Discretionary Monetary Policy in the Calvo Model[*]

Willem Van Zandweghe[†]   Alexander L. Wolman[‡]

Abstract

We study discretionary equilibrium in the Calvo pricing model for a monetary authority that chooses the money supply, producing three main contributions. First, price-adjusting firms have a unique equilibrium price for a broad range of parameterizations, in contrast to earlier results for the Taylor pricing model. Second, a generalized Euler equation makes transparent how the monetary authority affects future welfare through its influence on the future state of the economy. Third, we provide global solutions, including welfare analysis, for the transitional dynamics that occur if the monetary authority gains or loses the ability to commit.

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Keywords: Time-consistent optimal monetary policy; Discretion; Markov-perfect equilibrium; Sticky prices; Relative price distortion

[†]Corresponding author. Research Department, Federal Reserve Bank of Kansas City, 1 Memorial Drive, Kansas City, MO 64198. Tel: 816-881-2766. E-mail: `willem.vanzandweghe@kc.frb.org`.

[‡]Research Department, Federal Reserve Bank of Richmond. E-mail: `alexander.wolman@rich.frb.org`.

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1 Introduction

Over the last two decades New Keynesian models have become the dominant framework for applied monetary policy analysis. This framework is characterized by optimizing private-sector behavior in the presence of nominal rigidities, typically Calvo (1983) pricing as described by Yun (1996). The fact that some prices are predetermined in these models leads to a time-consistency problem for monetary policy, and there is a vast literature studying aspects of discretionary, i.e. time-consistent, optimal policy in New Keynesian models with Calvo pricing. While the typical practice, exemplified by Clarida, Galí, and Gertler (1999) and Woodford (2003a), has been to work with models approximated around a zero-inflation steady state, a growing literature studies the discretionary policy problem with global methods. This paper contributes to that literature in three ways. First, for a broad range of parameterizations, it shows that under discretionary policy the Calvo model delivers a unique equilibrium price for adjusting firms, in contrast to earlier results for the Taylor model. This equilibrium price in turn determines a unique discretionary equilibrium. Second, it derives a generalized Euler equation (GEE), as in Krusell, Kuruscu, and Smith (2002) and Klein, Krusell, and Rios-Rull (2008), and uses the GEE to decompose the dynamic policy trade-offs facing a discretionary policymaker. Third, it conducts global welfare analysis of the transitional dynamics that occur when a policymaker gains or loses the ability to commit.

The first contribution relates to an existing literature which has identified discretionary policy as a potential source of multiple equilibria in a broad range of contexts. Private agents make decisions, such as saving or price setting, based on expectations of future policy. Those decisions in turn are transmitted to the future through state variables, creating the potential for a form of complementarity between future policy and expected future policy when policy is chosen under discretion. Viewed from another angle, the fact that policy will react to endogenous state variables can be a source of complementarity among private agents' actions. The link between discretionary policy and multiple equilibria has been especially prominent in the monetary policy literature. Khan, King, and Wolman (2001) and King and Wolman (2004) show that in Taylor-style models with prices set for three and two periods respectively, under discretion there are multiple equilibrium values of the price

set by adjusting firms.[1] Calvo and Taylor models are similar in many ways, yet we find no evidence that discretionary policy generates equilibrium multiplicity in the Calvo model. Although we do not prove the uniqueness of discretionary equilibrium, we show that a policy analogous to the optimal policy in the Taylor model guarantees a unique equilibrium in the Calvo model, despite its greater potential for complementarity than the optimal discretionary policy. We trace the contrasting behavior of the two models to differences in how current pricing decisions affect the overall price level, and how the future policymaker responds to a measure of the dispersion in predetermined relative prices, which is an endogenous state variable in the Calvo model.

Uniqueness of the equilibrium price set by adjusting firms opens up the possibility of deriving a GEE, which represents the dynamic trade-off facing a discretionary policymaker in equilibrium. While the GEE has been extensively studied in fiscal policy applications, and recently extended to the Rotemberg sticky price model by Leeper, Leith, and Liu (2018), to the best of our knowledge it has not previously been derived for the Calvo model.[2] Under discretion, the policy problem is dynamic only to the extent that endogenous state variables affect future welfare. The GEE highlights three distinct channels through which the measure of dispersion in predetermined relative prices links current policy to future welfare. First, the state variable affects welfare directly because higher relative price dispersion effectively reduces productivity. Second, the state variable shifts the future policymaker's trade-off

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[1]Albanesi, Chari and Christiano (2003) show that multiple equilibria arise under discretionary policy in a model in which a fraction of firms have predetermined prices. Siu (2008) extends King and Wolman's (2004) analysis and Barseghyan and DiCecio (2007) extend Albanesi, Chari and Christiano's (2003) analysis, by incorporating elements of state-dependent pricing and showing that Markov-perfect discretionary equilibrium is unique. Those papers assume that monetary policy is conducted with a money supply instrument. Dotsey and Hornstein (2011) show that with an interest rate instrument there is a unique Markov-perfect discretionary equilibrium in a Taylor model with two-period pricing.

[2]Our paper is closely related to Anderson, Kim, and Yun (2010). They study optimal allocations without commitment in the Calvo model. Their approach cannot be used to investigate the possibility of multiple equilibrium prices for a given policy action, or to derive a GEE. Their solution method, like ours, is based on Chebyshev collocation. While they study a slightly different region of the parameter space, the nature of their solutions is consistent with our findings. Ngo (2014) extends their analysis to a stochastic environment with the zero bound on nominal interest rates, and Leith and Liu (2016) use their approach to compare the Calvo and Rotemberg models.

between consumption and leisure. Third, the state variable enters price-setting firms' optimization problems, so that even if future policy did not respond to the state the current policymaker would have some leverage over future equilibrium prices.

When any aspect of public policy suffers from a time-consistency problem, it is important to know the value of commitment. Thus, the third contribution of the paper is to provide global solutions to the transitional dynamics that occur (i) when an economy that had converged to a discretionary steady state unexpectedly finds itself with a policymaker who can commit to future policy, and (ii) when an economy that had been operating with optimal policy under commitment unexpectedly finds itself with a policymaker who cannot commit to future policy. In both cases, we find that the welfare gain or loss from the transition is quite close to the steady-state welfare difference between discretion and commitment. However, the transitions differ qualitatively. The transition from discretion to commitment involves a gradual decline in inflation, reminiscent of the Volcker disinflation. The transition from commitment to discretion, in contrast, involves an initial overshooting in inflation.

The paper proceeds as follows. The next section contains a description of the Calvo model. Section 3 defines a discretionary equilibrium. Section 4 contains the numerical results for the discretionary equilibrium, emphasizing the issue of multiplicity or lack thereof. Section 5 describes the GEE approach. Section 6 presents the results on transitional dynamics and welfare. Section 7 relates our analysis to the early literature on discretionary monetary policy and concludes. Secondary material is contained in appendixes.

2   The Calvo model

The model is characterized by a representative household that values consumption and dislikes supplying labor, a money demand equation, a competitive labor market, a continuum of monopolistically competitive firms producing differentiated goods, and a monetary authority that chooses the money supply. Each firm faces a constant probability of price adjustment. We assume the model's exogenous variables are constant.

2.1 Households

There is a large number of identical, infinitely-lived households. They act as price-takers in labor and product markets, and they own shares in the economy's monopolistically competitive goods-producing firms. Households' preferences over consumption (c_t) and labor (n_t) are given by

$$\sum_{j=0}^{\infty} \beta^j \left(\ln c_{t+j} - \chi n_{t+j} \right), \quad \beta \in (0,1), \; \chi > 0,$$

where consumption is taken to be the Dixit-Stiglitz aggregate of a continuum of differentiated goods with elasticity of substitution $\varepsilon > 1$,

$$c_t = \left[\int_0^1 c_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{1}$$

The consumer's flow budget constraint is

$$P_t w_t n_t + R_{t-1} B_{t-1} + \int_0^1 d_t(z)\, dz \geq P_t c_t + B_t,$$

where w_t is the real wage, R_t is the one-period gross nominal interest rate, B_t is the quantity of one-period nominal bonds purchased in period t, $d_t(z)$ is the dividend paid by firm z, and P_t is the nominal price of a unit of consumption. The aggregator (1) implies the demand functions for each good,

$$c_t(z) = \left[\frac{P_t(z)}{P_t} \right]^{-\varepsilon} c_t, \tag{2}$$

where $P_t(z)$ is the price of good z. The price index is given by

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}. \tag{3}$$

From the consumer's intratemporal and intertemporal problems we have the efficiency conditions:

$$\begin{aligned}
w_t &= \chi c_t, \\
\frac{c_{t+1}}{c_t} &= \beta \left(\frac{R_t}{\pi_{t+1}} \right),
\end{aligned} \tag{4}$$

where $\pi_t \equiv P_t/P_{t-1}$ denotes the gross inflation rate between periods $t-1$ and t. We assume there is a money demand equation such that the quantity of money is equal to the nominal value of consumption,

$$M_t = P_t c_t. \tag{5}$$

This constant-velocity money demand equation simplifies the model by abstracting from any distortions arising from money demand, and enables a straightforward comparison with the previous literature (e.g. King and Wolman, 2004). It will be convenient to write the money demand equation normalizing by the lagged price level, which serves as an index of the predetermined nominal prices:

$$m_t \equiv \frac{M_t}{P_{t-1}} = \pi_t c_t. \tag{6}$$

We will refer to m_t as the normalized money supply.

2.2   Firms

Each firm $z \in [0,1]$ produces output $y_t(z)$ using a technology that is linear in labor $n_t(z)$, the only input, with a constant level of productivity that is normalized to unity: $y_t(z) = n_t(z)$. A firm adjusts its price with constant probability $1 - \alpha$ each period, as in Calvo (1983).[3] As firms are owned by households, adjusting firms solve the following problem:

$$\max_{X_t} \left\{ \sum_{j=0}^{\infty} (\alpha\beta)^j \left(\frac{P_t}{P_{t+j}} \right) \left(\frac{c_t}{c_{t+j}} \right) \left[X_t \left(\frac{X_t}{P_{t+j}} \right)^{-\varepsilon} c_{t+j} - P_{t+j} w_{t+j} \left(\frac{X_t}{P_{t+j}} \right)^{-\varepsilon} c_{t+j} \right] \right\}.$$

The factor α^j is the probability that a price set in period t will remain in effect in period $t + j$. We will denote the profit-maximizing value of X_t by $P_{0,t}$ and we will denote by $p_{0,t}$ the nominal price $P_{0,t}$ normalized by the previous period's price level, $p_{0,t} \equiv P_{0,t}/P_{t-1}$. Thus, we write the first-order condition as

$$\frac{P_{0,t}}{P_t} = \frac{p_{0,t}}{\pi_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{\sum_{j=0}^{\infty} (\alpha\beta)^j \left(P_{t+j}/P_t \right)^{\varepsilon} w_{t+j}}{\sum_{j=0}^{\infty} (\alpha\beta)^j \left(P_{t+j}/P_t \right)^{\varepsilon - 1}}. \tag{7}$$

With the constant elasticity aggregator (1) a firm's desired markup of price over marginal cost is constant and equal to $\varepsilon/(\varepsilon - 1)$. The optimal pricing equation (7) indicates that the firm chooses a constant markup over an appropriately defined weighted average of current and future marginal costs. Because firm-level productivity is assumed constant and equal to one, real marginal cost is equal to the real wage. The economy-wide average markup is then simply the inverse of the real wage.

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[3]In Yun's (1996) version of the Calvo model there is price indexation, whereas the version in King and Wolman (1996) has no indexation. We analyze the Calvo model without indexation.

The optimal pricing condition can be written recursively by defining two new variables, $\tilde{S}_t$ and $\tilde{F}_t$, that are related to the numerator and denominator of (7), respectively:

$$\tilde{S}_t = w_t + \alpha\beta\pi_{t+1}^{\varepsilon}\tilde{S}_{t+1}, \tag{8}$$

$$\tilde{F}_t = 1 + \alpha\beta\pi_{t+1}^{\varepsilon-1}\tilde{F}_{t+1}, \tag{9}$$

then,

$$p_{0,t} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\pi_t\frac{\tilde{S}_t}{\tilde{F}_t}. \tag{10}$$

We can eliminate future inflation from (8) and (9) by defining $S_t = \pi_t^{\varepsilon}\tilde{S}_t$ and $F_t = \pi_t^{\varepsilon-1}\tilde{F}_t$, such that

$$S_t = \pi_t^{\varepsilon}(w_t + \alpha\beta S_{t+1}), \tag{11}$$

$$F_t = \pi_t^{\varepsilon-1}(1 + \alpha\beta F_{t+1}), \tag{12}$$

and

$$p_{0,t} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{S_t}{F_t}. \tag{13}$$

Because of Calvo pricing, the price index (3) is an infinite sum,

$$P_t = \left[\sum_{j=0}^{\infty}(1-\alpha)\,\alpha^j P_{0,t-j}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{14}$$

but it can be simplified, first writing it recursively as

$$P_t = \left[(1-\alpha)P_{0,t}^{1-\varepsilon} + \alpha P_{t-1}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{15}$$

and then dividing by the lagged price level:

$$\pi_t = \left[(1-\alpha)\,p_{0,t}^{1-\varepsilon} + \alpha\right]^{\frac{1}{1-\varepsilon}}. \tag{16}$$

2.3 Market clearing

Goods market clearing requires that the consumption demand for each individual good is equal to the output of that good, $c_t(z) = y_t(z)$, and labor market clearing requires that the supply of labor by households equal the labor input into the production of all goods:

$$n_t = \int_0^1 n_t(z)dz. \tag{17}$$

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A firm's labor input is determined by its output demand, which depends on its relative price. Let $n_{j,t}$ denote the labor input employed in period t by a firm that set its price in period $t-j$. Because each period a fraction $1-\alpha$ of firms adjusts its price, the labor market clearing condition is

$$n_t = \sum_{j=0}^{\infty} (1-\alpha)\,\alpha^j n_{j,t}.$$

Combining this expression with the individual goods market clearing conditions, then using the demand curve (2) for each good yields

$$\frac{n_t}{c_t} = \sum_{j=0}^{\infty} (1-\alpha)\,\alpha^j \left(\frac{P_{0,t-j}}{P_t}\right)^{-\varepsilon}.$$

This can be written recursively as

$$\Delta_t = \pi_t^\varepsilon \left[(1-\alpha)\,p_{0,t}^{-\varepsilon} + \alpha\Delta_{t-1}\right], \tag{18}$$

where

$$\Delta_t \equiv \frac{n_t}{c_t}. \tag{19}$$

When all firms charge the same price, $\Delta = 1$. Values greater than one reflect inefficiency due to price dispersion, which translates into low average productivity. We call Δ_{t-1} the inherited relative price distortion.

2.4   Monetary authority and timing

The monetary authority chooses the money supply, M_t. We assume the sequence of actions within a period is as follows:

1. Predetermined prices $(P_{0,t-j},\ j > 0)$ are known at the beginning of the period.

2. The monetary authority chooses the money supply.

3. Firms that adjust in the current period set their prices, and simultaneously all other period-t variables are determined.

 Timing assumptions are important for equilibrium with discretionary policy. Transposing items 2 and 3 or assuming that firms and the monetary authority act simultaneously would change the nature of the policy problem and the properties of equilibrium.

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3 Discretionary equilibrium in the Calvo model

We are interested in studying Markov-perfect equilibrium (MPE) with discretionary monetary policy. In an MPE, outcomes depend only on payoff-relevant state variables; trigger strategies and any role for reputation are ruled out. Hence, it is important to establish what the relevant state variables are. Although there are an infinite number of predetermined nominal prices $(P_{0,t-j}, j = 1, 2, ...)$, for an MPE a state variable is relevant only if it affects the monetary authority's set of feasible real outcomes. All the predetermined variables vanish from the price index (14) when we write it in terms of inflation (16). And the recursive formulation of the labor market clearing condition shows that the inherited relative price distortion, rather than the predetermined nominal prices individually, is relevant. It follows that in an MPE the normalized money supply and all other equilibrium objects are functions of the single state variable Δ_{t-1}. A discretionary policymaker chooses the money supply as a function of that state, taking as given the behavior of future policymakers. In equilibrium, the future policy that is taken as given is also the policy chosen by the current policymaker.

3.1 Equilibrium for arbitrary monetary policy

As a preliminary to studying discretionary equilibrium, it is useful to consider stationary equilibria for arbitrary monetary policy—that is, for an arbitrary function $m = \Gamma(\Delta)$. To describe equilibrium for arbitrary policy we use recursive notation, eliminating time subscripts and using a prime to denote a variable in the next period. The nine variables that need to be determined in equilibrium are S, F, p_0, π, Δ', c, n, w, and m, and the nine equations are the recursions for S (11) and for F (12); the optimal pricing condition (13); the transformed price index (16); the law of motion for the relative price distortion (18); the definition of the relative price distortion (19); the labor supply equation (4); the money demand equation (6); and the monetary policy rule $m = \Gamma(\Delta)$.

A stationary equilibrium can be expressed as two functions of the endogenous state variable. The two functions $S(\Delta)$ and $F(\Delta)$ must satisfy the two functional equations

$$S(\Delta) = \pi^{\varepsilon} \left[w + \alpha\beta S(\Delta') \right], \tag{20}$$

$$F(\Delta) = \pi^{\varepsilon-1} \left[1 + \alpha\beta F(\Delta') \right], \tag{21}$$

where the other variables are given successively by the following functions of Δ:

$$p_0 = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{S(\Delta)}{F(\Delta)}, \tag{22}$$

$$\pi = \left[(1 - \alpha) p_0^{1-\varepsilon} + \alpha\right]^{1/(1-\varepsilon)}, \tag{23}$$

$$\Delta' = \pi^\varepsilon \left[(1 - \alpha) p_0^{-\varepsilon} + \alpha\Delta\right], \tag{24}$$

$$c = \frac{\Gamma(\Delta)}{\pi} \tag{25}$$

$$n = \Delta'c, \tag{26}$$

$$w = \chi c. \tag{27}$$

For an arbitrary policy of the form $m = \Gamma(\Delta)$, functions $S()$ and $F()$ that satisfy $(20)-(27)$ represent a stationary equilibrium.

3.2 Discretionary equilibrium defined

A discretionary equilibrium is a particular stationary equilibrium with policy $m = \Gamma^*(\Delta)$, in which the following property holds: If the monetary authority and private agents in the current period take as given that all future periods will be described by a stationary equilibrium associated with $\Gamma^*(\Delta)$, then the monetary authority maximizes welfare by choosing $m = \Gamma^*(\Delta)$ for every Δ.

More formally, a discretionary equilibrium is a policy function $\Gamma^*(\Delta)$ and a value function $v^*(\Delta)$ that satisfy

$$\Gamma^*(\Delta) = \arg\max_m \left\{\ln c(\Delta; m) - \chi n(\Delta; m) + \beta v^*(\Delta'(\Delta; m))\right\} \tag{28}$$

and

$$v^*(\Delta) = \ln c(\Delta; \Gamma^*()) - \chi n(\Delta; \Gamma^*()) + \beta v^*(\Delta'(\Delta; \Gamma^*())), \tag{29}$$

where $v^*(\Delta)$ is the value function associated with the policy $\Gamma^*(\Delta)$, and correspondingly, consumption $c(\Delta; \Gamma^*())$, labor $n(\Delta; \Gamma^*())$, and the future state $\Delta'(\Delta; \Gamma^*())$ in (29) are functions of Δ determined by the stationary equilibrium associated with $\Gamma^*(\Delta)$. The maximand in (28) can be seen to be a function of m by noting that $c = m/\pi$ and then combining (23), (24), (26), and (27) with optimal pricing by adjusting firms,

$$p_0 = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{\pi^\varepsilon \left[w + \alpha\beta S(\Delta')\right]}{\pi^{\varepsilon-1} \left[1 + \alpha\beta F(\Delta')\right]}, \tag{30}$$

where the functions $S()$ and $F()$ satisfy (20) and (21) in the stationary equilibrium associated with $\Gamma^*(\Delta)$. Note the subtle difference between (30) and (20)$-$(22): in (30), which applies in the current period, we have not imposed a stationary equilibrium. The monetary authority takes as given that the future will be described by a stationary equilibrium. It is an equilibrium outcome, not a constraint, that current policy is identical to that which generates the stationary equilibrium in the future.

4   Properties of discretionary equilibrium

We use a projection method to compute numerical solutions, restricting attention to equilibria that are limits of finite-horizon equilibria. This restriction may further reduce the number of discretionary equilibria, and allows us to derive a useful analytical result for the case of a monetary policy that holds the normalized money supply m constant.[4] Even though the discretionary equilibrium does not involve holding m constant, analyzing that policy provides some intuition for our numerical results.

The quarterly baseline calibration is common in the applied monetary policy literature: $\alpha = 0.5$, $\beta = 0.99$, $\varepsilon = 10$, $\chi = 4.5$. Prices remain fixed with probability $\alpha = 0.5$, which means that the expected duration of a price is two quarters. The demand elasticity $\varepsilon = 10$ implies a desired markup of approximately 11 percent. Given the value for ε, $\chi = 4.5$ is chosen to target a steady-state level of labor in the flexible-price economy of $n = 0.2$. The baseline calibration is chosen to facilitate comparison with King and Wolman (2004), but many other examples were computed that cover a wide range of structural parameter values. Computational details are provided in Appendix A.

Equilibrium is characterized by the value function $v^*(\Delta)$ and the associated monetary policy function, $m = \Gamma^*(\Delta)$, along with the transition function for the state variable and equilibrium functions for the other endogenous variables. For given values of the state and the money supply, firms' price setting is characterized by the fixed point of a best-response function. We use that best-response function to study uniqueness of an adjusting firm's

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[4]This restriction follows Krusell, Kuruscu, and Smith (2002). Krusell and Smith (2003) show that the infinite horizon can admit a large number of Markov-perfect equilibria that are non-differentiable. See also the discussion in Martin (2009, Appendix C).

optimal price in discretionary equilibrium. Non-uniqueness of that price would give rise to different discretionary equilibria that depended on the price firms coordinated on.[5]

4.1 Equilibrium as a function of the state

Figure 1, Panel A plots the transition function for the state variable as well as the function mapping from the state to the inflation rate in a discretionary equilibrium.[6] The first thing to note is that there is a unique steady-state inflation rate of 5.4 percent annually. Two natural benchmarks against which to compare the steady state of the discretionary equilibrium are the inflation rate with highest steady-state welfare and the inflation rate in the long run under optimal policy with commitment. For our baseline parameterization, the inflation rate that maximizes steady-state welfare is just barely positive (less than one tenth of a percent) and the long-run inflation rate under commitment is zero. The latter result is parameter-independent; we return to it in Section 6.

In addition to showing the steady state, Panel A illustrates the dynamics of the state variable, which exhibit monotonic convergence to the steady state. This means that a policymaker inheriting a relative price distortion that is large relative to steady state finds it optimal to bequeath a smaller relative price distortion to her successor. Together with the monotone downward-sloping equilibrium function for inflation, it follows that inflation dynamics in the transition from a large relative price distortion (as would be implied by a high inflation rate) involve an initial discrete fall in inflation and a subsequent gradual increase to the steady state.[7]

Panel B of Figure 1 displays the policy variable (m) and welfare (v) as functions of the state variable in the discretionary equilibrium (m is plotted on the left scale and welfare on

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[5]In King and Wolman (2004), multiple equilibrium prices set by adjusting firms (that is, multiple fixed points of the best-response function for a given value of m) form the basis for multiple MPE, each one indexed by a different distribution over the equilibrium prices.

[6]Note that in the model π is a gross quarterly inflation rate, but the figures and the text refer to annualized net inflation rates obtained as $100(\pi^4 - 1)$ percent.

[7]Yun's (2005) analysis of the Calvo model with a subsidy to offset the markup distortion displays similar transition dynamics of inflation. But in his model, the steady-state inflation rate under optimal policy is zero, so the transition from a steady state with positive inflation inevitably entails a period of deflation.
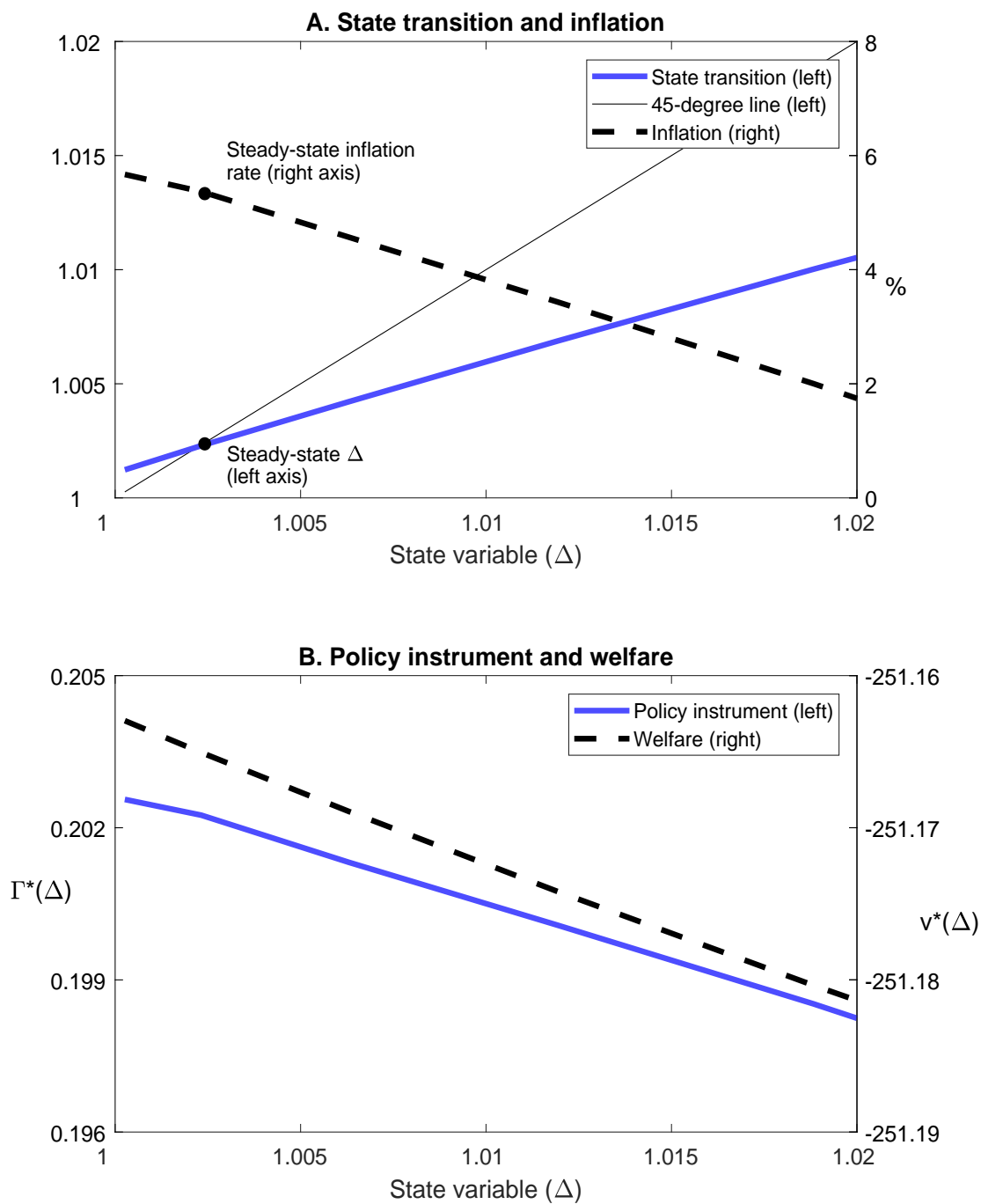
Figure 1: Equilibrium as a function of the state

the right scale).[8] Both functions are downward sloping. Intuition for the welfare function's downward slope is straightforward. By definition, the current relative price distortion represents the inverse of average productivity. But the current relative price distortion is also a summary statistic for the dispersion in relative prices. The higher is the inherited relative price distortion, the higher is the inherited dispersion in relative prices, and through (24) this contributes to a higher dispersion in current relative prices. Higher dispersion in current relative prices in turn reduces current productivity, reducing welfare.

Turning to the monetary policy function $m = \Gamma^*(\Delta)$, the fact that the future state is decreasing in Δ leads m to be decreasing in Δ. If the initial state is high, then equilibrium involves the relative price distortion declining. In this case, the large inherited relative price distortion needs to be met with a relatively low normalized money supply, so that newly adjusting firms do not exacerbate the relative price distortion. Looking in more detail, the essential intratemporal trade-off is that the policymaker has an incentive to raise the money supply in order to bring down the markup, but this incentive is checked by the cost of increasing the relative price distortion. It appears that the short-run trade-off shifts toward containing the relative price distortion as the state variable increases. That is, in equilibrium the policymaker chooses lower m at larger values of Δ because the value of the decrease in the markup that would come from holding m fixed at higher Δ is more than offset by welfare costs of a higher relative price distortion.[9]

Although we have not proved uniqueness of equilibrium, our computations have found only one equilibrium in every case, and we provide an argument in the next subsection that the numerical results do generalize. If, as we suppose, MPE is unique, the nature of the equilibrium ought to be invariant to (i) the policy instrument and (ii) whether we use an alternative approach to solving the policy problem, either by solving the GEE or solving the

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[8]In Panel B of Figure 1 we have not converted welfare into more meaningful consumption-equivalent units. We defer a quantitative discussion of welfare to Section 6.

[9]While the intratemporal trade-off between the relative price distortion and the markup is central to the policy problem, there is also an intertemporal element because the current relative price distortion is the endogenous state variable inherited by the future policymaker. The policymaker chooses "too low" a money supply with respect to current utility, because future value is decreasing in the current relative price distortion.

planner's problem as in Anderson, Kim, and Yun (2010). For our baseline parameterization we have confirmed that the same steady-state inflation rate obtains whether the policy instrument is the money supply or the nominal interest rate. In addition, we have replicated the steady-state inflation rate of 2.2 percent for Anderson, Kim, and Yun's baseline case with $\alpha = 0.75$, $\varepsilon = 11$, and a unit labor supply elasticity, for both interest rate and money supply instruments. Finally, we have computed equilibrium for our benchmark example using the GEE approach, which we discuss in Section 5.

4.2 Price setting and the lack of complementarity

Our computational approach has found no evidence of multiple equilibria, neither for the baseline calibration, whose properties are highlighted above, nor in the many other examples described in Appendix A. This is in stark contrast to the Taylor model with two-period price setting, in which King and Wolman (2004) *proved* the existence of multiple discretionary equilibria, which they traced to multiplicity of the equilibrium price set by adjusting firms. To help explain why such multiplicity does not appear in any of our numerical solutions for the Calvo model, we turn to the best-response function for price-adjusting firms.

4.2.1 The best-response function

The best-response function $p_0 = r(p_0; m, \Delta, \Gamma())$ describes an individual firm's optimal price as a function of the price set by other adjusting firms, given the state and the money supply, and conditioning on some arbitrary policy and associated stationary equilibrium that will hold in all future periods. The best-response function is represented by (30), but we rewrite it here to highlight the explicit dependence of the right-hand side on the price set by adjusting firms:

$$p_0 = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{\pi(p_0)^\varepsilon \left[w(p_0; m) + \alpha\beta S(\Delta'(\Delta, p_0); \Gamma())\right]}{\pi(p_0)^{\varepsilon-1} \left[1 + \alpha\beta F(\Delta'(\Delta, p_0); \Gamma())\right]}, \tag{31}$$

where $\pi(p_0) = \left[(1 - \alpha) p_0^{1-\varepsilon} + \alpha\right]^{1/(1-\varepsilon)}$, $w(p_0; m) = \chi m/\pi(p_0)$, and the numerator and denominator functions $S()$ and $F()$ depend on the current price chosen by adjusting firms and the current state via the law of motion for the state. The left hand side of (31) can be viewed as the individual firm's (normalized) optimal price given the actions of other price-setters and the monetary authority. The right hand side of (31) is $r(p_0; m, \Delta, \Gamma())$; it captures

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the influence of all other firms' pricing behavior on the individual firm's current and future marginal cost and marginal revenue. In a symmetric equilibrium, an individual firm chooses the same price that it sees all other adjusting firms charging.
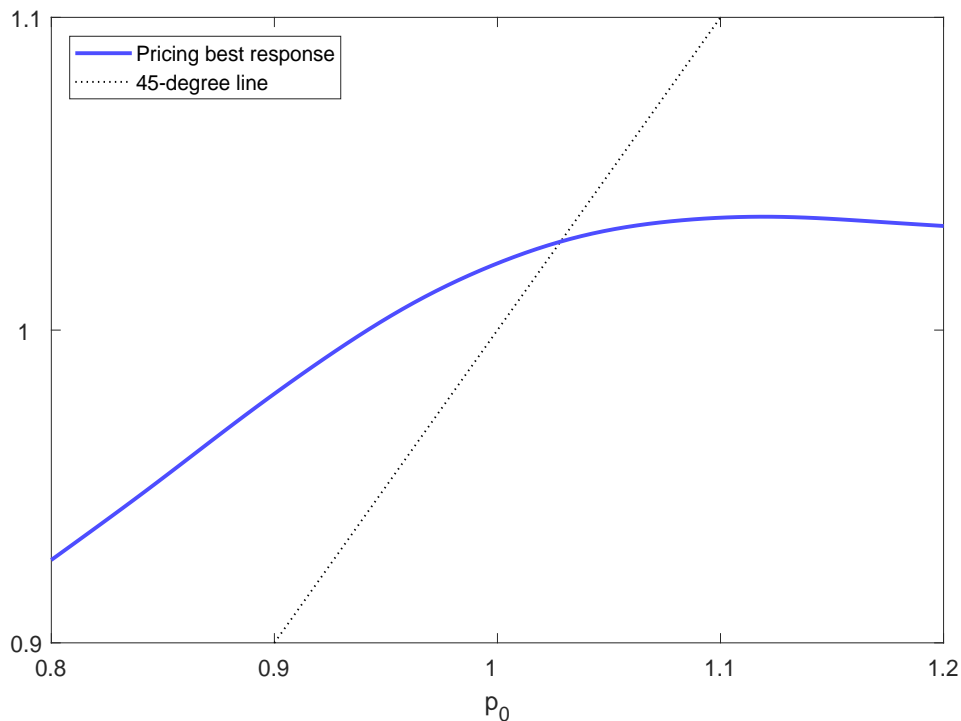

Figure 2: Pricing best-response function in the steady state: $\Delta = 1.002$, $\Gamma^*(\Delta) = 0.202$

We compute the best-response function for each available value of the state and the money supply, and find the fixed points by interpolating adjacent values of p_0 for which the sign of $r(p_0; m, \Delta, \Gamma()) - p_0$ changes. Figure 2 plots $r(p_0; m = 0.202, \Delta = 1.002, \Gamma^*())$, which is the best-response function in the steady state of the discretionary equilibrium for the baseline calibration.[10] It has a unique fixed point, and is concave in a neighborhood of the fixed point. In contrast, the best-response function in the two-period Taylor pricing model is upward sloping, strictly convex and generically has either two fixed points or no fixed points (see King and Wolman, 2004, Figure I).[11]

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[10]Although the figure shows the best-response function for a range of values of p_0 around the fixed point, the computations consider possible fixed points in the larger interval from 0 to 2.

[11]Our computations have not revealed multiple fixed points in discretionary equilibrium. However, under

The starkly different best-response functions in the two models reflect differences in how future monetary policy reacts to the nominal price firms set in the current period. This relationship is linear in the two-period Taylor model, where the current period's optimal price (P_0) is precisely the index of predetermined nominal prices that normalizes the future money supply, and the normalized money supply is constant in discretionary equilibrium. The response of the future money supply to the price set in the current period is nonlinear in the Calvo model, for two reasons. First, the relationship between P_0 and the future index of predetermined prices (P) is nonlinear. Second, the normalized money supply is not constant; as shown in Figure 1.A it responds to the real state variable, which in turn is affected by P_0. We consider next how both these factors weaken the complementarity in price setting.

4.2.2 Explaining the weak complementarity

Focusing first on the nonlinear relationship between P_0 and P, assume for now that the future policymaker sets a constant m, raising the nominal money supply in proportion to the index of predetermined prices. In the Taylor model, where the discretionary policymaker chooses such a policy, the optimal price *is* the index of predetermined prices, so the future nominal money supply rises linearly with the optimal price. Understanding that this future policy response will occur, and that the price it sets today will also be in effect in the future, an individual firm's best response is to choose a higher price when all other adjusting firms choose a higher price.

In the Calvo model, in contrast, next period's index of predetermined prices—today's price index (15)—is affected by today's index of predetermined prices as well as today's optimal price. Under a constant-m policy, the effect of an increase in today's optimal price on next period's nominal money supply depends on how the increase affects next period's index of preset prices. That index of preset prices is highly sensitive to low levels of today's optimal price and relatively insensitive to high levels of today's optimal price, because goods with higher prices have a lower expenditure share and thus receive a smaller weight in the price index. As the optimal price goes to infinity, it has no effect on the index of preset

some alternative calibrations we have encountered instances of multiple fixed points for values of m well below optimal. In such cases, there is a convex region of the best-response function to the left of the fixed point that intersects the 45-degree line twice, with a third fixed point located on the concave portion.

prices and no effect on tomorrow's nominal money supply.

Thus, in the Calvo model a constant-m policy would lead to a nominal money supply that is increasing and concave in the optimal price. Because a higher future money supply leads firms to set a higher price today, concavity of the future money supply corresponds to decreasing complementarity of the prices set by adjusters. This intuition is confirmed by the following result.

Proposition 1 *Suppose the normalized money supply is constant, and therefore independent of the state. Then the Calvo model has a unique equilibrium price set by adjusting firms.*

Proof. *See Appendix B.* ∎

The second reason for weaker complementarity in the Calvo model is that the relationship between the optimal price and the future nominal money supply depends on the future state variable. The discretionary policymaker does not hold m constant, instead lowering it with the state as illustrated in Figure 1.B. The response of next period's normalized money supply to the price set by adjusting firms today therefore depends on the relationship between p_0 and Δ'. Combining the transformed price index (23) and the market clearing condition (24) yields

$$\Delta' = \frac{(1-\alpha)p_0^{-\varepsilon} + \alpha\Delta}{\left[\alpha + (1-\alpha)p_0^{1-\varepsilon}\right]^{\varepsilon/(\varepsilon-1)}},$$

which implies that for high (low) values of p_0 the future state is increasing (decreasing) in p_0, holding fixed the current state:

$$\frac{\partial \Delta'}{\partial p_0} = \frac{\varepsilon\alpha\left(1-\alpha\right)p_0^{-\varepsilon-1}}{\left[\alpha + (1-\alpha)p_0^{1-\varepsilon}\right]^{1+[\varepsilon/(\varepsilon-1)]}}\left(\Delta p_0 - 1\right).$$

Given that equilibrium m is decreasing in Δ, future m is decreasing in p_0 for high values of p_0 and increasing in p_0 for low values of p_0. That is, a higher price set by adjusting firms—if it is greater than $1/\Delta$—translates into a higher value of the future state, and thus a lower value of the future normalized money supply.[12]

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[12]This relationship is reversed at low values of p_0 ($p_0 < 1/\Delta$): increases in p_0 would reduce the future state, and the policymaker would respond by raising future m. Such low values of p_0 are not relevant for understanding the properties of equilibrium, however, because they are associated with suboptimally low values of m. Indeed, if m were low enough that raising m would reduce both the markup and the relative price distortion, the policymaker would choose a higher m.

Summarizing the argument: in the Taylor model the normalized money supply is constant in equilibrium, and this results in an increasing convex best-response function with multiple fixed points. In the Calvo model, if policy kept the normalized money supply constant there would be a unique equilibrium: complementarity would be weaker at high p_0 than in the Taylor model, because next period's index of predetermined prices responds only weakly to p_0 at high levels of p_0. Because the normalized money supply is not constant in the discretionary equilibrium of the Calvo model, complementarity is weakened even further; m is decreasing in the state, and future m is decreasing in p_0 for high p_0. As both parts of this argument rely on the fact that there are many cohorts of firms with predetermined prices, this feature appears key to explaining why the Calvo model does not have the same tendency toward multiple discretionary equilibria as the Taylor model with two-period pricing.[13]

4.3   The effect of price rigidity on steady-state outcomes

To gain further insight into the trade-off facing the monetary authority we compare the inflation rate and the two distortions in discretionary equilibrium at different degrees of price rigidity. Table 1 displays the inflation rate, the normalized price of adjusting firms, the two distortions, and the normalized money supply in steady state for four values of the probability of no price change, α, while keeping the other parameters at their baseline values.[14] The table shows non-monotonic relationships between the variables and α. The inflation rate rises if α increases from a low level, but declines if α increases from higher levels. The normalized price of adjusting firms and the two distortions are increasing in α over a wide range, but are decreasing for very large α. The normalized money supply mimics the inverse of the two distortions.

To explain the non-monotonicity of the inflation rate, we focus first on the range of values for α from 0.5 to 0.8, which is most consistent with microeconomic evidence on price rigidity,

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[13]This reasoning suggests, however, that a Taylor model with longer duration pricing might not have multiplicity, because the same opportunities to substitute would be present. Khan, King, and Wolman (2001) find multiplicity is still present with three-period pricing. Unfortunately, it is computationally impractical to study discretionary equilibrium in a Taylor model with long-duration pricing.

[14]The alternative values considered for Table 1, $\alpha = 0.7$, 0.8, and 0.9, correspond to an average price duration of 3.3 quarters, 5 quarters, and 10 quarters, respectively.

Table 1: Steady state of discretionary equilibrium, varying price rigidity

Degree of price rigidity, α	0.5	0.7	0.8	0.9
Annualized inflation rate, %	5.4	9.7	7.2	3.3
Normalized price of adjusting firms, p_0	1.028	1.116	1.154	1.143
Relative price distortion, Δ	1.002	1.074	1.182	1.162
Markup distortion, $1/w$	1.113	1.179	1.255	1.200
Normalized money supply, m	0.202	0.193	0.180	0.187

before turning to the more extreme range from 0.8 to 0.9.[15] In the more moderate range of price rigidity a higher value of α leads adjusting firms to set a higher price, as they anticipate their nominal price will remain fixed for a longer time during which inflation will continually erode its real value. The higher optimal price of adjusters generates a larger relative price distortion by increasing the price dispersion across adjusters and non-adjusters, and a larger average markup distortion by raising adjusters' markup. The two distortions increase as α increases from 0.5 to 0.8, thus leading to a larger welfare cost of discretion. However, the inflation rate is non-monotonic, rising as α increases from 0.5 to 0.7 but declining as α increases to 0.8.[16] The transformed price index (23) implies that a higher α (a larger fraction of non-adjusting firms) and a higher normalized price of adjusting firms have offsetting effects on inflation: a higher p_0 raises inflation, but the higher α reduces the weight of p_0 in the price index, dampening the effect of p_0 on inflation.[17] Thus, when α is large enough the direct effect of increasing α outweighs the indirect effect through a higher p_0. The disparate consequences of the level of α for inflation and the two distortions in steady state show that the size of the inflation bias can be a misleading gauge of the welfare cost of discretion.

Once the degree of price rigidity becomes very large, further increases in α lead the

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[15]See Klenow and Malin (2010) and Nakamura and Steinsson (2013) for literature reviews of microeconomic evidence on price rigidity.

[16]Furthermore, the steady-state inflation rate is decreasing in the demand elasticity ε. Anderson, Kim, and Yun (2010) point out similar relationships between the model's structural parameters and the steady-state inflation rate.

[17]The transformed price index (23) implies that $\partial \pi(p_0; \alpha)/\partial \alpha < 0$ as long as $p_0 > 1$.

normalized price of adjusters to decline as shown in the last two columns of Table 1. The direct effect of a higher α reduces the inflation rate in (23), but the lower steady-state inflation rate also influences the price set by adjusting firms because it implies slower erosion of their real price. At high levels of price rigidity the reduced real price erosion stemming from lower inflation outweighs the increased average duration of the nominal price stemming from a higher α. The lower p_0 in turn reduces the relative price and average markup distortions. Therefore, even the degree of nominal price rigidity itself can be a misleading indicator of the welfare cost of discretion.

The last line of the table shows the normalized money supply mimics the inverse of the two distortions. As α rises from 0.5 to 0.8 and the magnitude of the two distortions increases, the monetary authority increasingly acts to curb the increase in the relative price distortion with a lower money supply, even though that means accepting a larger markup distortion. A further increase in α to 0.9 reduces the two distortions and brings about a larger money supply, indicating the monetary authority's concern shifts back toward the markup.

5   Generalized Euler equation

Until this point, we have been careful to allow for the possibility of multiple fixed points to a firm's best-response function. This has meant eschewing a first-order approach to the policy problem, as we needed to check for uniqueness of the price set by adjusting firms for all feasible values of m. For the broad range of parameter values that we have studied, however, we have found that this price is always unique at the optimal choice of m. Therefore, the first-order approach described by Krusell, Kuruscu, and Smith (2002) and Klein, Krusell, and Rios-Rull (2008, henceforth KKR) is appropriate for our problem, ought to yield equivalent results to those described above, and may provide additional insight into the nature of equilibrium. In this section we describe the discretionary equilibrium in terms of the policymaker's optimality condition (GEE), assuming the monetary policy function is differentiable and there is a unique fixed point to an adjusting firm's best-response function.

To state the GEE we define the firm's "pricing wedge" $\eta\left(\Delta, m, p_0\right)$, which is the (out-of-

equilibrium) deviation from the optimal pricing condition:

$$\eta(\Delta, m, p_0) = p_0 \left[1 + \alpha \beta F(\Delta'(\Delta, p_0))\right] - \left(\frac{\varepsilon}{\varepsilon - 1}\right) \pi(p_0) \left[-\frac{u_n}{u_c} + \alpha \beta S(\Delta'(\Delta, p_0))\right].$$

This expression is written in more generality than we allowed for above, where $u_n = -\chi$ and $u_c = 1/c$. Following KKR, given an equilibrium and under some regularity conditions, the implicit function theorem guarantees that there exists a unique function $H(\Delta, m)$, defined on some neighborhood of the steady state, satisfying $\eta(\Delta, m, H(\Delta, m)) \equiv 0$ in that neighborhood. The function H gives the price of an adjusting firm if the current state is Δ, current money is m, and firms expect that future money will be determined by the equilibrium policy function Γ^*. Thus, H describes an adjusting firm's response to a one-time deviation of monetary policy from the equilibrium policy, and it implies that $H_m = -\eta_m/\eta_{p_0}$ and $H_\Delta = -\eta_\Delta/\eta_{p_0}$.

The GEE is the first-order condition for the policymaker, incorporating all other equilibrium conditions:

$$\Theta + \beta H_m D_{p_0} \left[u'_n N'_\Delta - \frac{\eta'_\Delta}{\eta'_m} \left(u'_c C'_m + u'_n N'_m\right) + \frac{D'_\Delta}{D'_{p_0}} \frac{\eta'_{p_0}}{\eta'_m} \Theta'\right] = 0, \tag{32}$$

where

$$\Theta \equiv u_c C_m + u_n N_m + H_m \left(u_c C_{p_0} + u_n N_{p_0}\right).$$

Here we use the shorthand notation $c = C(p_0, m)$, $n = N(\Delta, p_0, m)$, and $\Delta' = D(\Delta, p_0)$ for the functions in Eqs. (24)$-$(26), and prime always denotes the next period, never derivative. The derivation of the GEE is provided in Appendix C. The GEE states that in equilibrium, a marginal change in the current money supply leaves welfare unchanged. The variable Θ represents the change in current utility with respect to a change in the current money supply. The term in brackets consists of three effects on future welfare. First, the future state variable affects welfare directly because higher relative price dispersion means lower productivity. Second, the future state changes the future money supply, which affects consumption and leisure. Third, the future state changes the future price of adjusting firms, which has a separate effect on consumption and leisure. The coefficient on future marginal value, $\beta H_m D_{p_0}$, represents discounting and the mapping from a change in current m to a change in the future state.

The GEE highlights both how the lack of commitment affects optimal policy and the fact that the discretionary policymaker does have some ability to affect expectations about future policy. In contrast to policy under commitment, the optimality condition (32) for the current money supply incorporates a response of future policy to the endogenous state variable, captured by the terms in C'_m, N'_m, and H'_m (some of these terms are contained in Θ'). Under commitment, future policy actions would be a function only of the initial state: the significance of commitment is precisely that policy will not respond in the future to the evolution of the endogenous state.[18] As under discretion, there would be a direct effect of current policy on future welfare through the state variable, but no indirect effect through future policy.

In addition to its analytical value, the GEE can be used as the basis for an alternative approach to computing equilibrium. In a reassuring check on our results above, using the GEE approach we computed an identical steady-state inflation rate of 5.4 percent to that reported in Section 4, although away from steady state the equilibrium differed slightly.[19]

6   Transitions to and from discretion

Having concluded earlier that the inflation rate can be a misleading gauge of the welfare cost of discretion—equilibrium can be characterized by relatively high inflation but relatively small distortions or the other way around, depending on the degree of price rigidity—in this section we examine the welfare cost of discretion explicitly. The simplest way is by comparing the steady-state levels of welfare under commitment and discretion. However, both empirical and theoretical considerations suggest that the steady state comparison may be incomplete. Empirically, large changes in the inflation rate rarely occur instantaneously. For example, the famous Volcker disinflation played out over a period of at least three years. Theoretically, we have emphasized the presence of a state variable in the discretionary equilibrium, and

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[18]With commitment and exogenous shocks, the future money supply would respond to the shock realizations.

[19]In an application to fiscal policy, Azzimonti, Sarte, and Soares (2009) also find that different computational approaches produce identical steady states under discretionary policy, but somewhat different dynamics.

commitment induces additional policy inertia, as emphasized by Woodford (2003b).

Thus, we examine the cost and benefit, respectively, of losing and gaining the ability to commit. Aside from giving a more complete welfare comparison, the transitional dynamics can be of independent interest. In particular, they indicate whether losing the ability to commit simply reverses the inflation dynamics induced by acquiring commitment, or whether the two transitions are qualitatively different. Acquisition of ability to commit involves the transitional dynamics under commitment, starting from the steady state under discretion. Loss of ability to commit involves the transitional dynamics under discretion, starting from the steady state under commitment.

6.1 Optimal allocations with commitment

To analyze optimal policy under commitment, we solve a social planner's problem to avoid the issue of how the policy is implemented. That is, we consider the problem of a planner who can choose current and future prices and quantities, subject to the conditions that characterize optimal behavior by households and firms, and subject to markets clearing. The planner's problem can be written as

$$\max_{\left\{c_t, n_t, w_t, \pi_t, \tilde{S}_t, \tilde{F}_t, p_{0,t}, \Delta_t\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \chi n_t\right),$$

subject to the labor supply equation (4), the conditions related to optimal pricing (8)−(10), the price index (16), and the relative price distortion's law of motion (18) and definition (19), for $t = 0, 1, \ldots$. Recall that without commitment, there was a single state variable, Δ_{t-1}. With commitment, the presence of future realizations of variables in the constraints means that there are two additional "artificial" state variables ϕ_{t-1} and ψ_{t-1} for $t = 1, 2, \ldots$, the lagged Lagrange multipliers on the constraints (8) and (9). The first-order conditions for this problem can be simplified to a system of nine nonlinear difference equations in the nine variables $\left\{c_t, \tilde{S}_t, \tilde{F}_t, \pi_t, \Delta_t, \phi_t, \psi_t, \xi_t, \gamma_t\right\}$, where ξ_t and γ_t are the Lagrange multipliers on (16) and (18). The nine-equation system is derived in Appendix D.

Comparing the steady states under commitment and discretion gives a rough estimate of the benefit (cost) of gaining (losing) the ability to commit to future policies. Whereas computing the steady state under discretion required solving for the functions describing

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equilibrium dynamics, the steady state under commitment is simply the time-invariant solution to the nine-equation system implied by the planner's first-order conditions. As shown in Appendix D, the steady-state inflation rate is zero; that is, $\pi = \Delta = p_0 = 1$.[20] Using the baseline calibration, both consumption and leisure are slightly higher in the commitment steady state, with zero inflation, than in the discretionary steady state. The welfare difference between the two steady states is equivalent to 0.221 percent of consumption every quarter.[21] In present value terms, the consumption increment represents 5.53 percent of annual consumption. Next we analyze the transitions between steady states under the baseline calibration.

6.2 Gaining the ability to commit

If a policymaker previously operating with discretion gains the ability to commit, the economy behaves according to the dynamics under commitment, beginning in the discretionary steady state and—presumably—ending in the commitment steady state. The dynamics under commitment are represented by the aforementioned nine-variable system of nonlinear difference equations.

To compute the transition path, we conjecture that convergence to the zero-inflation steady state is complete after $T = 40$ quarters. We then have a system of $9 \times T$ equations in the $9 \times T + 5$ variables $\left\{ c_t, \tilde{S}_t, \tilde{F}_t, \pi_t, \Delta_t, \phi_t, \psi_t, \xi_t, \gamma_t \right\}_{t=0}^{T-1}$ and $\left\{ \Delta_{-1}, \tilde{S}_T, \tilde{F}_T, \pi_T, \gamma_T \right\}$. If we assume there is a unique transition path, then to solve the system of $9 \times T$ equations we need to specify values for the initial condition Δ_{-1} and the terminal conditions $\left\{ \tilde{S}_T, \tilde{F}_T, \pi_T, \gamma_T \right\}$. Under the conjecture, the terminal conditions are given by the steady state under commitment (zero inflation). The initial condition is given by the steady-state value of Δ under

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[20]We use the term "steady state" informally in the case of a policymaker with commitment. It is more accurate to refer to this allocation as the limit point in the long run under commitment. A planner who inherited only the state variable $\Delta = 1$ would choose some initial inflation before converging in the long run back to $\Delta = 1$ and zero inflation; this reflects the time-consistency problem.

[21]The welfare calculation involves comparing the discretionary steady state to an allocation on the same indifference curve as the commitment steady state, but with the same wage as the discretionary steady state. The number 0.221 percent represents the parallel rightward shift of the budget constraint (consumption on the horizontal axis).

discretion. From the properties of the Jacobian matrix at the steady state, we know that locally there is a unique stable solution that converges to the steady state. Indeed, we computed a global solution satisfying the conjecture.

The solid lines in Figure 3 represent the paths of inflation, the relative price distortion, the markup, and the money growth rate along the transition from the steady state with discretion to the steady state with commitment. The transition contains an element of discretionary behavior: in the initial period (labeled zero), as the policy change is unexpected the policymaker has an incentive to exploit the fixed prices of non-adjusters by increasing the money supply (panel A). Hence the markup declines temporarily before settling at its steady-state level under commitment (panel C). However, because the long-run policy involves lower inflation, adjusting firms do not offset the temporary stimulus by frontloading larger price increases. Instead, they frontload smaller future price increases, more than offsetting any inflationary effects of the temporary monetary stimulus. As a result, the inflation rate declines gradually from the steady-state level under discretion to zero (panel B), as does the relative price distortion (panel D).

The welfare benefit of the transition from discretion to commitment is well approximated by the steady-state welfare comparison: the representative household would require a 0.216 percent increase in consumption each quarter in order to willingly forego the transition from discretion to commitment (5.39 percent in present-value terms).[22]

6.3   Losing the ability to commit

If an optimizing policymaker loses the ability to commit, then the economy behaves according to the transitional dynamics under discretion with an initial condition of $\Delta = 1$, the steady state under commitment. Although these dynamics can be inferred from Figure 1, we plot them explicitly in Figure 3 (dashed lines). Unlike the case where commitment is gained, the inflation rate overshoots in the initial period and then declines smoothly to the discretionary steady-state level. The money growth rate essentially mimics the inflation rate. The transition to the discretionary steady state does not involve any transitory benefits: along

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[22]The steady-state welfare comparison is not as good an approximation to the transition at high degrees of price stickiness.
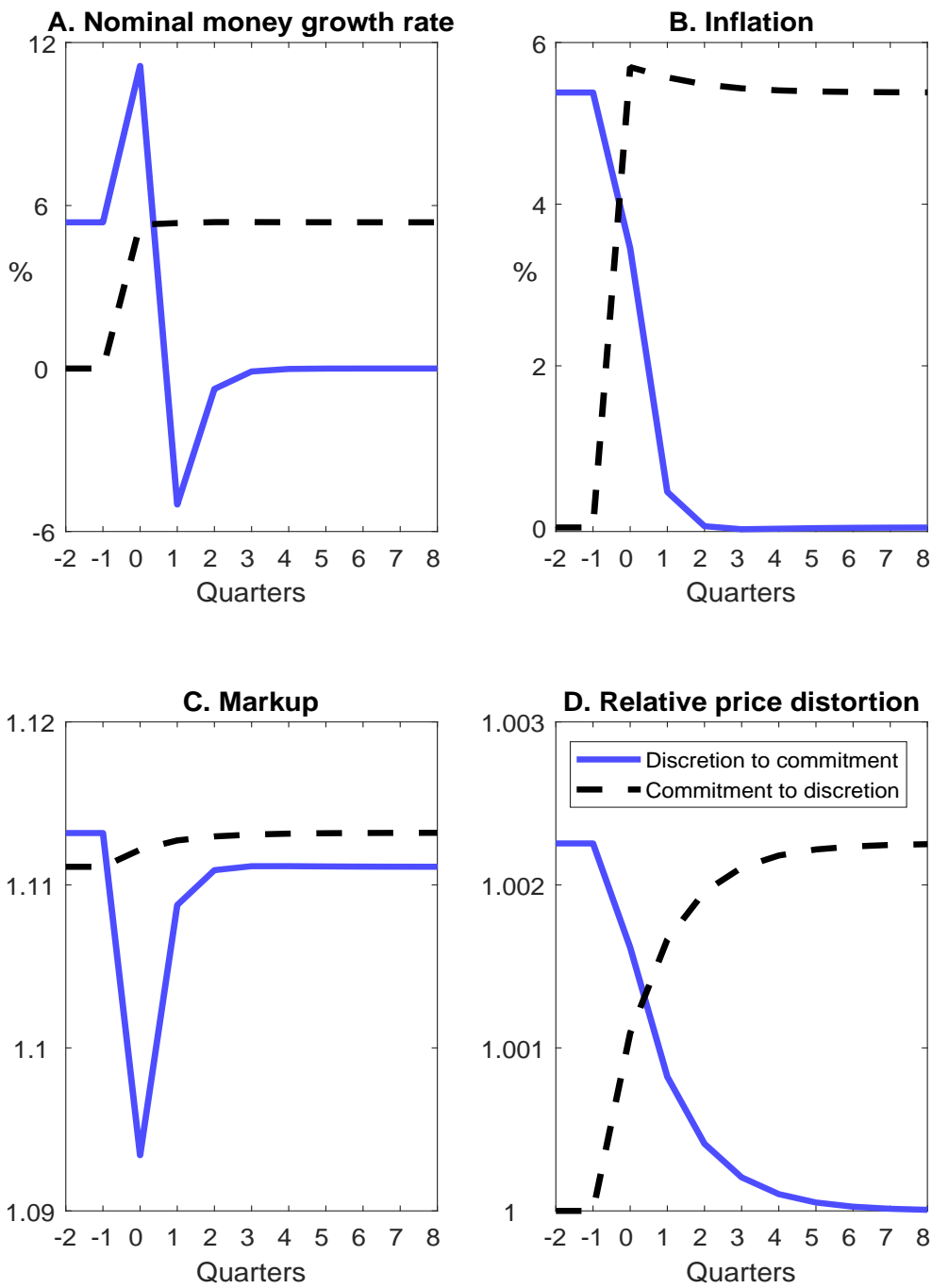
Figure 3: Transitions to and from discretion

the entire transition both the markup and the relative price distortion are increasing. One might have expected that in the initial period of the transition, the policymaker could effectively exploit preset prices and reduce the markup. It is indeed the case that the markup for non-adjusting firms falls substantially in the initial period. However, this decline is more than offset by an increase in the inflation rate that results from the behavior of adjusting firms. This reasoning uses the identity $P_t/MC_t = (P_t/P_{t-1}) \times (P_{t-1}/MC_t)$.[23]

The welfare decline associated with loss of commitment is again well-approximated by the steady-state welfare comparison: the representative household would be willing to give up 0.219 percent of consumption each quarter in order to avoid this transition (5.47 percent in present-value terms).[24]

Initial-period policy under commitment and discretion illustrates the difficulty of exploiting initial conditions. The discretionary monetary authority would like to exploit initial conditions, but in equilibrium it is unable to do so even in the short run because of firms' forward-looking behavior. Conversely, that same forward-looking behavior means that a policymaker who can commit is able to exploit initial conditions (once!) by combining short-run expansionary policy with lower money growth and inflation in the long run.

7   Concluding remarks

The vast literature on discretionary monetary policy with nominal rigidities is comprised of two seemingly disparate branches. Much of the profession's intuition is derived from the seminal work by Barro and Gordon (1983), hereafter BG, which in turn built on Kydland and Prescott (1977). They studied reduced-form macroeconomic models in which the frictions giving leverage to monetary policy were not precisely spelled out. In contrast, the staggered pricing models popularized in the last two decades are precise about those frictions. We conclude by summarizing the paper's three main contributions and then explaining how the

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[23]Although the equilibrium path involves both the markup and the relative price distortion rising, it is nonetheless the case that at each point in time the policymaker perceives a trade-off between reducing the markup and increasing the relative price distortion.

[24]Again, the steady-state welfare comparison is not as good an approximation to the transition at high degrees of price stickiness.

analysis relates to BG's early work on time-consistency problems for monetary policy.[25]

The Calvo model is the most influential model of staggered price setting for applied monetary policy analysis. Although it shares many features with the Taylor model, we find it does not share multiplicity of equilibrium under discretionary policy. Whereas previous literature based on the Taylor model shows discretionary policy induces complementarity among firms sufficient to generate multiple equilibria for their optimal price, we find no evidence of multiple equilibria in the Calvo model. The combination of a unique equilibrium and a real state variable allows us to analyze discretionary equilibrium using the GEE, a representation of the policymaker's first-order condition that highlights the various channels through which current policy can affect future welfare. We also use the steady-state and dynamic properties of the discretionary equilibrium together with the solution under commitment to study the processes of gaining and losing the ability to commit. The present-value welfare gain and loss considering the full transition paths are of similar magnitude as those based on steady-state welfare comparisons, though inflation dynamics differ qualitatively in the two transitions.

The time-consistency problem arises in both the staggered-pricing models and in BG from the interaction of two factors. First, there is a monopoly distortion. Second, some prices are determined before the monetary policy instrument. A discretionary policymaker therefore takes as given private agents' expectations—they are embedded in the predetermined prices— and has an incentive to reduce the monopoly distortion with a monetary surprise. But in equilibrium expectations accurately incorporate the policymaker's optimal behavior. In BG, the expectations just referred to are current expectations about current policy; dynamics only arise through serial correlation of exogenous shocks. Without other intertemporal links, the policy problem is a static one in BG: treating expectations as fixed, higher inflation is costly in its own right but brings about a beneficial reduction in unemployment. In equilibrium, private expectations are validated, and the policymaker balances the static marginal cost and marginal benefit of additional inflation. In contrast, in staggered pricing models prices set

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[25] A related strand of the literature studies the monetary policy time-consistency problem that arises when there is nominal debt, even if prices are flexible. Calvo (1978) is the seminal reference, and more recent contributions have been made by Díaz-Giménez et al. (2008), Martin (2009), and Niemann, Pichler, and Sorger (2013a). Niemann, Pichler, and Sorger (2013b) link the two strands of the literature by studying a model with nominal debt and Rotemberg-style sticky prices.

in the past incorporate expectations about current policy. Equilibrium requires that current policy actions be consistent with expectations formed in the past.[26]

The intertemporal nature of price setting also means that unlike BG, staggered pricing models generally contain one or more state variables that can be affected by a policymaker, even under discretion. Thus, the discretionary policymaker does not face a purely static trade-off between inflation and real activity. That trade-off is present, but it is complicated by the fact that the current policy action affects tomorrow's state, and thus tomorrow's value function. A key message of our paper is that the details of this intertemporal element differ across staggered pricing models, leading to different implications for the nature of equilibrium under discretionary monetary policy.

While staggered pricing models generate a static output-inflation trade-off superficially similar to the one in BG, forward-looking behavior means that the details of the policy trade-off depend critically on the entire path of expected future policy. For example, Section 6 has shown that an unexpected reduction in inflation can be stimulative, if it signals the transition to a permanently lower inflation rate (gaining commitment). Likewise, an unexpected increase in inflation can be contractionary if it signals the transition to a permanently higher inflation rate (losing commitment). Although these transitions may suggest that no output-inflation trade-off is present, in the discretionary equilibrium the policymaker perceives such a trade-off: a one-period deviation toward more expansionary policy would raise output and inflation, as it reduced the markup and raised the relative price distortion. The effects on welfare would be offsetting, and thus the policymaker does not deviate.

The properties of discretionary equilibrium are determined by the specifics of the model. The defining feature of the Calvo model is the assumption that a fraction of firms are prohibited from adjusting their price. This makes for a relatively tractable framework, undoubtedly the main reason the Calvo model has come to serve as the basis for so much applied work on monetary policy. It has recently become feasible to conduct some forms of policy analysis in models which allow firms to adjust their price by incurring a cost. Those models typically have a large number of state variables, currently rendering it impractical to perform the kind

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[26]With different timing assumptions in staggered pricing models the BG version of expectational consistency would also be required to hold.

of analysis conducted here (see for example Nakov and Thomas, 2014).[27] Nonetheless, we hope that our work can serve as a useful input for future research on discretionary policy in quantitative state-dependent pricing models.

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[27]While Barseghyan and DiCecio (2007) and Siu (2008) study discretionary policy in models with state-dependent pricing, both papers limit the state space by allowing firms to adjust costlessly after one and two periods, respectively.

Appendix

A   Computational details

This appendix describes how numerical solutions for the discretionary equilibrium are computed and how uniqueness of the equilibrium is verified in a large number of examples.

Computing a discretionary equilibrium

The value function and the expressions for $S()$ and $F()$ are approximated with Chebyshev polynomials. This computational method involves selecting a degree of approximation I, and then searching for values of v_i^* and Γ_i^*, for $i = 1 \ldots I$, that solve (28) and (29) at the grid points for the state variable Δ_i defined by the Chebyshev nodes. The optimization problem $(28)-(29)$ is solved using the following algorithm.

1. *Grids and initial values.* The example of the baseline calibration (i.e., $\alpha = 0.5$, $\beta = 0.99$, $\epsilon = 10$, and $\chi = 4.5$) uses a degree of approximation $I = 12$ on the interval $[1, 1.061]$ for the state variable. As an initial guess for $v()$, $S()$ and $F()$ the discretionary equilibrium for the static model is used, which is the final period of a finite-horizon model. To compute the private-sector response to an arbitrary policy, grids $\{m_1, \ldots, m_{I_m}\}$ and $\{p_{0,1}, \ldots, p_{0,I_p}\}$ are specified for the money supply and the optimal price. In the case of the baseline calibration, the grid for m consists of $I_m = 775$ evenly spaced points between 0.01 and 0.25 and the grid for p_0 consists of $I_p = 400$ evenly spaced points between 0 and 2.

2. *Private-sector responses.* For each possible value of Δ and m, compute the private-sector responses by solving (30) as a fixed-point problem. Specifically, compute the right hand side of (30) and call it $r(p_0)$. Then find the fixed points $p_0 = r(p_0)$ by linear interpolation of adjacent values of p_0 for which the sign of $r(p_0) - p_0$ changes (and check for values on the grid for p_0 that satisfy $r(p_0) = p_0$).

3. *Policy function and value function.* On each grid point Δ_i, select the value of m that maximizes the value function. If the value function and policy function that solve the

optimization problem are identical to the guess, then they form a discretionary equilibrium. Specifically, iteration j is the final iteration if $||v^{j+1} - v^j||_\infty$ and $||\Gamma^{j+1} - \Gamma^j||_\infty$ are smaller than the tolerance level $1.49 \cdot 10^{-8}$ (the square-root of machine precision). If not, the starting values are updated by pushing out the initial guess one period into the future, and assuming the one-period-ahead policy and value functions are the ones that solved the optimization problem.

To assess the accuracy of a solution, the difference between the left hand side and the right hand side of (29) is calculated using that solution on a grid of 100,000 points that do not include the Chebyshev nodes. With the baseline calibration, this residual function has a maximum absolute approximation error of order 10^{-6}.

Many other examples were computed that cover a wide range of values for α and ε. These include the values for $\alpha = 0.1$, 0.15 , 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.55, 0.6, 0.61, 0.62, 0.63, 0.64, 0.65, 0.66, 0.67, 0.68, 0.69, 0.70, 0.71, 0.72, 0.73, 0.74, 0.75, 0.76, 0.77, 0.78, 0.79, 0.8, 0.85, 0.9, and 0.95, and values for $\varepsilon = 6$, 7, 8, 9, and 11. In addition to these 38 solutions, which consider alternative values of one parameter at a time, solutions were computed for four combinations of extreme parameter values: $(\alpha, \varepsilon) = (0.1, 4)$, $(0.1, 11)$, $(0.9, 4)$, $(0.9, 11)$.

Uniqueness of the solutions

All the examples described above yield a unique solution. Step 2 in the model solution algorithm allows for the possibility of multiple fixed points at an arbitrary monetary policy at each value of the state. Suppose the money supply that maximizes the value function in iteration $j - 1$ induces multiple private-sector responses. Then the inherited relative price dispersion in iteration j is not uniquely determined and neither is the money supply in iteration j. Therefore, if j is the final iteration there are multiple discretionary equilibria. However, this hypothetical sequence of outcomes does not arise in any of our examples, where discretionary equilibrium is always unique. Multiple fixed points were only encountered for sub-optimal values of m, in which case the largest fixed point was arbitrarily selected (the same solutions were found when selecting the smallest fixed point). Specifically, for the alternative calibrations with values of α from 0.64 to 0.69, or with the value $\varepsilon = 6$, the final iteration of the solution algorithm exhibited multiple fixed points for values of m in a range

that is positive but smaller than the optimal value of m.

B Proof of Proposition 1

This appendix presents the proof of Proposition 1. Recall from equation (16) that inflation is the following function of the normalized price of an adjusting firm:

$$\pi(p_{0,t}) = \left[(1-\alpha)p_{0,t}^{1-\varepsilon} + \alpha\right]^{\frac{1}{1-\varepsilon}},$$

which is increasing and strictly concave:

$$\pi'(p_{0,t}) = (1-\alpha)\left[(1-\alpha) + \alpha p_{0,t}^{\varepsilon-1}\right]^{-\frac{\varepsilon}{\varepsilon-1}} = (1-\alpha)\left[\frac{\pi(p_{0,t})}{p_{0,t}}\right]^{\varepsilon} > 0 \tag{33}$$

$$\pi''(p_{0,t}) = -\alpha(1-\alpha)\varepsilon\pi(p_{0,t})^{2\varepsilon-1}p_{0,t}^{3(\varepsilon-1)} < 0, \tag{34}$$

and has a finite limit

$$\lim_{p_{0,t}\to\infty} \pi(p_{0,t}) = \alpha^{\frac{1}{1-\varepsilon}}. \tag{35}$$

Consistent with the computation of discretionary equilibrium as the stationary limit of the finite-horizon economy, we compute equilibrium with a constant-m policy as the limit of the finite-horizon economy. Let T denote the final period, so $S_{T+1} = F_{T+1} = 0$. Then:

$$S_T = \pi(p_{0,T})^{\varepsilon-1}\left[\chi m_T + \alpha\beta S_{T+1}\pi(p_{0,T})\right] = \chi\pi(p_{0,T})^{\varepsilon-1}m_T, \tag{36}$$

$$F_T = \pi(p_{0,T})^{\varepsilon-1}\left[1 + \alpha\beta F_{T+1}\right] = \pi(p_{0,T})^{\varepsilon-1}, \tag{37}$$

and the pricing best-response function is

$$\hat{p}_{0,T} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{S_T}{F_T} = \left(\frac{\varepsilon\chi}{\varepsilon-1}\right)m_T. \tag{38}$$

The outcomes $p_{0,T}$, S_T, and F_T do not depend on the state because monetary policy does not depend on the state. Moreover, there can be no complementarity in price setting in period T, because the pricing best-response function (38) of any given firm does not depend on other firms' price decisions.

Note from (36) that $S_T = S_T(m_T, p_{0,T})$ and from (37) that $F_T = F_T(p_{0,T})$. We can now analyze the period $T-1$ pricing best-response function to determine whether there is a unique fixed point. We have:

$$S_{T-1} = \pi(p_{0,T-1})^{\varepsilon-1}\left[\chi m_{T-1} + \alpha\beta S_T(m_T, p_{0,T})\pi(p_{0,T-1})\right]$$

$$F_{T-1} = \pi(p_{0,T-1})^{\varepsilon-1}\left[1 + \alpha\beta F_T(p_{0,T})\right],$$

so the period $T-1$ best-response function is

$$\hat{p}_{0,T-1} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\left[\frac{\chi m_{T-1} + \alpha\beta S_T(m_T, p_{0,T})\pi(p_{0,T-1})}{1 + \alpha\beta F_T(p_{0,T})}\right] \tag{39}$$

The optimal price does not depend on the state because the monetary policy function and the functions S_T and F_T do not depend on the state. To see that the best-response function has a unique fixed point, first write (39) as

$$\hat{p}_{0,T-1} = A_{T-1}(p_{0,T})m_{T-1} + B_{T-1}(m_T, p_{0,T})\pi(p_{0,T-1}),$$

where $A_{T-1}(p_{0,T}) > 0$ and $B_{T-1}(m_T, p_{0,T}) > 0$ because $m_T, p_{0,T} > 0$. It follows from $(33)-(35)$ that

$$\frac{\partial \hat{p}_{0,T-1}}{\partial p_{0,T-1}} = B_{T-1}\pi'(p_{0,T-1}) > 0$$

$$\frac{\partial^2 \hat{p}_{0,T-1}}{\partial p_{0,T-1}^2} = B_{T-1}\pi''(p_{0,T-1}) < 0$$

$$\lim_{p_{0,T-1}\to\infty}\hat{p}_{0,T-1} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\left[\frac{\chi m_{T-1} + \alpha^{\frac{2-\varepsilon}{1-\varepsilon}}\beta S_T(m_T, p_{0,T})}{1 + \alpha\beta F_T(p_{0,T})}\right].$$

Because the best-response function is always positive and concave and has a finite limit, it has a unique fixed point. Therefore, there exists a unique price set by adjusting firms in period $T-1$.

Write $S_{T-1} = S_{T-1}(m_{T-1}, m_T, p_{0,T-1}, p_{0,T})$ and $F_{T-1} = F_{T-1}(p_{0,T-1}, p_{0,T})$. In period $T-2$ we obtain

$$S_{T-2} = \pi(p_{0,T-2})^{\varepsilon-1}\left[\chi m_{T-2} + \alpha\beta S_{T-1}(m_{T-1}, m_T, p_{0,T-1}, p_{0,T})\pi(p_{0,T-2})\right]$$

$$F_{T-2} = \pi(p_{0,T-2})^{\varepsilon-1}\left[1 + \alpha\beta F_{T-1}(p_{0,T-1}, p_{0,T})\right].$$

Hence the period $T-2$ best-response function can be written as

$$\hat{p}_{0,T-2} = A_{T-2}(p_{0,T-1}, p_{0,T})m_{T-2} + B_{T-2}(m_{T-1}, m_T, p_{0,T-1}, p_{0,T})\pi(p_{0,T-2}),$$

where $A_{T-2} > 0$ and $B_{T-2} > 0$ because $m_T, m_{T-1}, p_{0,T}, p_{0,T-1} > 0$. By the same arguments as above there is a unique fixed point in period $T-2$.

Repeating the same steps, we can show that for period t,

$$S_t = \pi(p_{0,t})^{\varepsilon-1}\left[\chi m_t + \alpha\beta S(m_{t+1}, m_{t+2}, \ldots, p_{0,t+1}, p_{0,t+2}, \ldots)\pi(p_{0,t})\right]$$

$$F_t = \pi(p_{0,t})^{\varepsilon-1}\left[1 + \alpha\beta F(p_{0,t+1}, p_{0,t+2}, \ldots)\right].$$

The period-t best-response function can therefore be written as

$$\hat{p}_{0,t} = A_t(p_{0,t+1}, p_{0,t+2}, \ldots)m_t + B_t(m_{t+1}, m_{t+2}, \ldots, p_{0,t+1}, p_{0,t+2}, \ldots)\pi(p_{0,t}),$$

where $A_t > 0$ and $B_t > 0$ because $m_{t+j}, p_{0,t+j} > 0$ for $j = 1, 2, \ldots$. Therefore, by backward induction, there is a unique price set by adjusting firms associated with the constant-m policy.

C Derivation of the GEE

Discretionary equilibrium consists of a value function v, a monetary policy function Γ, and a pricing function h such that for all Δ, $m = \Gamma(\Delta)$ solves[28]

$$\max_m \{u(C(p_0, m), N(\Delta, p_0, m)) + \beta v(D(\Delta, p_0))\},$$

$p_0 = h(\Delta)$ satisfies the optimality condition for the price chosen by adjusting firms

$$p_0\left[1 + \alpha\beta F(D(\Delta, p_0))\right] = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left[(1 - \alpha)p_0^{1-\varepsilon} + \alpha\right]^{\frac{1}{1-\varepsilon}}\left[-\frac{u_n}{u_c} + \alpha\beta S(D(\Delta, p_0))\right],$$

and $v()$ is given by

$$v(\Delta) \equiv u(C(h(\Delta), \Gamma(\Delta)), N(\Delta, h(\Delta), \Gamma(\Delta))) + \beta v(D(\Delta, h(\Delta))). \tag{40}$$

Under the assumption of uniqueness, this description of equilibrium is equivalent to that in Section 3.

Assuming differentiability of the policy function $\Gamma()$, we derive a simplified representation of the policymaker's first-order condition by using the envelope condition to eliminate the derivative of the value function, as in KKR. Using the definition of $H(\Delta, m)$, which implies that $H_m = -\eta_m/\eta_{p_0}$ and $H_\Delta = -\eta_\Delta/\eta_{p_0}$, the first-order condition for the monetary authority is

$$u_c(C_{p_0}H_m + C_m) + u_n(N_{p_0}H_m + N_m) + \beta v'_\Delta D_{p_0}H_m = 0. \tag{41}$$

The next step is to get an expression for v'_Δ. Begin by differentiating (40) with respect to Δ, replacing $h(\Delta)$ with $H(\Delta, m)$, using the fact that in equilibrium $h(\Delta) = H(\Delta, \Gamma(\Delta))$:

$$v_\Delta = u_n N_\Delta + H_\Delta(u_c C_{p_0} + u_n N_{p_0} + \beta D_{p_0}v'_\Delta) + \beta D_\Delta v'_\Delta.$$

---

[28]We denote the value function and policy function in discretionary equilibrium by v and Γ, dropping for simplicity the asterisk notation used in Section 3.

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Using the monetary authority's first order condition (41) we can eliminate v_Δ', writing the value function derivative in purely static terms:

$$v_\Delta = u_n N_\Delta - \frac{\eta_\Delta}{\eta_m}\left(u_c C_m + u_n N_m\right) - \frac{D_\Delta}{D_{p_0}}\left[u_c C_{p_0} + u_n N_{p_0} - \frac{\eta_{p_0}}{\eta_m}\left(u_c C_m + u_n N_m\right)\right]. \quad (42)$$

Pushing (42) one period forward, we use it to eliminate the value function derivative from the monetary authority's first-order condition (41), therefore writing that first-order condition as the GEE (32).

D   Derivation of commitment solution

Here we derive the equations characterizing optimal policy with commitment using the following Lagrangian:

$$
\begin{aligned}
\mathcal{L} &= \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \chi n_t\right) + \sum_{t=0}^{\infty} \beta^t \zeta_t \left[p_{0,t} - \left(\frac{\varepsilon}{\varepsilon-1}\right)\pi_t \frac{\tilde{S}_t}{\tilde{F}_t}\right] \\
&\quad + \sum_{t=0}^{\infty} \beta^t \phi_t \left[\tilde{S}_t - \left(w_t + \alpha\beta\pi_{t+1}^{\varepsilon}\tilde{S}_{t+1}\right)\right] + \sum_{t=0}^{\infty} \beta^t \psi_t \left[\tilde{F}_t - \left(1 + \alpha\beta\pi_{t+1}^{\varepsilon-1}\tilde{F}_{t+1}\right)\right] \\
&\quad + \sum_{t=0}^{\infty} \beta^t \xi_t \left\{\pi_t - \left[(1-\alpha)p_{0,t}^{1-\varepsilon} + \alpha\right]^{\frac{1}{1-\varepsilon}}\right\} + \sum_{t=0}^{\infty} \beta^t \gamma_t \left\{\Delta_t - \pi_t^{\varepsilon}\left[(1-\alpha)p_{0,t}^{-\varepsilon} + \alpha\Delta_{t-1}\right]\right\} \\
&\quad + \sum_{t=0}^{\infty} \beta^t \Omega_t \left(n_t - \Delta_t c_t\right) + \sum_{t=0}^{\infty} \beta^t \theta_t \left(w_t - \chi c_t\right).
\end{aligned}
$$

The first order conditions are as follows:

$$p_0: \quad \zeta_t - \xi_t\left[(1-\alpha)p_{0,t}^{1-\varepsilon} + \alpha\right]^{\frac{\varepsilon}{1-\varepsilon}}(1-\alpha)p_{0,t}^{-\varepsilon} + \gamma_t \varepsilon(1-\alpha)\pi_t^{\varepsilon}p_{0,t}^{-\varepsilon-1} = 0 \quad (43)$$

$$\tilde{S}: \quad \phi_t - \phi_{t-1}\alpha\pi_t^{\varepsilon} - \zeta_t\left(\frac{\varepsilon}{\varepsilon-1}\right)\pi_t \frac{1}{\tilde{F}_t} = 0 \quad (44)$$

$$\tilde{F}: \quad \psi_t - \psi_{t-1}\alpha\pi_t^{\varepsilon-1} + \zeta_t\left(\frac{\varepsilon}{\varepsilon-1}\right)\pi_t \frac{\tilde{S}_t}{\tilde{F}_t^2} = 0 \quad (45)$$

$$
\begin{aligned}
\pi: \quad &-\phi_{t-1}\varepsilon\pi_t^{\varepsilon-1}\alpha\tilde{S}_t - \psi_{t-1}(\varepsilon-1)\pi_t^{\varepsilon-2}\alpha\tilde{F}_t - \zeta_t\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\tilde{S}_t}{\tilde{F}_t} \\
&+\xi_t - \gamma_t\varepsilon\pi_t^{\varepsilon-1}\left[(1-\alpha)p_{0,t}^{-\varepsilon} + \alpha\Delta_{t-1}\right] = 0 \quad (46)
\end{aligned}
$$

$$\Delta: \quad -\Omega_t + \gamma_t - \beta\gamma_{t+1}\alpha\pi_{t+1}^{\varepsilon} = 0 \quad (47)$$

$$c: \quad \frac{1}{c_t} - \Omega_t \frac{n_t}{c_t^2} = 0 \quad (48)$$

$$n: \quad -\chi + \frac{\Omega_t}{c_t} = 0 \quad (49)$$

$$w: \quad -\theta_t + \phi_t = 0, \quad (50)$$

as well as the constraints (4), (8)−(10), (16), and (18)−(19) in the main text. Note that there is an initial condition Δ_{-1} given by history, and initial conditions $\psi_{-1} = \phi_{-1} = 0$ are implied by the fact that commitment does not extend to the past. This is a system of 15 equations, which can be reduced to nine equations as follows. First, eliminate ζ_t using (43); second, eliminate $p_{0,t}$ using (10); third, eliminate θ_t using (50); fourth, eliminate n_t using (19); fifth, eliminate w_t using (4); finally, eliminate Ω_t using (48) and (19), as

$$\Omega_t = \left(\frac{1}{\Delta_t} + \theta_t \chi c_t \Delta_t^{-1} \right).$$

The nine remaining equations are as follows, where the variables $(n_t, w_t, p_{0,t}, \theta_t, \Omega_t, \zeta_t)$ should be understood to be substituted out as described above:

$$
\begin{aligned}
\tilde{S}_t &= w_t + \alpha\beta\pi_{t+1}^{\varepsilon}\tilde{S}_{t+1} \\
\tilde{F}_t &= 1 + \alpha\beta\pi_{t+1}^{\varepsilon-1}\tilde{F}_{t+1} \\
\pi_t &= \left[(1-\alpha)\, p_{0,t}^{1-\varepsilon} + \alpha \right]^{\frac{1}{1-\varepsilon}} \\
\Delta_t &= \pi_t^{\varepsilon} \left[(1-\alpha)\, p_{0,t}^{-\varepsilon} + \alpha\Delta_{t-1} \right] \\
0 &= -\chi + \frac{\Omega_t}{c_t} \\
0 &= \phi_t - \phi_{t-1}\alpha\pi_t^{\varepsilon} - \zeta_t \left(\frac{\varepsilon}{\varepsilon-1} \right) \pi_t \frac{1}{\tilde{F}_t} \\
0 &= \psi_t - \psi_{t-1}\alpha\pi_t^{\varepsilon-1} + \zeta_t \left(\frac{\varepsilon}{\varepsilon-1} \right) \pi_t \frac{\tilde{S}_t}{\tilde{F}_t^2} \\
0 &= -\phi_{t-1}\varepsilon\pi_t^{\varepsilon-1}\alpha\tilde{S}_t - \psi_{t-1}(\varepsilon-1)\pi_t^{\varepsilon-2}\alpha\tilde{F}_t - \zeta_t \left(\frac{\varepsilon}{\varepsilon-1} \right) \frac{\tilde{S}_t}{\tilde{F}_t} \\
&\quad + \xi_t - \gamma_t\varepsilon\pi_t^{\varepsilon-1} \left[(1-\alpha)\, p_{0,t}^{-\varepsilon} + \alpha\Delta_{t-1} \right] \\
0 &= -\Omega_t + \gamma_t - \beta\gamma_{t+1}\alpha\pi_{t+1}^{\varepsilon}
\end{aligned}
$$

This is the system of nonlinear difference equations that we solve to compute the transition path in Section 6.2. It is straightforward to show that zero inflation ($\pi = 1$) solves the steady-state system of equations.

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