o) + \varepsilon_{kt}\}]|x\right]. \tag{28}$$

The preference shock ε_{kt} is associated with the discrete choices in period t and not the continuous choice variables; therefore it is still indexed with k. As before, we can write the value function of the problem, which represents the expected present discounted value of lifetime utility from following I^o and A_t^o, given z_t and ε_t, as

$$\begin{aligned} V(z_{t+1}, \varepsilon_{t+1}) \quad = \quad & \max_{I_{t+1}, A_{t+1}} E_{I,A}\left(\left\{\sum_{t'=t+1}^{T}\beta^{t'-t}\sum_{k=0}^{17} I_{kt'}[u_{kt'}(z_{t'}, A_{t'}) + \varepsilon_{kt'}]\right.\right. \\ & \left.\left. +\beta^{T-t'}\lambda N(z_T)^{-\nu}\sum_{n=1}^{N} E_T[U_{g+1,n}(x_n')|z_T]\right\}|z_{t+1}, \varepsilon_{t+1}\right). \end{aligned} \tag{29}$$

By Bellman's principle of optimality, the value function can be defined recursively as

$$V(z_t, \varepsilon_t) = \sum_{k=0}^{17}\left(I_{kt}^o(z_t, \varepsilon_t)[u_{kt}(z_t, A_t^o(z_t)) + \varepsilon_{kt}] + \beta\int\left[\int V(z', \varepsilon)f_\varepsilon(\varepsilon)d\varepsilon\right]dF_k(z'|z_t, A_t)]\right),$$

where $f_\varepsilon(\varepsilon)$ is the continuously differentiable density of $F_\varepsilon(\varepsilon_{0t}, .., \varepsilon_{17t})$, and $F_k(z'|z_t, A_t)$ is a transition function for state variables that is conditional on choices $I_{kt}^o = 1$ and $A_t = A_t^0$. Note that $I_{kt}^o(z_t, \varepsilon_t)$ is a function of z_t and ε_t, while $A_t^o(z_t)$ is a function of only z_t. This is a consequence of the additive separability of the preferences shock, which will not affect the continuous choice as demonstrated below. The ex ante value function is then

$$V(z_t) = \sum_{k=0}^{17} p_k(z_t)\left[u_{kt}(z_t, A_t^o(z_t)) + E_\varepsilon[\varepsilon_{kt}|I_{kt} = 1, z_t] + \beta\int V(z')dF_k(z'|z_t, A_t)\right]. \tag{30}$$

In this form, $V(z_t)$ is now a function of the CCPs, the continuous choice decision rule, the expected value of the preference shock, the per-period utility, the transition function, and the *ex ante* continuation value. All components except the conditional probability, the continuous choice decision rule and the ex ante value function are primitives of the initial decision problem. By writing the CCPs and the continuous choice decision rule as a function of just the primitives and the ex ante value function, we can characterize the optimal solution of the problem (i.e., the ex ante value function) as implicitly dependent

on the primitives of the original problem. Let us define the conditional value function, $\upsilon_k(z_t, A_t)$, as

$$\upsilon_k(z_t) = \max_{A_t} \left[u_{kt}(z_t, A_t) + \beta \int V(z') dF_k(z'|z_t, A_t) \right]. \tag{31}$$

Therefore, the probability of observing choice k, conditional on z_t, $p_k(z_t)$, is still given by

$$p_k(z_t) = \int \left[\prod_{k \neq k'} 1\{\upsilon_k(z_t) - \upsilon_{k'}(z_t) \geq \varepsilon_{kt} - \varepsilon_{tk'}\} \right] f_\varepsilon(\varepsilon_t) d\varepsilon_t. \tag{32}$$

However, the optimal continuous choice is found in two steps. First, find the optimal choice conditional on $I_{kt} = 1$, which is defined as $A_{tk}(z_t)$. This is characterized by the following Euler equation:

$$\frac{\partial u_{kt}(z_t, A_t)}{\partial A_t} = -\beta \frac{\partial \int V(z') dF_k(z'|z_t, A_t)}{\partial A_t}. \tag{33}$$

Then substitute it into the conditional valuation function:

$$\upsilon_k(z_t) = \left[u_{kt}(z_t, A_{tk}(z_t)) + \beta \int V(z') dF_k(z'|z_t, A_t) \right], \tag{34}$$

and find the optimal discrete choice:

$$I^o(z_t, \varepsilon_t) = \arg\max_I \textstyle\sum_{k=0}^{17} I_{kt} [\upsilon_k(z_t) + \varepsilon_{kt}].$$

Finally, we obtain the optimal continuous choice by setting $A_t^o(z_t) = A_{tk}(z_t)$ if $I_{kt}^o(z_t, \varepsilon_t) = 1$.

We now can find an alternative value function that is a function of only $p_k(z_t)$, $A_{tk}(z_t)$, and the primitives of the model. We can now state a more general version of Proposition 1.

Proposition 2 *There exists an alternative representation for the ex ante conditional value function at time t that is a function of only the primitives of the problem and the CCPs as follows:*

$$\begin{aligned} \upsilon_k(z_t) \;=\; & u_{kt}(z_t, A_{tk}(z_t)) \\ & + \sum_{t'=t+1}^{T} \beta^{t'-t} \sum_{s=0}^{17} \int [p_s(z_{t'}) [u_{st'}(z_{t'}, A_{t'k}(z_t)) + E_\varepsilon(\varepsilon_{st'} | I_{st'} = 1, z_{t'})] dF_k^o(z_{t'}|z_t) \\ & + \lambda \beta^{T-t} \int N_T(z_{_T})^{-\nu} \sum_{n=1}^{N_T} \sum_{x'} V(x') \textstyle\sum_{s=0}^{K_T} [M_k^n(x'|z_{_T}) p_s(z_{_T})] dF_k^o(z_{_T}|z_t), \end{aligned} \tag{35}$$

where $F_k^o(z_{t'}|z_t)$ *is the* $t'-t$ *period-ahead optimal transition function, recursively defined as*

$$F_k^o(z_{t'}|z_t) = \begin{cases} F(z_{t'}|z_t, I_{kt}=1, A_{tk}(z_t)) & \text{for } t'-t=1 \\ \sum_{r=0}^{17} \sum_{z_{t'-1}} p_r(z_{t'-1}) F(z_{t'}|z_{t'-1}, I_{rt'-1}=1, A_{t'-1k}(z_{t'-1})) F_k^o(z_{t'-1}|z_t) & \text{for } t'-t>1, \end{cases}$$

where $N_T(z_T)$ *is the number the children induced from* z_T, K_T *is the number of possible choice combinations available to the individual in the terminal period (in which birth is no longer feasible) and* $M_k^n(x'|z_T) = M(x'|z_T)$ *conditional on* $I_{kT}=1$ *for the nth child born in a parent's life-cycle.*

This representation is similar to the one in Proposition 1 except for the inclusion of $A_{tk}(z_t)$ and the replacement of an integral for a summation deal with the continuous state variables over the life-cycle. The inversion – and hence the estimation – follows through as before except we now need a first-stage consistent estimate of $A_{tk}(z_t)$ as well. This is obtained as $A_{tk}(z_t) = E[A_t|z_t, I_{kt}=1]$[21].

4.2 Household and Gender

We extend the basic framework to include household decisions and gender. To the best of our knowledge, no other paper estimates dynastic models with household decisions. There are many models of household decisions; here we show how to extend the model to incorporate a unitary decisions-makers. The framework can be extended to deal with collective household decisions: see Gayle, Golan, and Soytas (2014) for an application of this estimation technique to a non corporative collective model of household behavior. Let an individual's gender, subscripted as σ, take the value of m for a male and f for a female: $\sigma = \{f, m\}$. Gender is included in the vector of invariant characteristics x_σ. Let K describe the number of possible combinations of actions available to each household. Individuals get married at time 0, and for simplicity we assume there is no divorce (see Gayle, Golan, and Soytas, 2014 for an application with marriage and divorce). Households are assumed to live for T periods and die together. Time 0 is normalized to account for the normal age gap between married couples, which would imply that men have a longer childhood than women. All individual variables and earnings are indexed by the gender subscript σ. We omit the gender subscript when a variable refers to the household (both spouses). The state variables are extended to include the gender of the offspring. Let the vector ζ_t indicate the gender

[21] We are assuming that there is no additional stochastic element in the determination of $A_{tk}(z)$.

of a child born at age t, where $\zeta_t = 1$ if the child is a female and $\zeta_t = 0$ otherwise. The vector of state variables is expanded to include the gender of the offspring is as follows:

$$z_t = (\{I_{k1}\}_{k=0}^{K}, ..., \{I_{kt-1}\}_{k=0}^{K}, \zeta_0, .., \zeta_{t-1}, x_f, x_m).$$

We assume households invest time and money in the children in the household. The function $w_{\sigma t}(z_t, h_{\sigma t})$ denotes the earnings function; the only difference from the single-agent problem is that gender is included in z_t and can thus affect earnings. The total earnings is the sum of individual earnings as $w_t(z_t, h_t) = w_{1t}(z_t, h_{ft}) + w_{2t}(z_t, h_{mt})$, where $h_t = (h_{ft}, h_{mt})$. The educational outcome of the parents' offspring is mapped from the same parental inputs as the single-agent model: income and time investment, number of older and younger siblings, and parental characteristics such as education, race, and labor market skills. In this extension, gender is also included as a parental characteristic. Thus, the production function is still denoted by $M(x'|z_{_{T+1}})$, where $z_{_{T+1}}$ represents the state variables at the end of the parents' life-cycle, T.

In the household, the total per-period expenditures cannot exceed the combined income of the spouses. The budget constraint for the household is given by

$$w_t \geq c_t + \alpha_{Nc}(z_t)(N_t + b_t)w_t(z_t, h_t). \tag{36}$$

The right-hand side of equation (36) represents expenditures on personal consumption of the parents, c_t, and on children. Parents pay for the children living in their household, regardless of the biological relationship, and do not transfer money to any biological children living outside the household.

As in the single-agent model, we can eliminate the continuous choice in the lifetime utility problem so that households face a purely discrete choice problem. Recall that the budget constraint for the household, assuming no borrowing or saving, is

$$w_t(z_t, h_t) - \alpha_N(z_t)(N_t + b_t)w_t(z_t, h_t) = c_t, \tag{37}$$

and, as in the single-agent problem, we may substitute for consumption in u_2 and obtain the following

household utility function:

$$u_{kt}(z_t) = \theta_k(z_t) + u_t[w_t(z_t, h_t)(1 - \alpha_N(z_t)(N_t + b_t)), z_t]. \tag{38}$$

$\theta_k(z_t)$ is the explicit functional form we assumed for the $u_{1kt}(z_t)$ in equation (8). In this formulation each discrete choice k corresponds to a utility level characterized by the parameter $\theta_k(z_t)$.

For notational simplicity, let $x_f \in \{f\}_{f=1}^{F}$, $x_m \in \{m\}_{m=1}^{M}$, and P_{fm} be the probability that a type-f female marries a type-m male at age 0. We can then define the expected lifetime utility for a type-(f, m) household at age 0, excluding the dynastic component, as:

$$U_T(f, m) = E_0 \left[\sum_{t=0}^{T} \beta^t \sum_{k=0}^{K} I_{kt}^0 \{u_{kt}(z_t) + \varepsilon_{kt}\} \right], \tag{39}$$

and the expected lifetime utility for a type-(f, m) household at age 0 as

$$U(f, m) = U_T(f, m) + \beta^T \lambda E_0 \left[N^{-\nu} \sum_{n=1}^{N} \sum_{f'=1}^{F} \sum_{m'=1}^{M} P_{f'm'} U_n(f', m') | f, m \right]. \tag{40}$$

As in the single individual version of the model, we can define the expected present discounted value of the lifetime utility of the household at any period t as

$$V(z_t, \varepsilon_t) = \max_I E_I \left(\sum_{s=t+1}^{T} \beta^{s-t} \sum_{k=0}^{K} I_{ks}[u_{ks}(z_s) + \varepsilon_{ks}] + \beta^{T-s} \lambda N^{-\nu} \sum_{n=1}^{N} \sum_{f'=1}^{F} \sum_{m'=1}^{M} P_{f'm'} U_n(f', m') | z_t, \varepsilon_t \right). \tag{41}$$

This can be written recursively as

$$V(z_t, \varepsilon_t) = \sum_{k=0}^{K} I_{kt}^o(z_t, \varepsilon_t)[u_{kt}(z_t) + \varepsilon_{kt}] + \beta \sum_{z'} \int V(z', \varepsilon) f_\varepsilon(\varepsilon) d\varepsilon F(z' | z_t, I_{kt}^o = 1)],$$

where $f_\varepsilon(\varepsilon)$ is the continuously differentiable density of $F_\varepsilon(\varepsilon_{0t}, .., \varepsilon_{17t})$, $F(z' | z_t, I_{kt} = 1)$ is a transition function for state variables conditional on choice k, and $I_{kt}^o(z_t, \varepsilon_t)$ is the optimal household decision rule. Similar to equation (13), we can define the conditional choice household probability as $p_k(z_t) = E[I_{kt}^o = 1 | z_t]$ and the ex ante value function as

$$V(z_t) = \sum_{k=0}^{K} p_k(z_t) \left[u_{kt}(z_t) + E_\varepsilon[\varepsilon_{kt} | I_{kt} = 1, z_t] + \beta \sum_{z'} V(z') F(z' | z_t, I_{kt} = 1) \right]. \tag{42}$$

The rest of the estimation carries through as in the single individual case.

The addition of the two household members to the model captures important issues of the degree of specialization in housework and labor market work in households with different composition of education levels between it members. The importance of which spouse spends time with the children (and the amount of time) depends on the production function of the education of children and whether the time of spouses are complements or substitutes. Furthermore, we capture patterns of assortative mating that may amplify the persistence of income across generations relative to a more random matching pattern. Since in our model there is a potential correlation of the cost of transfers to children (time input) with both parents' characteristics, assortative mating patterns imply that if children of more educated parents are more likely to be more educated, they are also more likely to have a more educated spouse, which increases the family resources and their children's educational outcomes.

5 Empirical Application

To illustrate the estimation method, we estimate the unitary household model developed in the previous section and use it to analyze the effect of Nature versus Nurture on intergenerational mobility. We estimate the model using a dataset compiled from the Panel Study of Income Dynamics (PSID). The PSID provides a large panel of matched data on individuals' labor market hours, earnings, housework hours, marriage, and childbirth histories for overlapping cohorts and generations.

Table 2 presents the summary statistics for our sample. Column (1) summarizes the overall sample, Column (2) shows data only for parents, and Column (3) summarizes data of their children. The first generation is on average 7 years older than the second generation. As a consequence, a higher proportion are married in the first generation. The male-to-female ratio is similar across generations (about 55 percent female), and this ratio is higher in our sample than in the general population because females are more likely to maintain responsibility for children in cases of divorce. Our sample contains a higher proportion of blacks than the general population, which is consistent with PSID survey procedures, and the second generation has an even higher proportion of blacks than the first generation (about 29 percent in the second and 20 percent in the first generation) because of higher fertility rates among blacks in our sample. There are no significant differences across generations in completed years of education. The second generation in our sample has a lower average age than the first generation, so the second generation

also has a lower marriage rate and a lower average number of children, annual labor income, labor market hours, housework hours, and mean time spent with children. Our second-generation sample spans the same age range, 17 to 55, as the first sample. For the estimation, we retain only married households and include the married individuals as of age 25 with all the individual years of observations whenever the family is intact up to age 40. Further, to account for the time and monetary investments during the early years of the child's life after birth, we exclude individuals who already have a child by age 25.

5.1 Empirical Implementation

This section describes the choice set specifications and functional forms of the model that we estimate. We assume that all individuals enter the first period of the life-cycle married. That is, they transition into a married household immediately after becoming adults. When individuals transition into a married household, their spouses' characteristics are drawn from the known matching function $G(x_{-\sigma}|\ x_{\sigma})$. The matching function depends on the individual's state variables – for example, it separately captures the effect of the number of children and past actions that affect labor market experience on the spouse's characteristics.

We set the number of an adult's periods in each generation to $T = 30$ and measure the individual's age where $t = 0$ is age 25 because at this age most individuals would have completed their education and started their family. As discussed earlier, we assume that parents receive utility from adult children, whose educational outcome is revealed at the last period of their life regardless of the birth date of the children. This assumption is similar to the Barro-Becker assumptions. We avoid situations where the outcome of an older child is revealed while parents make fertility and time investment decisions to ensure that (i) these decisions are not affected by adult children outcomes and (ii) adult children's behavior and choices do not affect investment in children and fertility of the parents, in which case solutions to the problems are significantly more complicated and it is not clear whether a solution exists.

The three levels of labor supply correspond to working 40 hours a week; individuals working fewer than 3 hours per week are classified as not working, individuals working between 3 and 20 hours per week are classified as working part-time, while individuals working more than 20 hours per week are classified as working full-time. There are three levels of parental time spent with children corresponding to no time, low time, and high time. To control for the fact that females spend significantly more time with children

than males, we use a gender-specific categorization. We use the 50th percentile of the distribution of parental time spent with children as the threshold for low versus high parental time with children, and the third category is 0 time with children. This classification is done separately for males and females. Finally, birth is a binary variable; it equals 1 if the mother gives birth in that year and 0 otherwise. Therefore, the household choices are a combination of labor supply and time with children for males and females in the household plus the birth decision.

Labor Market Earnings An individual's earnings depend on the subset of his or her characteristics, $z_{\sigma t}$. These include age, age squared, and dummy variables indicating whether the individual has completed high school, some college, or college (or more) education interacted with age, respectively; the omitted category is less than high school. Let η_σ be the individual-specific ability, which is assumed to be correlated with the individual-specific time-invariant observed characteristics. Earnings are assumed to be the marginal productivity of workers and are assumed to be exogenous, linearly additive, and separable across individuals in the economy. The earnings equations are given by

$$w_{\sigma t} = \exp(\delta_{0\sigma} z_{\sigma t} + \textstyle\sum_{s=0}^{\rho} \delta_{\sigma,s}^{pt} \sum_{k_{t-s} \in \mathcal{H}_{P\sigma}} I_{k_{t-s}\sigma} + \sum_{s=1}^{\rho} \delta_{\sigma,s}^{ft} \sum_{k_{t-s} \in \mathcal{H}_{F\sigma}} I_{k_{t-s}\sigma} + \eta_\sigma), \tag{43}$$

where $\mathcal{H}_{P\sigma}$ and $\mathcal{H}_{F\sigma}$ are the set of choices for part-time and full-time work, respectively. Therefore, the earnings equation depends on experience accumulated while working part-time or full-time and the current level of labor supply. Thus, $\delta_{\sigma,s}^{pt}$ and $\delta_{\sigma,s}^{ft}$ capture the depreciation of the value of human capital accumulated while working part-time or full time, respectively. In the estimation, we assume $\rho = 4$ given that the effect of experience with higher lags is insignificant (Gayle and Golan, 2012; Gayle and Miller, 2004).

Production function of children We assume that race is transmitted automatically to children and rule out interracial marriages and fertility. This is done because of insufficient interracial births in our sample to study this problem. Therefore, parental home hours when the child is young affect the future educational outcome of the child, which is denoted by Ed'_σ[22], and innate ability, η'_σ, both of which affect the child's earnings (see equation (43)).The state vector for the child in the first period of the life-cycle

[22]Level of education, Ed_σ, is a discrete random variable in the model where it can take 4 different values: less than high school (LHS), high school (HS), some college (SC), and college (COL).

is determined by the intergenerational state transition function $M(x'|z_{T+1})$; specifically, we assume that

$$M(x'|z_{T+1}) = \left[\Pr(\eta'_\sigma \mid Ed'_\sigma), 1\right] \Pr(Ed'_\sigma \mid z_{T+1}). \tag{44}$$

Thus, we assume that the parental inputs and characteristics (parental education and fixed effects) determine educational outcomes according to the probability distribution $\Pr(Ed'_\sigma \mid z_{T+1})$. In our empirical specification, the state vector of inputs, z_{T+1}, contains the parental characteristics, the cumulative investment variables (low time and high time with children) of each parent up to period T, the permanent income of each parent, and the number of a child's siblings. In the data, we observe only total time devoted to children each period; thus, we assign each child age 5 or younger in the household the average time investment, assuming all young children in the household receive the same time input. Parental characteristics include the education of the father and mother, their individual-specific effects, and race. Once the education level is determined, it is assumed that the ability η'_σ is determined according to the probability distribution $\Pr(\eta'_\sigma \mid Ed'_\sigma)$. The above form of the transition allows us to estimate the equations separately for the production function of children given as the first two probabilities and the marriage market matching given as the last term.

Contemporaneous utility We assume that the per-period utility from consumption is linear; therefore, equation (38), the utility for a single parent from consumption and children (after substituting the budget constraint), becomes

$$u_{kt}(z_t) = \theta_k(z_t) + \alpha w_t(z_t, h_t) - \alpha\alpha_N(z_t)(N_t + b_t), \tag{45}$$

where $\theta_k(z_t)$ are the coefficients associated with each combination of time allocation choice, thus capturing the differences in the value of nonpecuniary benefits/costs associated with the different activities. The vector of decisions includes birth; thus, we allow the utility associated with different time allocations to depend on whether or not there is a birth. As discussed earlier, this utility captures not only the level of leisure but also the nonpecuniary costs/benefits associated with the different activities. For example, we do not rule out that time spent with children may be valued and that the nonpecuniary costs/benefits depend on birth events and levels of labor supply.

We assume no borrowing and saving, one consumption good with price normalized to 1, and risk neutrality. The first term represents the utility from a parent's own consumption. The second term, however, represents the net utility/costs from having young children in the household. In general, given our assumptions, we can use a budget constraint to derive the coefficients on income and number of children and a separate, nonpecuniary utility from children and monetary costs. However, since we do not have data on consumption or expenditures on children, the coefficients on the number of children also capture nonpecuniary utility from children and cannot be identified separately from the monetary costs of raising children. The interaction of income with the number of children and education captures differences in the costs of raising children by the socioeconomic status of the parents. By assuming a linear utility function, we abstract from risk aversion and insurance considerations that may affect investment in children, fertility, as well as the labor supply. For families, we ignore the insurance aspects of marriage and divorce. While these issues are potentially important, we abstract from them and focus on transmission of human capital. The no borrowing and savings assumption is extreme and allows us to test (i) whether income is important in the production function of education of children and (ii) whether the timing of income is important.

6 Empirical Results

This section presents results of estimation and analysis of the structural model. First, we present estimates from Step 1 of our estimation procedure. Second, we present estimates from Step 2 of the estimation, which is estimated using the Hotz et al. (1994) extension of the Hotz and Miller (1993) estimator[23]. Third, we present results that assess how well our model fits the data. Finally, we present the counterfactual values of each type of household, which also can be interpreted as the return to parental investment in children; the value function of the children includes the value of their education, earnings, as well as the spouse they married and his or her income.

[23] We use the Hotz et al. (1994) estimator instead of the original Hotz and Miller (1993) estimator because the forward simulation used in the former significantly reduces the computational burden involved in computing the life-cycle component of the dynastic model.

6.1 First-Stage estimation

The first-stage estimation include estimates of the earnings equation, the unobserved skills function, the intergenerational education production function, and the marriage assignment functions. All these functions are fundamental parameters of our model and are estimated outside the estimation of the preferences, discount factors, and the net costs of raising children. The first-stage estimates also include equilibrium objects such as the CCPs. Below we present estimates on the main earnings equation, the unobserved skills function, and the intergenerational education production function. The estimates of the marriage assignment functions and the CCPs are included in a supplementary appendix.

Earnings equation and unobserved skills Table 3 presents the estimates of the earnings equation and the function of unobserved (to the econometrician) individual skill (see also Gayle, Golan, and Soytas, 2014). The top panel of the first column shows that the age-earnings profile is significantly steeper for higher levels of completed education; the slope of the age-log-earnings profile for a college graduate is about 3 times that of an individual with less than a high school education. However, the largest gap is for college graduates; the age-log-earnings profile for a college graduate is about twice that of an individual with only some college. These results confirm that there are significant returns to parental time investment in children in terms of the labor market because parental investment significantly increases the likelihood of higher education outcomes, which significantly increases lifetime labor market earnings.

The bottom panel of the first column and the second column of Table 3 show that male full-time workers earn 2.6 times more than part-time male workers and female full-time workers earn 2.3 times more than females part-time workers (see also Gayle, Golan, and Soytas (2014)). It also shows that there are significant returns to past full-time employment for both genders; however, females have higher returns to full-time labor market experience than males. The same is not true for part-time labor market experience; males' earnings are lower if they worked part-time in the past, while there are positive returns to the most recent female part-time experience. However, part-time experiences 2 and 3 years in the past are associated with lower earnings for females; these rates of earnings reduction are, however, lower than those for males. These results are similar to those in Gayle and Golan (2012) and perhaps reflect statistical discrimination in the labor market in which past labor market history affects employers' beliefs

about workers' labor market attachment in the presence of hiring costs.[24] These results imply there are significant costs in the labor market in terms of the loss of human capital from spending time with children, if spending more time with children comes at the expense of working more in the labor market. These costs may be smaller for females than males because part-time work reduces compensation less for females than for males. If a female works part-time for 3 years, for example, she loses significantly less human capital than a male working part-time for 3 years instead of full-time. This difference may give rise to females specializing in child care; this specialization comes from the labor market and production function of a child's outcome, as is the current wisdom.

The unobserved skill (to the econometrician) is assumed to be a parametric function of the strictly exogenous time-invariant components of the individual variables. This assumption is used in other papers (e.g., MaCurdy, 1981; Chamberlain, 1986; Nijman and Verbeek, 1992; Zabel, 1992; Newey, 1994; Altug and Miller, 1998; and Gayle and Viauroux, 2007). It allows us to introduce unobserved heterogeneity to the model while still maintaining the assumption on the discreteness of the state space of the dynamic programming problem needed to estimate the structural parameters from the dynastic model. Also because the unobserved is estimated in the first step and hence is data in the subsequence steps this does not introduce the standard initial condition problem. The Hausman test statistic shows that we cannot reject this correlated fixed effect specification. Column (3) of Table 4 presents the estimate of skills as a function of unobserved characteristics; it shows that blacks and females have lower unobserved skills than whites and males. This could capture labor market discrimination. Education increases the level of skills but it increases at a decreasing rate with the level of completed education. The rates of increase for blacks and females with some college and a college degree are higher than those of their white and male counterparts. This pattern is reversed for blacks and females with a high school diploma. Notice that skills are another transmission mechanism through which parental time investment affects labor market earnings in addition to education.

Intergenerational education production function A well-known problem with the estimation of production functions is the simultaneity of the inputs (time spent with children and income). As is clear from the structural model, the intergenerational education production function suffers from a similar problem. However, because the output of the intergenerational education production (i.e., completed

[24] These results are also consistent with part-time jobs difffering more than full-time jobs for males than for females.

education level) is determined across generations while the inputs, such as parental time investment, are determined over the life-cycle of each generation, we can treat these inputs as predetermined and use instruments from within the system to estimate the production function.

Table 4 presents results of a three-stage least squares estimation of the system of individual educational outcomes; the estimates of the two other stages are in the supplementary appendix. The system includes the linear probabilities of the education outcomes, $\Pr(Ed'_\sigma \mid z_{T+1})$, as well as the labor supply, income, and time spent with children equations. The estimation uses the mother's and father's labor market hours over the first 5 years of the child's life as well as linear and quadratic terms of the mother's and father's age on the child's fifth birthday as instruments. The estimation results show that controlling for all inputs, a child whose mother has a college education has a higher probability of obtaining at least some college education and a significantly lower probability of not graduating from high school relative to a child with a less educated mother; while the probability of graduating from college is also larger, it is not statistically significant. If a child's father, however, has some college or a college education, the child has a higher probability of graduating from college. This is consistent with the findings of Rios-Rull and Sanchez-Marcus (2002).

We measure parental time investment as the sum of the parental time investment over the first 5 years of the child's life. The total time investment (i.e., the sum of the per-period investment of the first 5 years of a child's life) is a variable that ranges between 0 and 10 because low yearly parental investment is coded as 1 and high yearly parental investment is code as 2. The results in Table 5 show that while a mother's time investment significantly increases the probability of a child graduating from college or having some college, a father's time investment significantly increases the probability of the child graduating from high school or having some college. These estimates suggest that while a mother's time investment increases the probability of a high educational outcome, a father's time investment truncates low educational outcome. However, the time investment of both parents is productive in terms of their children's education outcomes. It is important to note that mothers' and fathers' hours spent with children are at different margins, with mothers spending significantly more hours than fathers. Thus, the magnitudes of the discrete levels of time investment of mothers and fathers are not directly comparable since low and high investment of time differs across genders.

6.2 Second-Stage estimation

This section presents estimates of the intergenerational and intertemporal discount factors, the preference parameters, and child care cost parameters. Table 6 presents the discount factors. It shows that the intergenerational discount factor, λ, is 0.795. This implies that in the second-to-last period of the parent's life, a parental valuation of their child's utility is 79.5% of their own utility. The estimated value is in the same range of values obtained in the literature calibrating dynastic model (Rios-Rull and Sanchez-Marcos, 2002; Greenwood, Guner, and Knowles, 2003). However, these models do not include the life-cycle. The estimated discount factor, β, is 0.81. The discount factor is smaller than typical calibrated values; however, the few papers that have estimated it find similar values (e.g., Arcidiacono, Sieg, and Sloan, 2006, find it to be 0.8).[25] Lastly, the discount factor associated with the number children, υ, is 0.25 which implies that the marginal increase in value from the second child is 0.68 and from the third child is 0.60. Identification of the discounts factors are nontrivial in dynamic discrete models. Here we have three discount factors to identify instead of one as in the standard discrete choice models. However, note that past home hours, when the children are young, affect only the transition functions and not the current utility, so we have the common exclusion restrictions used to identify dynamic discrete choice models (Magnac and Thesmar, 2002; Fang and Wang, 2015). Then the identification of the intergenerational discounts factors follows by a direct application of the proof of Proposition 2 in Fang and Wang (2015) to our setting.

Table 5 also presents the marginal utility of income, which is positive and increasing with the number of children except for a household with a college graduate wife and a husband with at least a high school education. Also, a husband's education decreases the marginal utility of income for families with children. The marginal utility of income for families with children is also lower for black families.

The right panel of Table 5 presents our estimates of the disutility/utility from various combination of household choices. As is usual in discrete choice models, these are estimated relative to an outside choice, which is both spouses not (i) working, (ii) giving birth, or (iii) spending any time with young children. We also use an additive specification in which the costs of birth, work and time with children are additively separable. First, every labor supply choice of the household carries with it a disutility relative to the reference choice except for households in which both spouses work full-time (which statistically is

[25] We are not aware of dynastic models in which the time discount factor is estimated.

no different from disutility/utility reference) and when the wife does not work and the husband works full-time. In the data, if both spouses spend low time with children and there is no birth, then both spouses are equally likely to be observed working full-time than not working – hence the equal utility for both sets of choices. Second, there are no distinct patterns to utility from time with children; these estimates are highly nonlinear, perhaps reflecting that it is a mixture of leisure and disutility. However, giving birth provides a positive utility. This implies that, although parents get utility from the quality of their children, they also get some instantaneous utility from a birth.

6.3 Model Fit

In this section, we first assess the ability of our model to reproduce the basic stylized facts by race, gender, and marital status as summarized in Section 2. We assess how well our model predicts the choices of labor supply, home hours with young children, and birth. Our model is over identified and passes the standard over identifying restrictions J-test. In the estimation, the CCPs are targeted; in the model fit analysis, we simulate a sample of individuals and determine whether the individuals in our simulated sample behave like the individuals in our data. In some regards, this exercise is equivalent to a graphical summary of our model's over identification test.

Table 6 presents the model's fit. The model matches the labor supply patterns between gender and across race well. While it also matches the variation across race and gender for parental time with children, the levels are not similar in all cases. In examining the birth decisions, the model produces the differences in birth rates across households of different race, but it underpredicts the fecundity of whites by about a half. This lower birth rate is partly rationalized by the lower time with children predicted by the model. Nevertheless, our empirical model specification is very parsimonious: We do not include race, education, or marital status in the preference parameters for the disutility/utility of the different choices. In addition, the only unobserved heterogeneity is estimated from the earnings equations. Still, the model performs well in replicating the data based primarily on the economic interactions embodied in it.

6.4 The Effect of Nature versus Nurture on Intergenerational Mobility

One of the major benefit of our approach is the ability to do full blown counterfactual analysis. An alternative approach which has been used in empirical structural models to incorporate intergenerational/altruistic concerns is to modify the standard dynamics structural estimation methods by introducing an approximation for the value parents place on their children's adults outcome as a function of some state variables, normally the educational outcome or test scores (See example Bernal (2008), Brown and Flinn (2011), and Del Boca, Flinn, and Wiswall (2013) among others.). The main advantage of this alternate approach is that the estimation is easier and standard techniques in the literature can be used. The major disadvantage is that welfare /counterfactual analysis is subject to the Lucas's Critique. That is, in a counterfactual environment the value parents place on their children quality changes in two ways, the value of the state variables and the functional form of the mapping between the state variables and the utility derived from those state variables. The is made obvious by an examination of equations (18) and (20). This alternative approach does not allow the functional form of the mapping to change.

From equation (18) and (20), one can see that our framework can nested this alternative approach, since the approximation used in the alternative approach is equivalent to conducting a welfare/counterfactual analysis holding fixed the CCPs and transition used in the calculation of the value of a child. To illustrate the bias induced by ignoring the fact that the children themselves will re-optimized when we change the economic environment we asked the counterfactual question of how much of the mobility across generation is due to the automatic transmission education across generation as oppose to differences in parent investment between parents of different educational background. To do this we eliminate the automatic transition of education in production function[26]. We report the results from two models. Model (1) is the model estimated in above. Model (2) is a model in which the discount factors (β and λ) are set to values commonly used in the literature ($\beta = 0.90$ and $\lambda = 0.95$) and all other parameters estimated.

Table 6 presents the summary of labor supply, time investment, and birth rate by gender and race for the data, baseline models and counterfactual simulations. It shows that if we eliminate the portion of parental education that is transmitted automatically across generation then parents will re-optimize and change labor supply, time investment, and fertility behaviors. Therefore, a pure statistical decomposition would be inappropriate for answer the question of how much mobility would change if there were no

[26] Operationally we set the effect on education in the intergenerational production function equal to that of high school graduate irrespective of the mother's and father's education level.

automatic (Nature) transmission of education from parents to children. The columns $NN1$ presents the counterfactual estimates of our model and $NN1'$ the estimation results of the approximated model; similarly, $NN2$ presents the results of the counterfactual from our model with ($\beta = 0.90$ and $\lambda = 0.95$) and $NN2'$ the results of the counterfactual of the approximated model. The columns $NN1'$ and $NN2'$ show that not taking into account that the all subsequent generations will also re-optimize induces significant bias with the bias being greater the larger the discount factors.

To obtain a number that summarize the impact on mobility, Figure 1 presents the probability that a children born in a family in the bottom 20 percent of the family income distribution will end up a family in with family income above the median of the next generation family income distribution. It shows that in the baseline model, i.e. model1, that only 30 percent of children born in the bottom 20 percent will end up with families earning above the median. However, if the automatic transition of education was eliminated that probability would increase by about 20 percent to about 40 percent. However, we not account for the fact that subsequent generation will re-optimize we would over estimate it impact by about 25 percent. Model (2) shows the similar qualitative patterns but shows that the overestimate of the impact of 'Nature' on mobility could be as high as 90 percent. Which illustrate the gain from using the approach outline in this paper.

7 Conclusion

This paper provides a new representation of the value function for discrete choice dynastic models that partially overcomes the curse of dimensionality of dynastic models by exploiting properties of the stationary decision rules. The representation can be used in multi-stage CCP estimators to estimate a rich class of dynastic models including investment in children's human capital, monetary transfers, unitary households, endogenous fertility, and a life-cycle within each generation. Under certain conditions, we show that the framework can also accommodate continuous choice variables. The paper extends methods used in the literature for the estimation of single-agent non-dynastic models to the dynastic setting. The paper compares the performance of a multi-stage CCP estimator based on the new value function representation with a modified version of the full solution MLE using simulations and finds that estimates are close to the nested fixed point estimates as the sample increases but the computation time is reduced substantially.

The paper then provides an application of a unitary household model in which households choose labor supply, time with children, and fertility; human capital is transmitted across generations by monetary and time investments of the parents. We then used the estimate model to conduct counterfactual simulations, investigating the role of the automatic transmission of education across generations (Nature effect) in accounting for the intergenerational immobility at the bottom of the income distribution. We find that without the Nature effect in the intergenerational education production function mobility at the bottom of the income distribution would have been 20 percent higher. Finally, not accounting for the re-optimization of sequent generations in the model, as is done in the approach outlined in this paper, will overstate the effect of the nature effect on mobility by between 20 and 90 percent.

A Appendix

Proof of Proposition 1. Recall the conditional value function in equation (14):

$$\upsilon_k(z_t) = u_{kt}(z_t) + \beta \sum_{z_{t+1}} V(z_{t+1}) F(z_{t+1}|z_t, I_{kt} = 1) \tag{46}$$

We begin by noting that

$$V(z_{t+1}) = \sum_{s=0}^{17} p_s(z_{t+1}) \left[u_{st+1}(z_{t+1}) + E_\varepsilon(\varepsilon_{st+1}|I_{st+1} = 1, z_{t+1}) + \beta \sum_{z_{t+2}} V(z_{t+2}) F(z_{t+2}|z_{t+1}, I_{st+1} = 1) \right]. \tag{47}$$

Combining equations (46) and (47) gives:

$$\begin{aligned} \upsilon_k(z_t) = u_{kt}(z_t) &+ \beta \textstyle\sum_{z_{t+1}} \sum_{s=0}^{17} p_s(z_{t+1}) \left[u_{st+1}(z_{t+1}) + E_\varepsilon(\varepsilon_{st+1}|I_{st+1} = 1, z_{t+1}) \right] F(z_{t+1}|z_t, I_{kt} = 1) \\ &+ \beta^2 \textstyle\sum_{z_{t+1}} \sum_{s=0}^{17} p_s(z_{t+1}) \left[\sum_{z_{t+2}} V(z_{t+2}) F(z_{t+2}|z_{t+1}, I_{st+1} = 1) \right] F(z_{t+1}|z_t, I_{kt} = 1). \end{aligned} \tag{48}$$

Similarly,

$$V(z_{t+2}) = \sum_{r=0}^{17} p_r(z_{t+2}) \left[u_{rt+2}(z_{t+2}) + E_\varepsilon(\varepsilon_{rt+2}|I_{rt+2} = 1, z_{t+2}) + \beta \sum_{z+3} V(z_{t+3}) F(z_{t+3}|z_{t+2}, I_{rt+2} = 1) \right]. \tag{49}$$

Substituting equation (49) into equation (48) gives

$$\begin{aligned} \upsilon_k(z_t) = u_{kt}(z_t) &+ \beta \textstyle\sum_{z_{t+1}} \sum_{s=0}^{17} p_s(z_{t+1}) \left[u_{st+1}(z_{t+1}) + E_\varepsilon(\varepsilon_{st+1}|I_{st+1} = 1, z_{t+1}) \right] F(z_{t+1}|z_t, I_{kt} = 1) \\ &+ \beta^2 \textstyle\sum_{z_{t+1}} \sum_{s=0}^{17} p_s(z_{t+1}) \sum_{z_{t+2}} \sum_{r=0}^{17} p_r(z_{t+2}) \left[u_{rt+2}(z_{t+2}) + E_\varepsilon(\varepsilon_{rt+2}|I_{rt+2} = 1, z_{t+2}) \right] \\ &\qquad \times F(z_{t+2}|z_{t+1}, I_{st+1} = 1) F(z_{t+1}|z_t, I_{kt} = 1) \\ &+ \beta^3 \textstyle\sum_{z_{t+1}} \sum_{s=0}^{17} p_s(z_{t+1}) \sum_{z_{t+2}} \sum_{r=0}^{17} p_r(z_{t+2}) \sum_{z+3} V(z_{t+3}) F(z_{t+3}|z_{t+2}, I_{rt+2} = 1) \\ &\qquad \times F(z_{t+2}|z_{t+1}, I_{st+1} = 1) F(z_{t+1}|z_t, I_{kt} = 1) \end{aligned} \tag{50}$$

Without loss of generality (WLOG) we assume $t+3=T$; then

$$V(z_T,\varepsilon_T)=\max_I E\left(\sum_{k=0}^{17} I_{kT}[u_{kT}(z_T)+\varepsilon_{kT}+\lambda N_k(z_T)^{-\nu}\sum_{n=1}^{N_k}\sum_{x_n}U_{g+1,n}(x_n)]|z_T,\varepsilon_T\right).$$

Now

$$\begin{aligned}
V(z_T) &= \int V(z_T,\varepsilon_T)f_\varepsilon(\varepsilon_T)d\varepsilon_T \\
&= \int \max_I E\left(\sum_{j=0}^{17} I_{jT}[u_{jT}(z_T)+\varepsilon_{jT}+\lambda N_j(z_T)^{-\nu}\sum_{n=1}^{N_j}\sum_{x_n}U_{g+1,n}(x_n)]|z_T,\varepsilon_T\right)f_\varepsilon(\varepsilon_T)d\varepsilon_T \\
&= \sum_{j=0}^{17} p_j(z_T)[u_{kT}(z_T)+E_\varepsilon(\varepsilon_{jT}|z_T,I_{jT}=1) \\
+ &\quad \lambda N_j(z_T)^{-\nu}\sum_{n=1}^{N_j}\sum_{x_n}U_{g+1,n}(x_n)M(x_n'|z_T,I_{jT}=1)]. \qquad (51)
\end{aligned}$$

We know from the value function representation that $U_{g+1,n}(x_n)=V(x_n)$; therefore,

$$V(z_T)=\sum_{j=0}^{17} p_j(z_T)[u_{jT}(z_T)+E_\varepsilon(\varepsilon_{jT}|z_T,I_{jT}=1]+\lambda N_j(z_T)^{-\nu}\sum_{n=1}^{N_j}\sum_{x_n}V(x_n)M(x_n|z_T,I_{jT}=1)]. \qquad (52)$$

Substituting the above into equation (50) and rearranging gives

$$\begin{aligned}
\upsilon_k(z_t) = u_{kt}(z_t) &+ \beta\sum_{z_{t+1}}\sum_{s=0}^{17} p_s(z_{t+1})\left[u_{st+1}(z_{t+1})+E_\varepsilon(\varepsilon_{st+1}|I_{st+1}=1,z_{t+1})\right]F(z_{t+1}|z_t,I_{kt}=1) \\
&+\beta^2\sum_{z_{t+2}}\sum_{r=0}^{17} p_r(z_{t+2})\left[u_{rt+2}(z_{t+2})+E_\varepsilon(\varepsilon_{rt+2}|I_{rt+2}=1,z_{t+2})\right] \\
&\times\sum_{z_{t+1}}\sum_{s=0}^{17} p_s(z_{t+1})F(z_{t+2}|z_{t+1},I_{st+1}=1)F(z_{t+1}|z_t,I_{kt}=1) \\
&+\beta^3\sum_{z_T}\sum_{j=0}^{17} p_j(z_T)[u_{jT}(z_T)+E_\varepsilon[\varepsilon_{jT}|z_T,I_{jT}=1]\sum_{r=0}^{17}\sum_{z_{t+2}} p_r(z_{t+2})F(z_{t+3}|z_{t+2},I_{rt+2}=1) \\
&\times\sum_{s=0}^{17}\sum_{z_{t+1}} p_s(z_{t+1})F(z_{t+2}|z_{t+1},I_{st+1}=1)F(z_{t+1}|z_t,I_{kt}=1) \\
&+\lambda\beta^3\sum_{z_T}\sum_{j=0}^{17} p_j(z_T)N_j(z_T)^{-\nu}\sum_{n=1}^{N_j}\sum_{x_n}V(x_n')M(x_n'|z_T,I_{jT}=1)\sum_{z_{t+2}}\sum_{r=0}^{17} p_r(z_{t+2})F(z_T|z_{t+2},I_{rt+2}=1) \\
&\times\sum_{z_{t+1}}\sum_{s=0}^{17} p_s(z_{t+1})F(z_{t+2}|z_{t+1},I_{st+1}=1)F(z_{t+1}|z_t,I_{kt}=1). \qquad (53)
\end{aligned}$$

Using the definition of the optimal transition function, the above simplifies to

$$\begin{aligned}
v_k(z_t) = u_{kt}(z_t) &+ \beta \sum_{s=0}^{17} \sum_{z_{t+1}} p_s(z_{t+1}) \left[u_{st+1}(z_{t+1}) + E_\varepsilon[\varepsilon_{st+1} | I_{st+1} = 1, z_{t+1}] \right] F^o(z_{t+1} | z_t, I_{kt} = 1) \\
&+ \beta^2 \sum_{s=0}^{17} \sum_{z_{t+2}} p_s(z_{t+2}) \left[u_{rs+2}(z_{t+2}) + E_\varepsilon[\varepsilon_{st+2} | I_{st+2} = 1, z_{t+2}] \right] F^o(z_{t+2} | z_t, I_{kt} = 1) \\
&+ \beta^3 \sum_{s=0}^{17} \sum_{z_T} p_s(z_T) [u_{sT}(z_T) + E_\varepsilon[\varepsilon_{sT} | z_T, I_{sT} = 1]] F^o(z_T | z_t, I_{kt} = 1) \\
&+ \lambda \beta^3 \sum_{s=0}^{17} \sum_{z_T} p_s(z_T) N_s^{-\nu}(z_T) \sum_{n=1}^{N_s} \sum_{x_n} V(x_n) M(x_n | z_T, I_{sT} = 1) F^o(z_T | z_t, I_{kt} = 1) \qquad (54)
\end{aligned}$$

The assumption that parents are infertile in the final period of their life-cycle simplifies to

$$\begin{aligned}
v_k(z_t) = u_{kt}(z_t) &+ \beta \sum_{s=0}^{17} \sum_{z_{t+1}} p_s(z_{t+1}) \left[u_{st+1}(z_{t+1}) + E_\varepsilon[\varepsilon_{st+1} | I_{st+1} = 1, z_{t+1}] \right] F^o(z_{t+1} | z_t, I_{kt} = 1) \\
&+ \beta^2 \sum_{s=0}^{17} \sum_{z_{t+2}} p_s(z_{t+2}) \left[u_{rs+2}(z_{t+2}) + E_\varepsilon[\varepsilon_{st+2} | I_{st+2} = 1, z_{t+2}] \right] F^o(z_{t+2} | z_t, I_{kt} = 1) \\
&+ \beta^3 \sum_{s=0}^{17} \sum_{z_T} p_s(z_T) [u_{sT}(z_T) + E_\varepsilon[\varepsilon_{sT} | z_T, I_{sT} = 1]] F^o(z_T | z_t, I_{kt} = 1) \\
&+ \lambda \beta^3 \sum_{z_T} N_T(z_T)^{-\nu} \sum_{n=1}^{N} \sum_{x_n} V(x_n) \sum_{s=0}^{K_T} M(x_n | z_T, I_{sT} = 1) p_s(z_T) F^o(z_T | z_t, I_{kt} = 1). \qquad (55)
\end{aligned}$$

■

Proof of Proposition 2.

This result follows immediately by combining the results in Proposition 1, with the replacement of the summation over z_{t+1} with the integral over z_{t+1}. ■

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Table 1: Simplified Discrete Choice Monte Carlo Simulation Results

	Pseudo Maximum Likelihood				Nested Fixed Point (ML)			
	Sample size				Sample size			
	1,000	10,000	20,000	40,000	1,000	10,000	20,000	40,000
				$\theta = 0.25$				
Mean	0.24473	0.24935	0.24886	0.24881	0.22714	0.24571	0.23320	0.24477
Std. Dev.	0.04991	0.01328	0.00915	0.00668	0.04884	0.01354	0.02135	0.01019
Bias	-0.00527	-0.00065	-0.00114	-0.00119	-0.02286	-0.00429	-0.01680	-0.00523
MSE	0.00249	0.00017	0.00008	0.00005	0.00288	0.00020	0.00073	0.00013
				$\lambda = 0.8$				
Mean	0.80425	0.79745	0.79797	0.79673	0.77538	0.78966	0.76934	0.78855
Std. Dev.	0.11241	0.03175	0.02157	0.01587	0.09211	0.03244	0.03656	0.02063
Bias	0.00425	-0.00255	-0.00203	-0.00327	-0.02462	-0.01034	-0.03066	-0.01145
M.S.E.	0.01253	0.00100	0.00046	0.00026	0.00901	0.00115	0.00226	0.00055
				$\beta = 0.95$				
Mean	0.94208	0.95245	0.95037	0.95136	0.93441	0.95227	0.94603	0.95027
Std. Dev.	0.06276	0.01893	0.01301	0.00934	0.05322	0.01983	0.01820	0.01236
Bias	-0.00792	0.00245	0.00037	0.00136	-0.01559	0.00227	-0.00397	0.00027
MSE	0.00396	0.00036	0.00017	0.00009	0.00305	0.00039	0.00034	0.00015
Avg. Comp. time	0.65	2.88	6.06	12.60	347.6	376.4	467.5	509.8

Note: The pseudo maximum likelihood corresponds to the estimation conducted by the new estimator using PML and maximum likelihood (ML) estimation is by the nested fixed point (NFXP). All simulations were conducted using the programming language GAUSS on a 2-CPU 1.66-GHz, 3-GB RAM laptop computer. The Unit of time is seconds. The mean, empirical standard deviation, bias, and mean squared error (MSE) of each parameter estimate are reported in the respective column for each sample size. The bias and the MSE are calculated relative to the original data-generating value of the parameter. The data-generating value of the parameter is also reported at the center of the summary statistics block for that parameter.

Table 2: Summary Statistics for Full Sample

Variable	Full sample N	Full sample Mean	Parents N	Parents Mean	Children N	Children Mean
	(1)		(2)		(3)	
Female	115,280	0.545	86,302	0.552	28,978	0.522
Black	115,280	0.223	86,302	0.202	28,978	0.286
Married	115,280	0.381	86,302	0.465	28,978	0.131
Age (yr)	115,280	26.155	86,302	27.968	28,978	20.756
		(7.699)		(7.872)		(3.511)
Education (years completed)	115,280	13.438	86,302	13.516	28,978	13.209
		(2.103)		(2.138)		(1.981)
No. of children	115,280	0.616	86,302	0.766	28,978	0.167
		(0.961)		(1.028)		(0.507)
Annual labor income ($ US 2005)	114,871	16,115	86,137	19,552	28,734	5,811
		(24,622)		(26,273)		(14,591)
Annual labor market hours	114,899	915	86,185	1078	28,714	424
		(1041)		(1051)		(841)
Annual housework hours	66,573	714	58,564	724	8,009	641
		(578)		(585)		(524)
Annual time spent on children (hr)	115,249	191	86,275	234	28,974	63.584
		(432)		(468)		(259)
Number of individuals	12,318		6,813		5,505	

Note: Standard deviations are listed in parentheses. Data are from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID) and include individuals surveyed between 1968 and 1997. Column (1) contains the summary statistics for the full sample; column (2) contains the summary statistics for the parents generation; column (3) contains the summary statistics of the offspring of the parents in column (2). There are fewer observations for annual housework hours than time spent on children because single individuals with no children are coded as missing for housework hours but by definition are set to 0 for time spent on children.

Table 3: Estimates of Earnings Equation: Dependent Variable: Log of Yearly Earnings

Variable	Estimate	Variable	Estimate	Variable	Estimate
Demographic Variables				Fixed Effect	
Age squared	-4.0e-4	Female x Full-time work	-0.125	Black	-0.154
	(1.0e-5)		(0.010)		(0.009)
Age x LHS	0.037	Female x Full-time work $(t-1)$	0.110	Female	-0.484
	(0.002)		(0.010)		(0.007)
Age x HS	0.041	Female x Full-time work $(t-2)$	0.025	HS	0.136
	(0.001)		(0.010)		(0.005)
Age x SC	0.050	Female x Full-time work $(t-3)$	0.010	SC	0.122
	(0.001)		(0.010)		(0.006)
Age x COL	0.096	Female x Full-time work $(t-4)$	0.013	COL	0.044
	(0.001)		(0.010)		(0.006)
Current and Lags of Participation		Female x Part-time work $(t-1)$	0.150	Black x HS	-0.029
Full-time work	0.938		(0.010)		(0.010)
	(0.010)	Female x Part-time work $(t-2)$	0.060	Black x SC	0.033
Full-time work $(t-1)$	0.160		(0.010)		(0.008)
	(0.009)	Female x Part-time work $(t-3)$	0.040	Black x COL	0.001
Full-time work $(t-2)$	0.044		(0.010)		(0.011)
	(0.010)	Female x Part-time work $(t-4)$	-0.002	Female x HS	-0.054
Full-time work $(t-3)$	0.025		(0.010)		(0.008)
	(0.010)	Individual specific effects	Yes	Female x SC	0.049
Full-time work $(t-4)$	0.040				(0.006)
	(0.010)			Female x COL	0.038
Part-time work $(t-1)$	-0.087				(0.007)
	(0.010)			Constant	0.167
Part-time work $(t-2)$	-0.077				(0.005)
	(0.010)				
Part-time work $(t-3)$	-0.070				
	(0.010)				
Part-time work $(t-4)$	-0.010	Hausman Statistics	2296		
	(0.010)	Hausman p-value	0.000		
No. of Observations			134,007		
No. of Individuals			14,018		
R^2			0.44		0.278

Note: Standard errors are listed in parentheses. LHS indicates completed education of less than high school; HS indicates completed education of high school but not college; SC indicates completed education of some college but not a graduate; COL indicates completed education of at least a college degree.

Table 4: Three- Stage Least Squares Estimation of the Education Production Function

Variable	High School	Some College	College
High school father	0.063	0.003	-0.002
	(0.032)	(0.052)	(0.0435
Some college father	0.055	0.132	0.055
	(0.023)	(0.038)	(0.031)
College father	-0.044	0.008	0.120
	(0.032)	(0.051)	(0.042)
High school mother	0.089	0.081	-0.019
	(0.040)	(0.065)	(0.052)
Some college mother	0.007	-0.041	0.017
	(0.030)	(0.049)	(0.039)
College mother	0.083	0.120	0.040
	(0.036)	(0.057)	(0.047)
Mother's time	-0.014	0.080	0.069
	(0.021)	(0.034)	(0.027)
Father's time	0.031	0.100	0.026
	(0.019)	(0.029)	(0.025)
Mother's labor income	-0.025	-0.013	0.005
	(0.009)	(0.014)	(0.011)
Father's Labor Income	0.001	0.001	0.002
	(0.003)	(0.004)	(0.003)
Female	-0.002	0.135	0.085
	(0.017)	(0.028)	(0.022)
Black	0.020	0.082	0.043
	(0.039)	(0.063)	(0.051)
No. of siblings under age 3	-0.014	-0.107	-0.043
	(0.017)	(0.027)	(0.022)
No. of siblings between age 3 and 6	-0.029	-0.047	-0.012
	(0.019)	(0.030)	(0.025)
Constant	0.855	-0.231	-0.359
	(0.108)	(0.172)]	(0.140)]
Observations	1335	1335	1335

Note: Standard errors are listed in parentheses; the excluded class is less than high school. Data are from the Family-Individual File of the Michigan Panel Study of Income Dynamics (PSID), and include individuals surveyed between 1968 and 1997. Instruments: Mother's and father's labor market hours over the child's first 8 years of life, linear and quadratic terms of mother's and father's age when the child was 5 years old.

Table 5: Structural Estimates of Discount factors and Utility Parameter

Variable	Estimates	Variable		Estimates
Discount factors		Disutility/Utility of Choices		
β	0.816	Wife	Husband	
	(0.002)	Labor supply		
λ	0.795	No work	Part -time	-0.512
	(0.200)			(0.005)
υ	0.248	No work	Full-time	0.207
	(0.168)			(0.009)
Marginal Utility of Income		Part-time	No work	-2.023
Family labor income	0.480			(0.003)
	(0.004)	Part-time	Part-time	-1.168
Children x Family labor income	-0.466			(0.009)
	(0.066)	Part-time	Full-time	-0.605
Children x HS x Family labor income	1.216			(0.008)
	(0.065)	Full-time	No work	-0.408
Children x SC x Family labor income	1.279			(0.007)
	(0.066)	Full-time	Part-time	-1.24532
Children x COL x Family labor income	1.300			(0.011)
	(0.065)	Full-time	Full-time	0.001
Children x HS spouse x Family labor income	-1.017			(0.010)
	(0.066)	Time with children		
Children x SC spouse x Family labor income	-0.995	Low	Medium	0.502
	(0.066)			(0.014)
Children x COL Sspouse x Family labor income	-0.992	Low	High	0.564
	(0.066)			(0.013)
Children x Black x Family Labor Income	-0.108	Medium	Low	-0.169
	(0.004)			(0.008)
		Medium	Medium	0.129
				(0.010)
		Medium	High	0.593
				(0.013)
		High	Low	-0.364
				(0.007)
		High	Medium	0.353
				(0.011)
		High	High	-0.140
				(0.012)
		Birth		0.701
				(0.025)

Note: Standard errors are listed in parentheses. LHS indicates completed education of less than high school; HS indicates completed education of high school but not college; SC indicates completed education of some college but not a graduate; COL indicates completed education of at least a college degree. The excluded choice is no work, no time with children, and no birth for both spouses.

TABLE 6: MODEL FIT AND COUNTERFACTUAL

LABOR SUPPLY

WHITES

Wife

	Data	Model1	NN1	NN1'	Model2	NN2	NN2'
No work	0.2634	0.2599	0.3918	0.3750	0.3649	0.3730	0.2896
Part time	0.1596	0.1622	0.1416	0.1451	0.1790	0.2193	0.2219
Full time	0.5770	0.5779	0.4666	0.4800	0.4561	0.4076	0.4886

Husband

	Data	Model1	NN1	NN1'	Model2	NN2	NN2'
No work	0.0290	0.0250	0.0233	0.0233	0.0245	0.0145	0.0201
Part time	0.0306	0.0361	0.0336	0.0329	0.0447	0.0295	0.0295
Full time	0.9404	0.9390	0.9432	0.9439	0.9307	0.9560	0.9505

TIME INVESTMENT

Wife

	Data	Model1	NN1	NN1'	Model2	NN2	NN2'
Low	0.6363	0.8999	0.8485	0.8464	0.9530	0.6004	0.6816
Medium	0.2257	0.0531	0.0808	0.0820	0.0251	0.2260	0.1771
High	0.1380	0.0470	0.0707	0.0716	0.0220	0.1736	0.1413

Husband

	Data	Model1	NN1	NN1'	Model2	NN2	NN2'
Low	0.8237	0.9592	0.9459	0.9457	0.9796	0.8688	0.8985
Medium	0.1008	0.0238	0.0310	0.0310	0.0113	0.0807	0.0608
High	0.0755	0.0170	0.0231	0.0232	0.0091	0.0505	0.0407

BLACKS

Wife

	Data	Model1	NN1	NN1'	Model2	NN2	NN2'
No work	0.1998	0.1309	0.1258	0.1232	0.3045	0.3309	0.2467
Part time	0.1002	0.2150	0.2145	0.2154	0.2449	0.2142	0.2257
Full time	0.7000	0.6541	0.6597	0.6614	0.4506	0.4550	0.5277

Husband

	Data	Model1	NN1	NN1'	Model2	NN2	NN2'
No work	0.0640	0.0596	0.0587	0.0575	0.0788	0.0473	0.0567
Part time	0.0423	0.0555	0.0471	0.0468	0.0553	0.0397	0.0403
Full time	0.8937	0.8850	0.8942	0.8957	0.8659	0.9130	0.9030

Wife

	Data	Model1	NN1	NN1'	Model2	NN2	NN2'
Low	0.6837	0.9046	0.8678	0.8694	0.7840	0.6550	0.7460
Medium	0.2192	0.0497	0.0690	0.0687	0.1210	0.1964	0.1371
High	0.0971	0.0457	0.0632	0.0618	0.0951	0.1486	0.1169

Husband

	Data	Model1	NN1	NN1'	Model2	NN2	NN2'
Low	0.8338	0.9729	0.9638	0.9641	0.9299	0.9124	0.9320
Medium	0.0744	0.0123	0.0158	0.0158	0.0326	0.0479	0.0336
High	0.0919	0.0148	0.0204	0.0201	0.0375	0.0397	0.0344

BIRTH

WHITES

	Data	Model1	NN1	NN1'	Model2	NN2	NN2'
No birth	0.9014	0.9551	0.9387	0.9383	0.9701	0.8148	0.8833
Birth	0.0986	0.0449	0.0613	0.0617	0.0299	0.1852	0.1167

BLACKS

	Data	Model1	NN1	NN1'	Model2	NN2	NN2'
No birth	0.8955	0.9249	0.9129	0.9135	0.8847	0.8349	0.8784
Birth	0.1045	0.0751	0.0871	0.0865	0.1153	0.1651	0.1216

Note:Model 1 is the baseline model with all parameters estimated. Model 2 is the model with discount factors calibrated ($\beta = 0.90, \lambda = 0.95$). $NNi, i = 1, 2$ removes Nature from the intergenerational educational production with full re-optimization. $NNi\prime, i = 1, 2$ removes Nature from the intergenerational educational production but *Do Not* allow for re-optimization of subsequent generations.

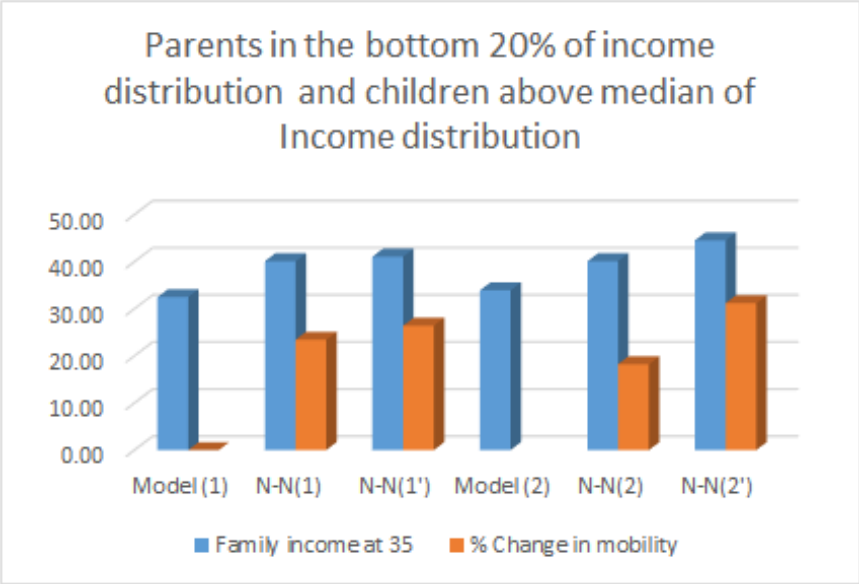

Figure 1: Counterfactuals and Mobility

Note:Model 1 is the baseline model with all parameters estimated. Model 2 is the model with discount factors calibrated ($\beta = 0.90, \lambda = 0.95$). $NN(i), i = 1, 2$ removes Nature from the intergenerational educational production with full re-optimization. $NN(i\prime), i = 1, 2$ removes Nature from the intergenerational educational production but *Do Not* allow for re-optimization of subsequent generations.