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The cyclical behavior of equity turnover

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We measure the extent to which the cyclical behavior of the turnover of equity shares generated by individual investors on the New York Stock Exchange can be accounted for by a single source of trade embedded in a neoclassical growth economy with dynamically complete markets. The source of trade is heterogeneity in agents' financial wealth. In the post-war United States, turnover has been more than seven times as volatile as output and has exhibited asynchronous cyclical characteristics: lagged turnover has co-varied positively with output and led turnover negatively. The baseline model, calibrated to match the mean behavior of asset returns and the distribution of wealth across households, accounts for 29% of the level of turnover observed in the data and 22% of the volatility. The asynchronous relationship observed between turnover and output is puzzling.

Keywords. Asset trade, dynamically complete markets, time- and wealth-varying risk aversion, production economies.

JEL classification. E32, G12.

1. Introduction

Substantial literatures have been devoted to theoretical and empirical characterizations of the behavior of aggregate asset prices, yet relatively little is known about the corresponding behavior of quantities traded. Indeed, as Lo and Wang (2009, p. 1) noted in their comprehensive survey of the literature devoted to trading volume, "... the intersection of supply and demand determines not only equilibrium prices but also equilibrium quantities, yet quantities have received far less attention, especially in the asset-pricing literature."

Here we seek to add to our understanding of quantities by conducting a measurement exercise. The goal is to determine the extent to which the cyclical behavior of the

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turnover of equity shares generated by individual investors on the New York Stock Exchange can be accounted for by a single source of trade embedded in a simple asset-pricing model. The model is a one-sector neoclassical growth economy with dynamically complete markets. The source of trade is heterogeneity among agents along a single dimension: their levels of financial wealth. The upshot of this heterogeneity is that productivity shocks spur disparate responses in asset demand across agent types by generating changes in the stock of capital, which gives rise to trade in financial markets. The lead question we address is the following: To what extent can the level and cyclical behavior of equity-share turnover be attributed to the simple mechanism that generates trade in our model?

Our analysis relates to four literatures. The first examines empirical characteristics of trade. Beginning with studies focussed on aggregate data, Gallant, Rossi, and Tauchen (1992) characterized distributional properties of aggregate trading volume using daily data, Hiemstra and Jones (1994) identified a causal relationship between aggregate daily returns and volume, and Jones (2002) demonstrated that turnover is useful in predicting future returns using annual data. Additional evidence regarding the predictive power of turnover is provided by Campbell, Grossman, and Wang (1993), Chordia and Swaminathan (2000), Llorente-Alverez, Michaely, Saar, and Wang (2002), and Lo and Wang (2009). Turning to studies focussed on individual investors, there is a large literature devoted to the dynamic relationship between individual investor trading and returns. This literature indicates a general tendency toward sell-offs following periods of relatively high returns and purchases following periods of relatively low returns. For evidence on these tendencies and an extensive literature review, see Kaniel, Saar, and Titman (2008). As far as we are aware, the business-cycle properties of turnover remain unexplored.

The second literature focusses on asset trade from a theoretical perspective. This work follows Judd, Kubler, and Schmedders (2003), who established the inability of Lucas' (1978) asset-pricing model to generate nontrivial asset trading under dynamically complete markets. In particular, they showed that after some initial rebalancing in short- and long-lived assets, agents choose a fixed equilibrium portfolio that is independent of the aggregate state of economy. In so doing, equilibrium stock trading (and thus turnover) goes to zero from period 1 onward. This finding established as an open question of whether the nature of extensions to Lucas' environment are necessary or sufficient for generating changes in equilibrium portfolios.

In addressing this issue, Bossaerts and Zame (2006), Espino and Hintermaier (2009), and Espino (2007) established that when the model economy features changing degrees of heterogeneity across agents, fixed-portfolio trading strategies will not be optimal in equilibrium. By studying a stationary pure exchange economy with complete markets, Judd, Kubler, and Schmedders (2003) abstracted from changing heterogeneity and showed that equilibrium trading disappears. Bossaerts and Zame (2006) overturned this result by assuming that a crucial dimension of heterogeneity changes through time. In contrast, Espino and Hintermaier (2009) and Espino (2007) extended Lucas' (1978) model by introducing neoclassical production specifications, and established theoretical conditions under which changes in heterogeneity can arise endogenously as a function of the evolution of the capital stock. Consequently, equilibrium asset trading arises in these settings despite the absence of frictions or market imperfections.

The third literature focusses on the behavior of asset prices in production economies with complete markets amenable to analysis from the perspective of a representative agent (for a detailed overview, see Lettau (2003)). Early work in this area (Danthine, Donaldson, and Mehra (1992), Rouwenhorst (1995)) showed that relative to endowment economies featuring agents with constant relative risk aversion (CRRA) preferences over consumption, the equity premium underscored by Mehra and Prescott (1985), coupled with the risk-free rate puzzle underscored by Weil (1989), becomes all the more puzzling given the incorporation of a production sector. This result arises from the ability of agents to make adjustments in the production sector, which enhances the pursuit of consumption-smoothing objectives. However, Jermann (1998) showed that the addition of capital-adjustment costs, coupled with the specification of habit formation in consumption, is sufficient to account for return behavior given the incorporation of a production sector. Boldrin, Christiano, and Fisher (2001) obtained similar results by coupling habit formation with a multisector production specification with limited intersectoral factor mobility. For overviews of the equity premium and risk-free rate puzzles, see Kocherlakota (1996) and Mehra and Prescott (2003).

The fourth literature has sought to determine whether departures from the representative-agent framework, coupled with a particular breakdown in market completeness, hold the potential to help account for asset-pricing puzzles. Early investigations into this possibility (e.g., Telmer (1993), Den Haan (1996), Heaton and Lucas (1996), Krusell and Smith (1997)) led Krusell and Smith (1997, p. 388) to conclude that "…success in explaining asset prices with this endeavor has been partial at best." More recently, Guvenen (2009) developed a simple two-agent model featuring limited stock-market participation that is relatively successful in accounting for the behavior of asset prices, but that implies counterfactual business-cycle behavior for consumption and investment. The relationship of our paper to this literature is indirect. For agent heterogeneity to carry implications for asset prices, it is necessary to incorporate some sort of departure from the complete markets assumption. Note that in adhering to market completeness in our framework, we have not attempted to make headway in the characterization of asset prices using heterogeneity as a mechanism. Instead, our aim is to introduce heterogeneity to make headway in explaining asset turnover.

Our structural characterization of equity turnover takes as a point of departure the sufficiency conditions for asset trade established by Espino (2007). In particular, working in a frictionless framework, Espino showed that when initial differences in wealth serve as the sole source of heterogeneity across agents, two conditions are sufficient for generating trade: risk aversion that varies with fluctuations in wealth, and a lack of perfect collinearity across human and financial wealth (defined as the discounted present value of wage and nonwage income). The structure we study satisfies both conditions, while remaining parsimonious and transparent in terms of the mechanism that serves to generate trade.

Specifically, the model features a single good produced via a neoclassical production specification. The good may be either consumed or invested. There are two assets: a bond that delivers one sure unit of consumption in the next period and equity shares issued by a representative firm. Thus, the model characterizes trade as the exchange of

equity and bond holdings. Production is subject to a two-state shock to total factor productivity. This shock is the only source of uncertainty in the economy; thus markets are dynamically complete. The economy is populated by agents who differ only in terms of their financial wealth. Following the seminal work of Stone (1954) and Geary (1950), the agents have CRRA-type preferences, modified to feature a minimum consumption requirement. Regarding the sufficiency conditions established by Espino (2007), nonzero capital depreciation is sufficient to eliminate collinearity between human and financial wealth, and the minimum consumption requirement is sufficient to link variations in wealth with variations in risk aversion.[1]

The model has two additional features introduced to account for the equity premium and risk-free rate puzzles: capital adjustment costs and state-dependent preferences. The former is modelled following Jermann (1998). The latter is introduced to generate a relatively volatile pricing kernel using a relatively modest specification of risk aversion. In our setting, both features are needed to account jointly for the puzzles. We define as a baseline the special case under which these additional features are shut down. This is attractive because it provides the clearest understanding of the mechanism that serves to generate trade. We then generalize the model, imposing as discipline in the parameterization stage (in part) its characterization of mean return behavior. Subject to this constraint, we evaluate its characterization of turnover.[2]

Two factors are critical for determining the behavior of turnover in this setting. First, because agents differ in their holdings of financial wealth, productivity shocks generate differential impacts on the evolution of individual wealth and, thus, on changes in asset demand. Second, agents differ in their attitudes toward risk, since consumption enjoyed by wealthier agents is relatively distant from the minimum consumption requirement. Thus the impact of productivity shocks on these attitudes toward risk, and on asset demand, also differs: from the perspective of risk, relatively wealthy agents are better able to bear the brunt of productivity shocks. The interaction of these factors determines patterns of asset trade.

As this discussion suggests, differences in financial wealth provide a crucial channel through which turnover arises in the model. We impose discipline in characterizing these differences by working with a parameterization constrained to align steady state holdings of financial wealth with empirical patterns observed in the United States (Budria-Rodriguez, Diaz-Gimenez, Quadrini, and Rios-Rull (2002) report distributions of wealth holdings across U.S. households based on the 1998 Survey of Consumer Finances).

[1]Preference specifications that link variations in financial wealth with variations in risk aversion have proven useful for characterizing asset returns; for example, see Campbell and Cochrane (1999) and Gordon and St. Amour (2004).

[2]While we follow Espino (2007) in emphasizing wealth differences as a critical source of trade, the model we examine here differs from his along two dimensions. First, we have followed the asset-pricing literature in incorporating features designed to characterize return behavior (e.g., capital adjustment costs, etc.). Second, we have extended his simple asset-market structure (which included only Arrow–Debreu securities) so as to focus on patterns of equity trade in aggregate financial markets. Thus while Espino (2007) provides a point of departure for the empirical analysis we conduct, the model we consider is a significant extension of his framework that is intended to achieve empirical coherence.

In our post-war quarterly measure, turnover has exhibited stable but volatile fluctuations around its sample average of 15.6%: logged deviations of turnover from trend have been more than seven times as volatile as those exhibited by output. Moreover, turnover exhibits a distinct asynchronous relationship with output. Contemporaneously, turnover and output are virtually uncorrelated. However, lagged turnover co-varies positively with output (e.g., at the 4-quarter horizon, the correlation between detrended turnover and output is 0.33), while led turnover co-varies negatively with output (-0.22 at the 4-quarter horizon).

The baseline model, parameterized subject to the constraint that its steady state characterization of equity returns matches the sample mean of its empirical counterpart, accounts for 29.4% of the average level of turnover observed in the data and 22.2% of the volatility. The extended model, parameterized to match both the sample means of returns to equity and debt, also accounts for 29.4% of the level of turnover and up to 21.6% of the volatility. Regarding correlation patterns, each version of the model characterizes turnover as being closely correlated with output contemporaneously, and positively correlated at both leads and lags. Thus in the context of our framework, the asynchronous relationship observed in the data represents a puzzle. Another puzzling feature of the data relative to our extended model is the volatility of returns to both risk-free debt and equity, which are predicted to be far higher that we actually observe. As Mehra and Prescott (2003) noted, this empirical shortcoming is a common general feature of models designed to characterize the average levels of these returns; thus our model is not unique in this regard (e.g., see Boldrin, Christiano, and Fisher (2001) for a frank discussion of this shortcoming in their analysis of the business-cycle implications of their asset-pricing model).

Despite these empirical shortcomings, the models we examine indicate that differential fluctuations in asset demand arising from differences in individual wealth provide a nontrivial mechanism for generating fluctuations in equity turnover. We view this mechanism as complementary to many additional sources of asset trade from which we have abstracted. A partial listing of additional sources that have been emphasized, including example references, include noise trading (DeLong, Shleifer, Summers, and Waldmann (1990)), sequential information arrival (Copeland (1976), Jennings, Starks, and Fellingham (1981)), heterogeneous beliefs (Epps and Epps (1976)), heterogeneity in risk aversion with nonexpected utility preferences (Coen-Pirani (2004)), and urgency to save (Krusell, Kuruscu, and Smith (2002)). However, a caveat regarding these additional sources bears mentioning: their potential for generating trade in models featuring dynamically complete markets may be limited. For example, Beker and Espino (2010) showed in such a setting that asset trade resulting from heterogeneous beliefs is merely transitory. Clearly, a full account of asset trade remains a topic for future research. But as efforts to build upon our understanding of turnover continue, the mechanism we have explored warrants recognition as a building block.

2. Data description

A full description of the data is provided in the Data Appendix. All series are measured on a quarterly basis and span 1950:I through 2004:II. The series consist of annualized real

returns to equity (accruing to shares in the Standard and Poor's (S&P) 500 index, measured using a 12-month investment horizon) and government debt (3-month Treasury bills); real per capita consumption, investment, and output; and the turnover of shares on the New York Stock Exchange (NYSE). Consumption is personal consumption expenditures on nondurables and services. Investment is gross private domestic investment. Output is consumption plus investment, in accordance with the structure of our model.

Turnover is defined as trade volume (the number of total shares traded) measured as a percentage of shares outstanding. Our use of turnover as a measure of trade activity follows Lo and Wang (2009, p. 7), who argued that when focussing on "... the relation between volume and equilibrium models of asset markets... turnover yields the sharpest empirical implications and is the most natural measure." As detailed in the Appendix, differences in the cyclical behavior of trading volume and turnover are trivial: aggregate shares outstanding closely adhere to a log-linear trajectory (see Figure A.1); thus their temporal deviations from trend add little to the deviations from trend exhibited by turnover (see Figure A.2).

To align our empirical characterization of turnover with its theoretical counterpart, we have sought to isolate from the aggregate behavior of turnover the proportion attributable to individual investors.[3] Unfortunately, our ability to do so is imperfect: information on the breakdown between individual and institutional investors in contributing to trade volume and share ownership is available only through occasional surveys conducted by the NYSE. Thus in the sense of Prescott (1986), our theory is ahead of measurement. Using a total of 23 data points on individual/institutional contributions to volume and share ownership, we approximated the trend behavior of the breakdown for both variables via interpolation, and adjusted aggregate turnover to account for this trend behavior. Specifically, denoting by $\mathrm{Vol}_{\mathrm{agg}}$ and $\mathrm{SO}_{\mathrm{agg}}$ aggregate volume and shares outstanding, our measure of individual turnover T_{ind} is obtained via the adjustment

$$\begin{aligned} T_{\mathrm{ind}} &= \frac{\mathrm{Vol}_{\mathrm{agg}} \cdot \%\mathrm{Vol}_{\mathrm{ind}}}{\mathrm{SO}_{\mathrm{agg}} \cdot \%\mathrm{SO}_{\mathrm{ind}}} \\ &= T_{\mathrm{agg}} \cdot \frac{\%\mathrm{Vol}_{\mathrm{ind}}}{\%\mathrm{SO}_{\mathrm{ind}}}, \end{aligned}$$

where $\%\mathrm{Vol}_{\mathrm{ind}}$ and $\%\mathrm{SO}_{\mathrm{ind}}$ are the percentages of volume generated and shares held by individual investors, and T_{agg} is aggregate turnover. Unless $\%\mathrm{Vol}_{\mathrm{ind}}$ and $\%\mathrm{SO}_{\mathrm{ind}}$ exhibit cyclical behavior in addition to their secular trends, this adjustment should provide a good approximation of individual turnover. As a final note regarding alignment, recall that turnover in the model results from the exchange of equity and bond holdings. In turn, if the two-fund separation theorem serves as a good approximation to reality, then the turnover rate of all risky assets is the same, and thus is well summarized by the turnover of the market portfolio (Lo and Wang (2009)).

The series are depicted in Figure 1 (along with National Bureau of Economic Research (NBER)-dated business-cycle peaks and troughs, indicated by vertical lines).

[3]We thank the editor and an anonymous referee for prompting this alignment: previous versions of the paper examined the behavior of aggregate turnover.

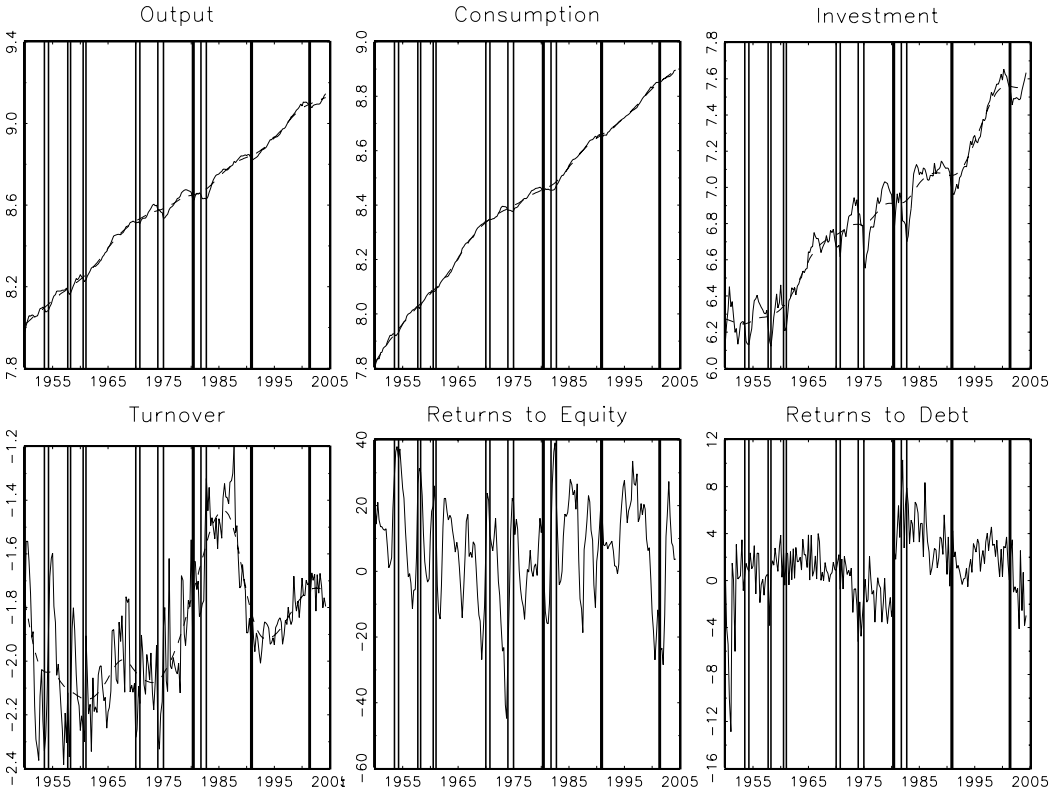

FIGURE 1. Time series observations.

Returns are represented in levels; the remaining series are represented in logs and are depicted along with their corresponding Hodrick–Prescott trends (calculated using $\lambda = 1600$). While our primary interest is in the cyclical characteristics of these series, note that returns and turnover exhibit no tendency toward long-term growth, while consumption and output exhibit roughly balanced growth.[4] Our model is designed to match these features of the data.

Returns exhibit the familiar patterns underscored by Mehra and Prescott (1985) and Weil (1989) as puzzling. The mean (standard deviation) return to equity is 7.15% (15.24%), while the mean return to debt is 1.16% (2.79%); the mean equity premium is 5.995% (15.05%) and the contemporaneous correlation of movements in these returns is 0.16.

Returns also exhibit patterns of conditional predictability that have been documented extensively (for textbook references, see Campbell, Lo, and MacKinlay (1997) and Cochrane (2001); for recent surveys, see Campbell (2000, 2002)). Focussing on serial correlation patterns, the first-order serial correlation between time-t and time-$(t+5)$ equity returns is 0.13 (the 5-quarter spread ensures that returns are nonoverlapping).

[4]While individual turnover exhibits no tendency toward long-term growth, aggregate turnover grows at the annual average rate of 3.9%. This growth is due to the behavior of institutional investors: while both $\%\text{Vol}_{\text{ind}}$ and $\%\text{SO}_{\text{ind}}$ have declined over time, $\%\text{Vol}_{\text{ind}}$ has declined relatively rapidly.

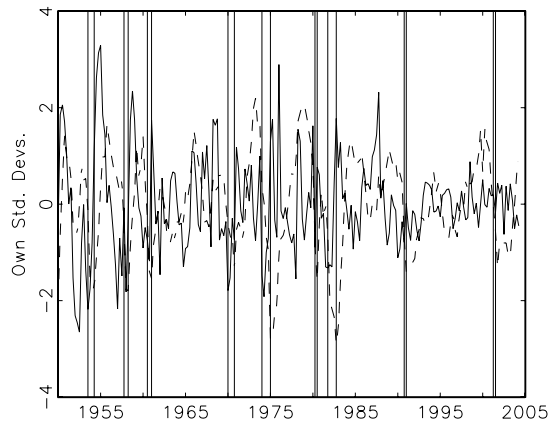

FIGURE 2. Interactions between turnover (solid line) and output for HP-filtered data.

For debt returns, the correlation between time-t and time-$(t+2)$ returns (again eliminating overlap) is 0.6 (and 0.47 between time-t and time-$(t+5)$). Comparable figures were reported, for example, by Campbell (2002, Table 1).

The sample mean of turnover is 15.6% (compared to 44% for aggregate turnover); its business-cycle characteristics are summarized in Figures 2–4. Figure 2 illustrates time-series observations of turnover and output for HP-filtered data. To aid the comparison, each series is reported in standard deviation units. As the figure illustrates, the relationship between turnover and output is systematic but unsynchronized. In particular, depending on perspective, peaks in turnover tend to precede peaks in output; alternatively, peaks in output tend to precede troughs in turnover.

Regarding standard deviations, turnover is highly volatile relative to output: its standard deviation is 7.9 times that of output in the HP-filtered series. The comparable figure for investment is 4.2, the volatility of returns is as reported above, and the volatility of consumption is approximately half that of output.

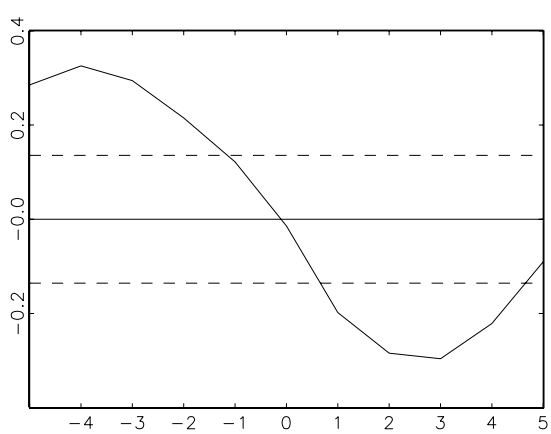

FIGURE 3. Correlations between output$_t$ and turnover$_{t-j}$ (– – – indicates 95% confidence bands).

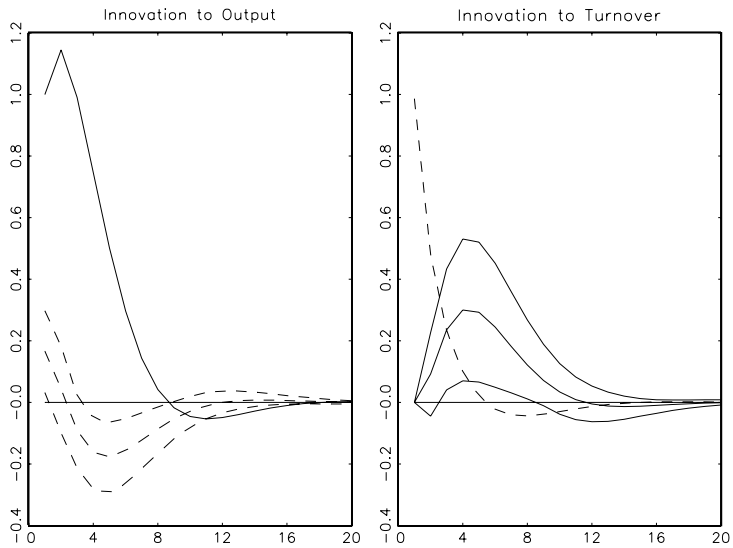

FIGURE 4. Impulse response functions, turnover (dashed lines) and output (solid lines).

Figure 3 provides a graphical characterization of the systematic but asynchronous relationship noted above by illustrating cross-correlations between output and turnover for up to five leads and lags. Note that turnover is positively correlated with output when lagged (0.33 at the four-quarter horizon), uncorrelated contemporaneously, and negatively correlated when led (−0.20 at the 4-quarter horizon).

Further information regarding the asynchronous relationship observed between turnover and output is provided in Figure 4, which illustrates impulse response functions calculated using a variable autoregression (VAR) specified for HP-filtered turnover and output. The responses were constructed using a Cholesky decomposition of the associated innovation variance–covariance matrix for the case in which output was ordered first (reversing the ordering yields similar response patterns). The left-hand panel illustrates responses of both output and turnover to output innovations; the right-hand panel shows responses to turnover innovations only. Responses are reported in own innovation standard deviation units. 95% confidence intervals are reported for responses of variable j to innovations to variable i (intervals for own responses are suppressed to reduce clutter).

Given a positive innovation to output, turnover lies above trend in the initial period (i.e., innovations to output and turnover are positively correlated). Turnover then responds negatively over the next four quarters, bottoming out at roughly −20% of its own innovation standard deviation. It then overshoots its steady state in climbing between the 4- and 12-quarter horizons, and follows dampening oscillations around its steady state thereafter. In turn, given a positive innovation to turnover, output responds by climbing steadily over the following four quarters, peaking at roughly 30%. It then overshoots its steady state in falling between the 4- and 12-quarter horizons, and follows dampening oscillations around its steady state thereafter. Note that peaks in both responses are significantly different from zero.

Impulse responses between turnover and the remaining series (not depicted) appear as follows. Response patterns observed between turnover and output are qualitatively similar to those obtained by replacing output with consumption and investment. Given the strong procyclicality of consumption and investment, this similarity is unsurprising. In particular, turnover falls over a 4–5-quarter horizon in response to innovations to each series, while each series climbs over a 4–7-quarter horizon in response to an innovation in turnover. The response of turnover is most distinct given an innovation to consumption (-20% at the 4-quarter horizon, compared with roughly -15% for investment). In turn, the response of investment is most distinct given an innovation to turnover (peaking at roughly 38%, compared with roughly 20% for consumption).

For the relationship between turnover and returns to equity, the general pattern illustrated above is roughly reversed. In this case, given an innovation to returns, turnover is initially below trend (with a response of nearly -20%) and then rises over the next four quarters, peaking at roughly 30%. In turn, returns fall over a 4-quarter horizon following an innovation to turnover, bottoming out at roughly -30%. Finally, the relationship between turnover and returns to debt is weak. In particular, cross-responses of each variable are weak, lying in roughly a $\pm 15\%$ band. Regarding the systematic relationship noted between turnover and returns to equity, this reflects the stylized fact that turnover is useful in predicting future returns (e.g., Hiemstra and Jones (1994), Jones (2002)). Previous explanations for this phenomenon include asymmetries in investors' information sets (e.g., Copeland (1976)), heterogeneity in beliefs (e.g., Epps and Epps (1976)), and noise trading (DeLong et al. (1990)).

In sum, the behavior of turnover has several notable aspects. (i) Its level is stable over time, with a sample average of 15.6%. (ii) Measured as logged deviations from trend, the volatility of turnover is roughly 7.9 times that of output. (iii) Lagged observations of turnover co-vary positively with output; lead observations co-vary negatively. (iv) Positive innovations to output correspond with negative responses in turnover over roughly a 1-year horizon; positive innovations to turnover correspond with positive responses in output over roughly a 1-year horizon. (v) Of the components of output, innovations to consumption correspond with relatively strong responses in turnover and innovations to turnover correspond with relatively strong responses in investment. (vi) The relationship between turnover and returns to equity is roughly reverse that observed between turnover and output, while the relationship between turnover and returns to debt is nondistinct.

3. The economy

The economy is populated by H (types of) infinitely lived agents, where $h \in \mathcal{H} = \{1, \dots, H\}$. Time is discrete and denoted by $t = 0, 1, 2, \dots$. Agents are endowed with one unit of time per period, which they supply inelastically in the production sector; aggregate labor supply is thus H. Production yields a single good that can either be consumed or invested to produce new capital.

There is aggregate uncertainty in the form of shocks to total factor productivity (TFP), denoted as s_t; $\{s_t\}$ follows a first-order stationary Markov process with transition probabilities $\pi(s_t, s_{t+1}) > 0$, where $s_t \in S_t = \{\underline{s}, \overline{s}\}$ for all t. Let $s^t = (s_0, \dots, s_t) \in S^t =$

$\times_{k=0}^{t} S_k$ represent the partial history of aggregate shocks realized through date t and let $X(s^t)$ denote the value of X chosen at node s^t. These histories are observed by all agents.

There is an aggregate production technology that takes as inputs the capital good K and the labor input H. This technology features labor-improving technological progress with growth rate g. Aggregate output is given by

$$Y_t = F(s_t, K, (1+g)^t H). \tag{1}$$

For all s, $F(s, \cdot, \cdot)$ is homogeneous of degree 1 (HD1), strictly increasing, strictly concave, and satisfies Inada conditions. In our empirical implementation, F will be specified as Cobb–Douglas.

Let $\Psi(K, I)$ be the total cost of investment I; $\Psi(\cdot, \cdot)$ is HD1, convex, and increasing in I. Following Jermann (1998), we assume

$$\frac{\Psi(K, I)}{K} = \left(\frac{b_0}{1-\kappa}\left(\frac{I}{K}\right)^{1-\kappa} + b_1\right), \tag{2}$$

with $b_i \geq 0$ and $\kappa \geq 0$. Note that for $(b_0, b_1, \kappa) = (1, 0, 0)$, $\Psi(K, I) = I$.

The law of motion of capital at s^t is given by

$$K(s^t) = (1-\delta)K_j(s^{t-1}) + I(s^t), \tag{3}$$

where $\delta \in (0, 1)$ is the depreciation rate. $K^0 > 0$ is the initial capital stock.

A consumption bundle for agent h is a sequence of functions $\{c_t\}_{t=0}^{\infty}$ such that $c_t : S^t \to [(1+g)^t\gamma, +\infty)$ for all t and $\sup_{(t,s^t)} c(s^t) < \infty$. Agent h's consumption set, C_h, is the set of all consumption plans. In turn, agent h's preferences are represented by expected, state-dependent , time-separable, discounted utility, where for $C \in C_h$,

$$U_h(C) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t) \xi(s_t) \frac{(C_h(s^t) - (1+g)^t\gamma)^{1-\sigma}}{1-\sigma}, \tag{4}$$

where $\beta \in (0, 1)$, $\sigma > 0$, and $\gamma(1+g)^t$ is a minimum consumption requirement. The growth component $(1+g)^t$ is included so that the model is consistent with balanced growth (see, for instance, Alvarez-Pelaeza and Diaz (2005)). The requirement induces attitudes toward risk that vary with wealth, a mechanism that provides a critical source of equity turnover in this environment (see Section 4.2). Empirical support for the variation of attitudes toward risk as a function of wealth is provided by, for example, Atkeson and Ogaki (1996).

The inclusion of $\xi(s^t)$ in (4) renders preferences as state-dependent; it is specified as

$$\xi(s_t) = (s_t)^{-\mu}, \tag{5}$$

where $\mu \geq 0$. Note that for $\mu = 0$, $\xi(s_t) = 1$ for all t, but for $\mu > 0$, $\xi(s_t)$ intensifies the volatility of the pricing kernel for a given specification of risk aversion. Coupled with the capital-adjustment-cost specification (2), the inclusion of $\xi(s_t)$ suffices to account

for the equity premium and risk-free rate puzzles in this setting. As noted, we consider two cases below: a baseline under which the capital-adjustment cost and the state-dependence of preferences are shut down, and a generalized calibration disciplined by the return puzzles. We emphasize that our purpose in evaluating the generalized model is not to provide an account of the return puzzles, but to evaluate the behavior of turnover subject to the empirical discipline imposed by the puzzles.

Since the technologies $\{F, \Psi\}$ are both HD1 and preferences are represented by (4), this framework enables growth detrending. Hereafter we normalize to eliminate trend components: the detrended component of any $X(s^t)$ at s^t is denoted as $x(s^t) = X(s^t)/(1+g)^t$.

Written in terms of detrended variables, (4) can be expressed as

$$U_h(c) = \sum_t^{\infty} \sum_{s^t} \rho^t \pi(s^t) \xi(s_t) \frac{(c_h(s^t) - \gamma)^{1-\sigma}}{1-\sigma}, \tag{6}$$

where $\rho = \beta(1+g)^{1-\sigma} \in (0,1)$. Corresponding feasibility constraints are given by

$$\begin{aligned} &c(s^t) + \Psi(k(s^{t-1}), i(s^t)) = F(s_t, k(s^{t-1}), H), \\ &k(s^t)(1+g) = (1-\delta)k(s^{t-1}) + i(s^t), \\ &s_0 \text{ and } k_0 = K^0 \text{ are given.} \end{aligned}$$

3.1 *Competitive equilibrium*

Every period, agents meet to trade the consumption good and two assets. There is a risk-free bond held in zero net supply that pays 1 unit of consumption next period. Let a'_h denote the holdings of this asset chosen by agent h; the initial endowment $a_h(s_{-1}) = 0$ for all h. Agents can also trade equity shares. Let θ'_h denote the number of shares chosen by agent h, where $\theta' = \sum_h \theta'_h$ is the total number of shares outstanding issued at the current period by the representative firm. Agents are endowed with θ^0_h shares at time 0, where $\sum_h \theta^0_h = 1$.

The Markovian structure of this economy allows us to study *recursive competitive equilibria* (RCE) directly. Consider the set of state variables. At the consumer level, the state is described by individual financial wealth, denoted by ϕ_h and defined below. At the firm level, the state is described by the firm's stock of capital $\varkappa$. Finally, let Φ, k, and θ describe the distribution of financial wealth, aggregate capital, and outstanding shares. At the aggregate level, the state is (s, θ, Φ, k), where $\Phi' = J(s, \theta, \Phi, k)$ and $k' = Z(s, \theta, \Phi, k)$ denote laws of motion for the distribution of financial wealth and aggregate capital, respectively. Additionally, all agents (including the firm) take the law of motion of outstanding shares $\theta' = G(s, \theta, \Phi, k)$ as given. Note that the aggregate shock ξ is uniquely determined by s through (5). The price system is given by $(p, q^{\mathrm{RF}}, w) : S \times \mathbb{R}_+ \times \mathbb{R}^I \times \mathbb{R}^J_+ \to \mathbb{R}_{++}$, representing the ex-dividend price of equity, the price of the risk-free bond, and wages, respectively.

Observe that markets are dynamically complete with this asset-market structure, since $s' \in \{\underline{s}, \overline{s}\}$. By no arbitrage, this implies that at each (s, Φ, K) there is a unique *state price vector*, denoted by $(q(s, \Phi, K)(s'))_{s' \in \{\underline{s}, \overline{s}\}}$.

Firm's production plans and financial policy Production decisions are made by a representative firm that maximizes its value

$$\begin{aligned} &v_F(\varkappa, s, \theta, \Phi, k) \\ &\quad = \max_{(\varkappa', i, l)} \left\{ d(\varkappa, s, \Phi, k) + \sum_{s'} q(s, \Phi, k)(s') v_F(\varkappa', s', \theta', \Phi', k') \right\}, \\ &\quad d(\varkappa, s, \Phi, k) = F(s, \varkappa, l) - H(\varkappa, i) - w(s, \Phi, k) l, \\ &\quad \varkappa'(1+g) = (1-\delta)\varkappa + i, \\ &\quad l_h \geq 0 \quad \forall h. \end{aligned} \tag{7}$$

The Modigliani–Miller theorem states that the firm's financial policy does not affect equilibrium real allocations, hence it is typically assumed that there is one share outstanding every period. However, changes in shares outstanding do affect the amount of assets traded in equilibrium and, thus, the extent of equity trade. Thus, for our purposes it is important to incorporate the specification of a financial policy that is empirically relevant, in that it admits growth in shares outstanding, and makes explicit how dividends per share are determined in light of this growth. Regarding the growth of shares outstanding, this is observed to be remarkably stable over time, as illustrated in Figure A.1. Therefore, we assume the growth of shares obeys

$$\theta' = G(s, \theta, \Phi, k) = (1 + g_s)\theta \quad \forall (s, \theta, \Phi, k), \tag{8}$$

with the growth rate of issued shares g_s representing an important parameter to be calibrated to match its empirical counterpart. Then, given that $(\theta' - \theta)$ new shares are issued in the current period, dividends per share d_f are defined according to the firm's budget constraint:

$$\theta d_f(\varkappa, s, \theta, \Phi, k) = d(\varkappa, s, \theta, \Phi, k) + p(s, \theta, \Phi, k)(\theta' - \theta) \tag{9}$$

such that $\theta' = G(s, \theta, \Phi, k)$. Thus an agent holding θ_h shares will receive the dividend payment $\theta_h d_f(\varkappa, s, \Phi, k)$ at the beginning of the period. Any (θ', θ, d_f) satisfying (8) and (9) is a financial policy.

Given a price system $(p, q^{\mathrm{RF}}, w_h)$ and laws of motion (G, J, H), agent h's problem is

$$\begin{aligned} &v_h(\phi_h, s, \theta, \Phi, k) \\ &\quad = \sup_{(c_h, a'_h, \theta'_h)} \left\{ \xi(s) \frac{(c_h - \gamma)^{1-\sigma}}{1-\sigma} + \rho \sum_{s'} \pi(s, s') v_h(\phi'_h, s', \theta', \Phi', k') \right\} \end{aligned}$$

subject to

$$\begin{aligned} &c_h + p(s, \theta, \Phi, k)\theta'_h + q^{\mathrm{RF}}(s, \theta, \Phi, k) a'_h = \phi_h + w_h(s, \theta, \Phi, k), \\ &\quad \phi'_h = [p(s', \theta', \Phi', k') + d(s', \theta', \Phi', k')]\theta'_h + a'_h, \\ &\quad c_h \geq \gamma, \end{aligned}$$

where $\theta' = G(s, \theta, \Phi, k)$, $\Phi' = J(s, \theta, \Phi, k)$, and $k' = Z(s, \theta, \Phi, k)$.

Definition. A RCE is a set of value functions for the individuals $(v_h)_{h\in\mathcal{H}}$, a value function for the firm v_F, a set of policy functions for the individuals $(c_h, a'_h, \theta'_h)_{h\in\mathcal{H}}$, policy functions for the firm $(\varkappa', i, l)$, a financial policy (θ', θ, d_f), a price system (p, q^{RF}, w), a corresponding set of state prices q, and laws of motion for the aggregate state variables $\Phi' = J(s, \theta, \Phi, k)$, $k' = Z(s, \theta, \Phi, k)$, and $\theta' = G(s, \theta, \Phi, k)$ such that the following statements hold:

RCE 1. Given the price system (p, q^{RF}, w) and the aggregate laws of motion $\Phi' = J(s, \theta, \Phi, k)$, $k' = Z(s, \theta, \Phi, k)$, and $\theta' = G(s, \theta, \Phi, k)$, $(v_h, c_h, a'_h, \theta'_h)$ solve (RAP) for each h.

RCE 2. Given the price system (p, q^{RF}, w), its corresponding state prices q and the aggregate laws of motion $\Phi' = J(s, \theta, \Phi, k)$, $k' = Z(s, \theta, \Phi, k)$, and $\theta' = G(s, \theta, \Phi, k)$, then $(v_F, \varkappa', i, l)$ solves (7) and (θ', θ, d_f) is the corresponding financial policy.

RCE 3. All markets clear:

$$\sum_{h\in\mathcal{H}} c_h(\phi_h, s, \theta, \Phi, k) + \Psi(k, i(\varkappa, s, \Phi, K)) = F(s, k, L(s, k, A, \Theta, K)),$$

$$\sum_{h\in\mathcal{H}} a'_h(\phi_h, s, \theta, \Phi, k) = 0,$$

$$\sum_{h\in\mathcal{H}} \theta'_h(\phi_h, s, \theta, \Phi, k) = \theta'.$$

RCE 4. Consistency. For all (s, θ, Φ, k) and each h,

$$\begin{aligned}\Phi'_h &= J_h(s, \theta, \Phi, k)\\ &= a'_h(\Phi_h, s, \theta, \Phi, k) + [p(s', \theta', \Phi', k') + d_f(\varkappa', s', \Phi', k')]\theta'_h(\Phi_h, s, \theta, \Phi, k),\\ k' &= Z(s, \theta, \Phi, k) = \varkappa'(k, s, \Phi, k).\end{aligned}$$

For notational convenience, the consistency condition (that aggregate levels be consistent with individual behavior) is imposed to avoid the need to express optimal decision rules as functions of individual state variables. Also, since financial policies affect neither equilibrium allocations nor equilibrium state prices $(q(s'))$, these will not include θ as an argument. For instance, we directly write $c_h(s, \Phi, k)$.

As noted, given the near-constant-growth trajectory for shares outstanding observed in the data, as illustrated in Figure A.1, we assume that the shares outstanding grow at the constant rate g_s. The following equilibrium property allows us to simplify the analysis.

Proposition 1. *Suppose* $((c_h, a'_h, \theta'_h)_{h\in\mathcal{H}}, (k', i, l), (p, q^{\mathrm{RF}}, w))$ *constitute a RCE and G obeys* (8). *Consider an alternative financial policy where* $\widehat{\theta}' = \widehat{\theta} = 1$. *Then* $((c_h, a'_h, \widehat{\theta}'_h)_{h\in\mathcal{H}}, (k', i, l), (\widehat{p}, q^{\mathrm{RF}}, w))$ *constitute a RCE, where*

$$\widehat{\theta}'_h(s, 1, \Phi, k) = \frac{\theta'_h(s, \theta, \Phi, k)}{(1+g_s)\theta},$$

$$\widehat{p}(s, 1, \Phi, k) = \theta(1+g_s)p(s, \theta, \Phi, k).$$

See the Technical Appendix for the proof.

Measuring trading volume: The turnover rate To see why Proposition 1 simplifies the analysis, recall that we follow Lo and Wang (2009) in quantifying stock trading volume as the turnover rate. This is defined as

$$\tau(s,\theta,\Phi,k)=\frac{1}{2}\frac{\sum_h|\theta'_h(s,\theta,\Phi,k)-\theta_h|}{\theta'},\tag{10}$$

where $\sum_h \theta'_h=\theta'$ in equilibrium. Proposition 1 implies that we can compute (10) using

$$\tau(s,\Phi,k)=\frac{1}{2}\sum_h|\theta'_h(s,1,\Phi,k)-\theta_h/(1+g_s)|,\tag{11}$$

where $\sum_h \theta_h=1$. That is, we can solve the model for the economy defined with $\theta=1$ and then compute turnover using (11) given g_s.

3.2 *Equilibrium portfolios*

To calculate the turnover rate, we must compute equilibrium portfolios explicitly. To do so, we follow an indirect strategy and implement a recursive version of Negishi's (1960) computational approach. Details are provided in the Technical Appendix.

Let $\alpha\in\mathbb{R}^I_+$ denote the vector of welfare weight assigned to the agents that parameterize some Pareto optimal (PO) allocation. Under our assumptions, the policy functions regarding agregate consumption, next period capital, and investment are independent of this distributional parameter α (i.e., $c(s,k)$, $k'(s,k)$, $i(s,k)$), while individual consumption is allocated according to

$$\begin{aligned}&(c_h(s,k,\alpha)-\gamma)=\omega_h(c(s,k)-\gamma_A),\\&\omega_h=\frac{(\alpha_h)^{1/\sigma}}{\sum_j(\alpha_j)^{1/\sigma}}\quad\forall h.\end{aligned}\tag{12}$$

It can be shown that there exists a unique welfare weight α^0 such that the corresponding PO allocation, denoted $((c_h(s,k))_{h\in\mathcal{H}},k'(s,k),i(s,k))$, can be decentralized as a RCE. Furthermore, given (s,k), the distribution of financial wealth that supports a RCE can be uniquely determined by some function $\Phi(s,k)$. This implies that the state space reduces to (s,k). For each agent h, his individual financial wealth, ϕ_h, is the unique solution to the functional equation

$$\phi_h(s,k)=c_h(s,k)-w(s,k)+\sum_{s'}q(s,k)(s')\phi_h(s',k'(s,k)).$$

Given the individual levels of financial wealth, $(\phi_h(s,k))_{h\in\mathcal{H}}$, the corresponding equilibrium portfolios are constructed as follows. For each k, let $[(a_h(k),\theta_h(k)]$ solve the 2×2 system

$$\phi_h(s,k)=a_h(k)+[p(s,k)+d(s,k)]\theta_h(k)\quad\text{for all } s\in\{\underline{s},\overline{s}\}.\tag{13}$$

This system (generically) has a unique solution for each h. Equilibrium portfolios for each h are determined by

$$\begin{aligned} a'_h(s,\Phi,k) &= a_h(k'(s,k)), \\ \theta'_h(s,\Phi,k) &= \theta_h(k'(s,k)), \end{aligned} \tag{14}$$

where $\Phi = \Phi(s,k) = [\phi_1(s,k),\dots,\phi_H(s,k)]$ represents the aggregate distribution of wealth uniquely determined by (s,k). An important feature of this framework is that the financial endogenous state variable Φ is determined uniquely by (13). This implies that equilibrium portfolios depend on (s,k) only through $k'(s,k)$, the law of motion of the endogenous physical state variable.

3.3 *Intuition regarding turnover*

We conclude this section by providing intuition regarding the behavior of turnover, particularly in response to innovations in s. To do so, we need to introduce some notation further discussed in the Technical Appendix.

Let $v_M(s,k)$ denote the value of the minimum consumption requirement γ which uniquely solves

$$v_M(s,k) = \gamma + \sum_{s'} q(s,k)(s')v_M(s',k'(s,k)).$$

Similarly, define $v_W(s,k)$ as the individual human wealth which uniquely solves

$$v_W(s,k) = w(s,k) + \sum_{s'} q(s,k)(s')v_W(s',k'(s,k)).$$

Finally, let $v_F(s,k) = p(s,k) + d(s,k)$ denote the value of the representative firm which satisfies

$$v_F(s,k) = \sum_{h\in\mathcal{H}} \phi_h(s,k);$$

that is, $v_F(s,k)$ is interpreted as the aggregate value of financial wealth. The welfare weight α^0 parameterizes the PO allocation that decentralizes as a RCE and is normalized such that $\sum_{h\in\mathcal{H}}(\alpha_h^0)^{1/\sigma} = 1$.

Note that (13) and (14) can be used to express agent h's demand for equity holdings, which is given by

$$\theta'_h(s,k) = \theta_h(k'(s,k)) = \frac{\phi_h(\overline{s},(k'(s,k)) - \phi_h(\underline{s},(k'(s,k))}{v_F(\overline{s},(k'(s,k)) - v_F(\underline{s},(k'(s,k))}. \tag{15}$$

Thus the impact of an innovation in s on equity demand is given by the ratio of this impact on the dispersion of agent h's financial wealth next period (the numerator) relative to the impact on the dispersion of aggregate financial wealth next period (the denominator).

In general, the impact on this ratio can be nonmonotonic in wealth. But under the calibrated structure we consider, the wealthier the agent is, the larger is the impact. Specifically, it follows from (30) in the Technical Appendix that

$$\frac{\phi_h(\overline{s}, k'(s,k)) - \phi_h(\underline{s}, k'(s,k))}{v_F(\overline{s}, k'(s,k)) - v_F(\underline{s}, k'(s,k))} = (\alpha_h^0)^{1/\sigma} + H\left((\alpha_h^0)^{1/\sigma} - \frac{1}{H}\right)R(k'(s,k)), \quad (16)$$

where

$$\begin{aligned} &R(k'(s,k)) \\ &\quad = \frac{[v_W(\overline{s}, k'(s,k)) - v_M(\overline{s}, k'(s,k))] - [v_W(\underline{s}, k'(s,k)) - v_M(\underline{s}, k'(s,k))]}{v_F(\overline{s}, k'(s,k)) - v_F(\underline{s}, k'(s,k))}. \end{aligned} \quad (17)$$

Thus for a given response of $R(k'(s,k))$ to an innovation in s, the response of equity demand is larger the larger is the welfare weight $(\alpha_h^0)^{1/\sigma}$ and thus the larger is wealth.

Regarding $R(k'(s,k))$, this is positive and increasing in s. To see why $R(k'(s,k))$ is positive, observe for the numerator that since the difference between wages and the minimum consumption requirement is increasing in s, so too is the difference in their values; this difference is interpretable as disposable human wealth. For the denominator, dividends are increasing in s; thus so too is the value of the firm.

To see why $R(k'(s,k))$ is increasing in s, note that k' is increasing in s due to standard consumption-smoothing arguments. Furthermore, the difference between wages and the minimum consumption requirement is increasing in k' due to the technological complementarity between capital and labor; thus so too is the difference in their values. So the numerator is increasing in k'. The denominator is also increasing in k' since dividends are increasing in k', but not by as much as the numerator. This appears to be the case because the firm increases investment when k' increases, thus smoothing its value. As a result, $R(k'(s,k))$ is both positive and increasing in k', and thus also in s.

Given this behavior for $R(k'(s,k))$, the equity demand of relatively wealthy agents (those with welfare weights $(\alpha_h^0)^{1/\sigma} > \frac{1}{H}$) is increasing in s. Through a similar derivation, the demand for risk-free bonds can be shown to be decreasing in s for the same agents. So following a positive productivity shock, equity flows from poor to rich agents, while bonds flow from rich to poor agents.

Following Espino (2007), note two factors that are critical in determining the responsiveness of $R(k'(s,k))$, and thus ultimately turnover, to innovations in s. The first is the correlation between v_W and v_F induced by innovations in s: the closer the correspondence, the less responsive will be turnover. Indeed, absent the minimum consumption requirement, if v_W and v_F were perfectly correlated, turnover would be zero. All else equal, the greater is the wedge between v_W and v_F, the greater is the volatility of turnover.

The second factor is the presence of the minimum consumption requirement: for a given correspondence between v_W and v_F, a nonzero minimum consumption requirement amplifies the response of $R(k'(s,k))$ to an innovation in s. To see why, note from

the specification of instantaneous utility that agent h's measure of relative risk aversion is given by

$$-\frac{c_h(s,k,)u''(c_h(s,k))}{u'(c_h(s,k))} = \sigma\left(1 - \frac{\gamma}{c_h(s,k)}\right)^{-1}. \tag{18}$$

Absent the minimum consumption requirement, (relative) risk aversion is equal across agent types, but the positive requirement renders relative risk aversion as wealth dependent: in particular, poorer agents are relatively risk averse. Moreover, for a given innovation in s, the subsequent response of risk aversion will be greater the poorer is the agent. Differential responses of risk aversion to innovations (i.e., differences in $(\alpha_h^0)^{1/\sigma}$) translate into differential portfolio rebalancing responses (see equation (17)). Thus the minimum consumption requirement along with σ both serve as potential sources of amplification in this environment.

So with relatively poor agents featuring a relatively strong consumption-smoothing incentive that, moreover, is particularly responsive to innovations in s, why then is equity demand increasing in s for rich agents and decreasing for poor agents? The reason is the dominance of a substitution over an income effect for poor agents. Regarding the latter, a positive innovation to s enriches all agents, thus decreasing their risk aversion and increasing their demand for equity. As noted, this effect is more intense the poorer is the agent. However, there is also a substitution effect: the decrease in risk aversion drives up the price of equity relative to debt. For rich agents, the income effect dominates; thus their demand for equity is increasing in s. For poor agents, the substitution effect dominates, thus their demand for equity is decreasing in s. (Recall that within a period, the supply of equity shares is fixed.) Again, rich agents are defined as having welfare weights $(\alpha_h^0)^{1/\sigma} > \frac{1}{H}$, as seen in (16).

We conclude this discussion with a note regarding the mapping of equity demand to turnover. Substituting for θ_h in (21) using (15)–(17), turnover may be expressed as

$$\begin{aligned}\tau(s,k) &= \frac{1}{2}\sum_h \left|\theta_h(k'(s,k)) - \theta_h(k)\right| \\ &= \frac{H}{2}\left|R(k'(s,k)) - R(k)\right| \sum_h \left|(\alpha_h^0)^{1/\sigma} - \frac{1}{H}\right|.\end{aligned} \tag{19}$$

The term $|R(k'(s,k)) - R(k)|$ is dependent solely on the structural specification of the model. Given an innovation s, the larger is the response in R to movements in aggregate capital, the larger is the technological component of turnover. In contrast, the term $\sum_h |(\alpha_h^0)^{1/\sigma} - 1/H|$ is purely distributional and independent of the aggregate state (s,k). This component reflects the impact of wealth dispersion on risk aversion and, thus, on turnover. Thus, through this mechanism, all else equal, a given innovation in s will have an amplified impact on turnover the greater is the wedge between degrees of risk aversion observed across agent types, parameterized by the dispersion of $((\alpha_h^0)^{1/\sigma})_{h\in\mathcal{H}}$.

3.4 *Quantifying asset returns and turnover*

To facilitate quantitative analysis, returns and turnover are defined to correspond with their empirical counterparts. For returns, three considerations affect alignment: a period corresponds to a quarter; returns are annualized; and while model variables are in detrended form, returns are calculated using trending data. Thus for each (s, k) and s', let

$$Q(s, K)(s') = \frac{1}{(1+g)} q(s, k)(s')$$

and define the price of the risk-free bond as

$$Q^{\mathrm{rf}}(s, K) = \sum_{s'} Q(s, K)(s').$$

Then the annualized risk-free rate is given by

$$R^{\mathrm{rf}}(s, K) = 400 \cdot \left(\frac{1}{Q^{\mathrm{rf}}(s, K)} - 1 \right)$$

and annualized equity returns are given by

$$\begin{aligned} R^{e}(s, K) &= 100 \cdot \ln \left(\frac{P(s, K)_{t+4} + \sum_{q=1}^{4} D(s, K)_{t+q}}{P(s, K)_t} \right) \\ &= 100 \cdot \ln \left(\frac{(1+g)^{t+4} p(s, k)_{t+4} + \sum_{q=1}^{4} (1+g)^{q} d(s, k)_{t+q}}{p(s, k)_t} \right). \end{aligned}$$

Regarding turnover, let α^0 be given by (32) and define

$$\phi_h(k, \alpha^0) = \begin{bmatrix} \phi_h(\underline{s}, k, \alpha^0) \\ \phi_h(\overline{s}, k, \alpha^0) \end{bmatrix},$$

where $\phi_h(s, k, \alpha)$ is given by (31). Likewise, let

$$p(k) = \begin{bmatrix} p(\underline{s}, k) \\ p(\overline{s}, k) \end{bmatrix}, \qquad d(k) = \begin{bmatrix} d(\underline{s}, k) \\ d(\overline{s}, k) \end{bmatrix},$$

where $d(s, k)$ and $p(s, k)$ are as defined in (28) and (29).

Then equilibrium portfolios can be constructed using

$$\begin{bmatrix} a_h(k'(s, k)) \\ \theta_h(k'(s, k)) \end{bmatrix} = [\, \mathbf{1} \quad (\mathbf{p} + \mathbf{d}) k'(s, k) \,]^{-1} \phi_h(k'(s, k)), \tag{20}$$

where $[\mathbf{1}(\mathbf{p}+\mathbf{d})k'(s,k)]^{-1}$ is the 2×2 matrix evaluated at $k'(s,k)$. In turn, given Proposition 1, turnover is given by

$$\tau(s,k)=\frac{1}{2}\sum_h\big|\theta_h(k'(s,k))-\theta_h(k)/(1+g_s)\big|. \tag{21}$$

4. Empirical implementation

4.1 *Calibration*

We specify five individual types ($I=5$) and a two-state Markov process for s parameterized to mimic the first-order autoregressive representation $s_t=(1-\lambda)+\lambda s_{t-1}+\varepsilon_t$. In turn, λ and the standard deviation of ε_t (σ_ε) were chosen so that the parameterized model matched the observed first-order serial correlation and standard deviation of output. Given values chosen for the additional parameters, the corresponding specification of $(\lambda,\sigma_\varepsilon)$ turned out to be $(0.7743, 0.00929)$ for the baseline model and $(0.6715, 0.0091)$ for the extended model. The difference in λ indicates that the extended model has a relatively strong internal propagation mechanism. Table 1 presents parameterizations for both models.

For both models, capital's share α was set at 0.33, the discount factor β was set at 0.99 (implying an annualized discount rate of approximately 4%), the depreciation rate δ was set at 0.025 (implying an annual depreciation rate of 10%), g was set at 0.00475 (matching the observed 1.9% annualized growth rate of output), and g_s was set at 0.0235 (matching the observed 9.4% annualized growth rate of shares outstanding). We take these specifications as standard and do not present results obtained using alternative specifications along these dimensions.

The welfare weights α_h were chosen so that the corresponding steady state distribution of financial wealth across individual types matched the distribution of wealth holdings across U.S. households reported by Budria-Rodriguez et al. (2002). Specifically, their Table 7 reports shares of total wealth across household quintiles constructed using the 1998 Survey of Consumer Finances. Note from our Table 1 that the match is close but not perfect: it reflects the minimized sum of squared differences across quintiles obtained using a numerical optimization routine. Reported as a fraction of the weight assigned to upper-quintile types, the fitted welfare weights we employ are $\alpha_h=(0.2, 0.22, 0.24, 0.29, 1.00)$, $h=1,\dots,5$. The steady states in both models are aligned with each other to aid in cross-model comparisons, thus so too are fitted welfare weights.

The remaining parameters to be assigned for the baseline model are the minimum consumption value γ and the curvature parameter σ specified for the instantaneous utility function. (The additional parameters associated with the extended model are discussed below.) As a benchmark, we set γ to 5% of the steady state level of consumption, and experiment with alternative specifications in the range of 0–8%. The latter value is an upper bound imposed by the condition (implicit in (32) of Proposition 2) that the combined value of human and financial wealth must exceed the value of the minimum consumption requirement for all individual types. Finally, σ was calibrated so that the steady state return to equity implied by the model matched the sample average observed

TABLE 1. Model parameterizations.[a]

	α	β	δ	σ	g	ρ	λ	σ_ε	γ	g_s
Baseline	0.33	0.99	0.025	1.732	0.00475	0.987	0.7743	0.00929	$0.05c^*$	0.0235
Extended	0.33	0.99	0.025	1.732	0.00475	0.987	0.6715	0.00910	$0.05c^*$	0.0235

	b_0	b_1	κ	μ
Extended	2.31E−07	0.03864	4.3478	25.17

Non-Human Wealth Shares

	Quintile				
	1	2	3	4	5
Targeted	−0.30%	1.30%	5.00%	12.20%	81.70%
Fitted	−0.83%	1.85%	5.03%	12.23%	81.73%

Distributional Characteristics of Consumption

	Quintile (i)				
	1	2	3	4	5
α_i/α_5	0.20	0.22	0.24	0.29	1.00
w_i	0.14	0.15	0.16	0.18	0.37
c_i^*	1.66	1.72	1.78	1.94	3.41
RRA_i	2.53	2.49	2.45	2.38	2.05

[a] c^* denotes the steady state value of aggregate consumption, $\rho = \beta(1+g)^{1-\sigma}$, α_i/α_5 is the weight the social planner assigns to the ith relative to the 5th quintile, $w_i = \alpha_i^{1/\sigma}/\sum(\alpha_j^{1/\sigma})$, c_i^* denotes the steady state consumption of individuals in quintile i, and RRA_i^* is the steady state measure of relative risk aversion of individuals in quintile i, The targeted distribution of non-human wealth is from Budria-Rodriguez et al. (2002, Table 7). Wealth shares and distributional characteristics are common across models.

in the data. (As noted, it is not possible to jointly match the returns to both equity and the risk-free asset in the baseline model.) The resulting value turned out to be 1.732.

Table 1 also reports the mapping of α_h into the consumption weights

$$w_h = \frac{(\alpha_h)^{1/\sigma}}{\sum_j (\alpha_j)^{1/\sigma}},$$

along with implied distributions of steady state consumption values and measures of relative risk aversion. Note that although steady state wealth holdings are highly uneven across quintiles (ranging from approximately 1% to 82%), the steady state distribution of consumption is relatively even: quintile values are $(1.66, 1.72, 1.78, 1.94, 3.41)$. Coupled with the specification $\sigma = 1.732$, corresponding measures of relative risk aversion across quintiles are $(2.53, 2.49, 2.45, 2.38, 2.05)$.

With the elasticity of intertemporal substitution (EIS) corresponding to the inverse of relative risk aversion, EIS ranges from 0.4 to 0.5 among the agents in our baseline model. In contrast, Vissing-Jorgensen (2002) estimated EIS to range from 0.3 to 0.4

among agents participating in the stock market; many subsequent studies obtain similar estimates (for a survey, see Guvenen (2009)). Holding σ fixed, if we increase γ to 7%, the range for EIS implied by our model shifts to 0.3–0.5; setting γ to the limit of 8% shifts this range to 0.2–0.5, which appears implausible. Thus 7% serves as an upper bound in the values of γ analyzed below.

For the extended model, the parameters that determine capital-adjustment costs and the behavior of the shock ξ were set as follows. Regarding the former, κ was set following Jermann (1998) at $1/0.23$, where 0.23 represents the elasticity of the investment-capital ratio with respect to Tobin's q, and (b_0, b_1) were set to equate steady state values of all variables across the baseline and extended models. The shock parameter μ was set to match the sample mean of returns to the risk-free asset. The required value turns out to be 25.17, which yields a steady state equity premium of 5.939 (compared with 5.995 in the data).

The results to which we now turn are based on simulated data generated using nonlinear model approximations. These are based on policy functions $c(s,k)$, $p(s,k), \ldots$ represented as Chebyshev polynomials. Polynomial approximations were constructed using the projection method outlined, for example, in Judd (1988) and DeJong with Dave (2007). Sample statistics calculated from simulated data are based on artificial sample sizes of 10,000, obtained after discarding 1000 burn-in observations (to eliminate the influence of initial conditions).

4.2 *Results*

Figure 5 illustrates impulse responses of turnover and output resulting from a 1-standard-deviation innovation to s_t. The responses were obtained using the baseline model; those associated with the extended model are comparable. Table 2 and Figure 6 present comparisons of theoretical and empirical moments for both models.

Consider first the baseline model. Regarding performance along familiar dimensions, on the positive side, note that the model provides a close characterization of

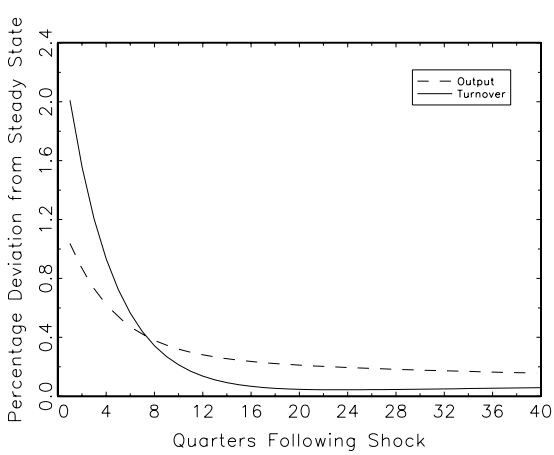

FIGURE 5. Model impulse response functions.

TABLE 2. Model and data comparisons.[a]

	E(Re)	Std(Re)	E(Rf)	Std(Rf)	E(Re − Rf)	Std(Re − Rf)	Corr(Re, Rf)	E(Turn.)
Data	7.152	15.244	1.156	1.156	5.995	15.050	0.161	15.60%
Baseline	7.152	0.290	7.382	0.221	−0.229	0.272	0.460	4.59%
Extended	7.095	35.692	1.156	40.539	5.939	31.552	0.664	4.59%

std(x)/ std(y)

	x		
	Con.	Inv.	Turnover
Data	0.460	4.263	7.883
Baseline	0.515	3.275	1.747
Extended	0.436	4.197	1.302

[a]For the data, Re and Rf are in levels; all other variables are HP-filtered. For model variables (where E = extended), Re and Rf are in levels; all other variables are logged deviations from steady state.

the procyclical nature of consumption and investment, and also closely captures their volatilities relative to output. On the negative side, note first that the risk-free-rate and equity premium puzzles are evident in this case: the steady state return to the risk-free asset is 7.382 (compared with 1.156 in the data), which corresponds with a steady state equity "premium" of −0.229 (compared with 5.995 in the data). Also, the standard deviations of returns fall far short of their empirical counterparts. These shortcomings are expected in light of Danthine, Donaldson, and Mehra (1992) and Rouwenhorst (1995).

Turning to the behavior of turnover, the steady state level of turnover in the model is 4.59%, while the average level of turnover in the data is 15.6%; thus the model accounts for 29.4% of the level of turnover. This result holds for all extensions of the model we consider. Regarding volatility, the ratio of standard deviations of turnover and output is 1.747 in the model, compared with 7.883 for the data. That is, the wealth-discrepancy channel that serves to generate equity trade in the model accounts for roughly 22% of observed fluctuations in turnover. However, note from Figure 6 that the model fails to

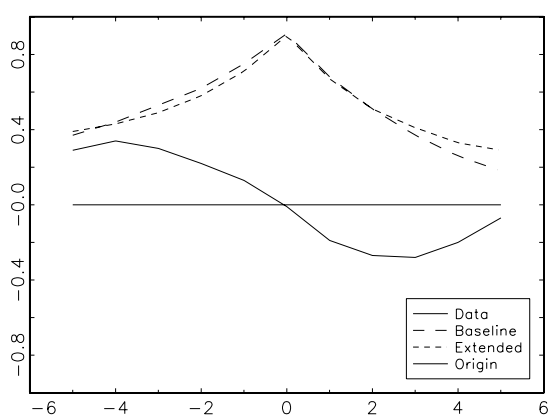

FIGURE 6. Comparison of correlations between output$_t$ and turnover$_{t-j}$.

TABLE 3. Sensitivity analysis.

	Baseline Model			Extended Model		
	$\sigma(t)/\sigma(y)$	E(Re)	E(Re − Rf)	$\sigma(t)/\sigma(y)$	E(Rf)	E(Re − Rf)
γ			$\sigma = 1.732$			
$0.00c^*$	0.980	7.152	−0.229	1.240	1.144	5.951
$0.05c^*$	1.750	7.152	−0.229	1.300	1.156	5.939
$0.07c^*$	2.370	7.152	−0.229	1.705	1.161	5.934
σ			$\gamma = 0.05c^*$			
1	0.043	5.822	−0.145	1.410	−0.250	6.011
1.732	1.750	7.152	−0.229	1.300	1.156	5.939
2	3.400	7.640	−0.264	1.270	1.646	5.934
3	14.01	9.433	−0.414	1.190	3.437	5.929
μ				$\sigma = 1.732, \gamma = 0.05c^*$		
20				1.150	3.510	3.620
25.17				1.300	1.156	5.751
30				1.430	−1.500	8.550

capture the asynchronous relationship between turnover and output evident in the data. (This failure is insensitive to model parameterization; its source is discussed below.) At the 5-quarter horizon, the correlation with output in the model is 0.37, compared with 0.29 in the data. However, the contemporaneous correlation between turnover and output is 0.91 in the model, but only −0.01 in the data. Additionally, while leads of turnover remain positively correlated with output in the model, they are negatively correlated in the data; for example, 0.26 compared with −0.22 at the 4-quarter horizon.

Consider now the sensitivity of this measure to changes in γ and σ, as reported in Table 3. Regarding γ, its main impact is on the volatility of turnover relative to output. Specifically, turnover volatility is monotonically increasing in γ, because as γ rises, differences in risk aversion are amplified across types. Setting $\gamma = 0$, differences in risk aversion are eliminated, and the volatility of turnover drops from the baseline measure of 1.745 to 0.98. Setting $\gamma = 0.07c^*$, steady state measures of risk aversion range from 3.4 to 2.1 in moving from lower- to upper-quintile types; in turn, turnover volatility increases to 2.4. Steady state returns and correlation patterns between turnover and output are unaffected by changes in γ, and the additional moments reported in Table 2 change only slightly. Thus the turnover volatility measure of 2.4, or 30% of that measured in the data, provides an upper-bound estimate under the baseline model and the measure of 0.98, or 12% of that measured in the data, provides a lower bound.

Regarding σ, note that the model's characterization of equity returns is sensitive to changes in this curvature parameter. For example, with $\sigma = 2$, equity returns increase from 7.152 to 7.64. So unlike γ, σ is tightly identified; thus we discount deviations from the baseline parameterization along this dimension.

Consider now the extended model. Beginning again with performance along familiar dimensions, on the positive side, the model once again does well in characterizing the

relative volatilities of consumption and investment relative to output. Also, the model is calibrated to exactly match the risk-free return, and does so while also nearly matching the equity premium. This is as expected, in light of Jermann (1998) and Campbell and Cochrane (1999). On the negative side, note that returns in this case are excessively volatile relative to their empirical counterparts. This is also as expected, following Jermann (1998), Campbell and Cochrane (1999), and Boldrin, Christiano, and Fisher (2001).

As in the baseline model, turnover and output are closely synchronized over the business cycle in the extended model (the contemporaneous correlation between turnover and output is 0.90 in this case, compared with 0.91 in the baseline model). The volatility of turnover is lower than in the baseline case: 1.302 relative to 1.747. That is, the extended model accounts for roughly 16.5% of observed fluctuations in turnover.

Regarding sensitivity to γ, at the upper bound of $\gamma = 0.07c^*$, Table 3 indicates that turnover volatility increases to 1.71; at the lower bound of zero, volatility falls to 1.24. As in the baseline case, note that the extended model's characterization of returns is relatively insensitive to changes in γ; for example, at $\gamma = 0.07c^*$, risk-free returns change by only 0.005, to 1.161. Thus the turnover volatility measurement of 1.71, or 22% of that measured in the data, provides an upper bound, and the measure of 1.24, or 16% of that measured in the data, provides a lower bound.

Regarding σ, note once again that this parameter is tightly identified. So too is the shock parameter μ: decreases in μ generate substantial increases in the steady state risk-free rate and corresponding reductions in the equity premium, leaving the measure of turnover volatility relatively unaffected. Thus we discount deviations from the baseline parameterization along the dimensions of σ and μ.

We close with a heuristic description of the strong positive contemporaneous correlation between output and turnover generated by the model. Consider for simplicity a two-agent specification wherein agent 1 has a relatively large welfare weight, so that her equity holdings θ_1 exceed 1 in the steady state and are increasing in s. Since $\theta_2 = 1 - \theta_1$, agent 2's equity holdings are negative in the steady state and are decreasing in s. (Agent 1 corresponds with the wealthiest agent in the calibrated models discussed above; agent 2 corresponds with the poorest agents.)

The contribution of agent h to aggregate turnover is given by

$$t_h = \left| \theta'_h - \frac{\theta_h}{1 + g_s} \right|,$$

where θ'_h denotes agent h's equity holdings in period $t+1$. The V-shaped t_h's are depicted in Figure 7 for the example described above. Note that in the steady state, $\theta'_h = \theta_h$ and thus turnover is positive (since $g_s > 0$): $t^*_h > 0$, $h = 1, 2$. For the agents to maintain their relative equity positions in the steady state, newly issued shares flow to agent 1.

Now consider the impact of an increase in s. In the figure, this moves equity holdings to $\theta'_h = \theta^A_h$, causing turnover to rise: relative to steady state, there is a greater flow of shares from agent 2 to agent 1. Of course, output also rises due to the increase in s; thus output and turnover co-vary positively. A decrease in s simply reverses the responses of both variables.

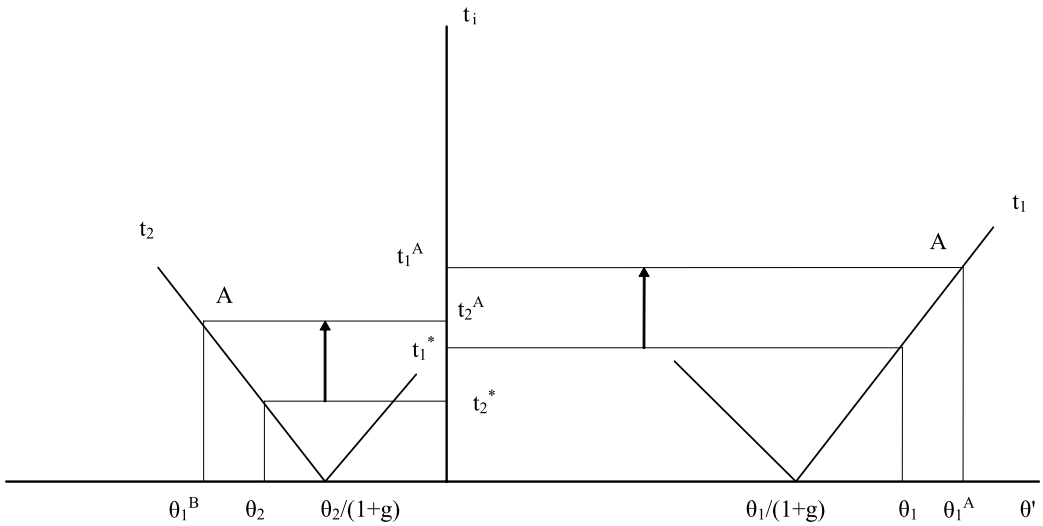

FIGURE 7. Heuristic characterization of turnover.

In both versions of the fully calibrated models on which our results are based, it turns out that the wealthiest agent always maintains equity holdings on the right arm of t_h and the poorest agents always maintain equity holdings on the left arm. This accounts for the strong positive correlation between output and aggregate turnover generated by the models. In the context of these models, the low contemporaneous correlation observed in the data is puzzling.

5. Conclusion

We have portrayed the cyclical behavior of the turnover of equity shares generated by individual investors on the New York Stock Exchange and have offered a theoretical characterization of this behavior. The theoretical characterization emphasizes differences in financial wealth across agents as giving rise to differential fluctuations in asset demand in response to productivity shocks; equity trade occurs as a result. We find that this simple mechanism accounts for 29% of the average level of turnover observed in the data and 22% of its volatility. As efforts to build on our understanding of turnover continue, this mechanism warrants recognition as an important building block.

Technical Appendix

Proof of Proposition 1. Let $p(s, 1, \Phi, k)$ denote the ex-dividend price of the firm, which uniquely solves the operator

$$p(s, 1, \Phi, k) = \sum_{s'} q(s, \Phi, k)(s')[p(s', 1, \Phi', k') + d(s', 1, \Phi', k')].$$

Since there is one outstanding share, this is also the ex-dividend price of the share. The stochastic return is given as usual by

$$\frac{p(s', 1, \Phi', k') + d(s', 1, \Phi', k')}{p(s, 1, \Phi, k)}. \tag{22}$$

Consider now the economy with financial policy (8). Let $p(s, \theta, \Phi, k)$ be the ex-dividend price of one share when there are θ outstanding shares at the beginning of the period. This price solves

$$\begin{aligned} p(s, \theta, \Phi, k) &= \sum_{s'} q(s, \Phi, k)(s')[p(s', \theta', \Phi', k') + d_f(s', \theta', \Phi', k')] \\ &= \sum_{s'} q(s, \Phi, k)(s')\Bigg[p(s', \theta', \Phi', k') \\ &\quad + \frac{d(s', \theta', \Phi', k') + p(s', \theta', \Phi', k')(\theta'(1+g_s) - \theta')}{\theta'}\Bigg] \\ &= \sum_{s'} q(s, \Phi, k)(s')\Bigg[p(s', \theta', \Phi', k')(1+g_s) + \frac{d(s', \theta', \Phi', k')}{\theta'}\Bigg]. \end{aligned}$$

Additionally, observe that

$$\begin{aligned} \theta' p(s, \theta, \Phi, k) &= \theta(1+g_s)p(s, \theta, \Phi, k) \\ &= \sum_{s'} q(s, \Phi, k)(s')[p(s', \theta', \Phi', k')\theta'(1+g_s) + d(s', \theta', \Phi', k')]. \end{aligned}$$

Consequently, it follows by uniqueness that

$$p(s, 1, \Phi, k) = \theta(1+g_s)p(s, \theta, \Phi, k) \tag{23}$$

for all θ (i.e., the ex-dividend price of the firm equals the value of the end-of-period shares outstanding). Using (23), it follows from (22) that equilibrium returns are unaffected by the firm's financial policy. Thus, returns can be computed directly using (22).

Finally, we check that the alternative policy function is the agent's optimal choice. Market clearing is satisfied by definition. It also follows by (9) and (23) that this alternative policy satisfies the budget constraint evaluated at the equilibrium allocation. To see this, note that

$$\begin{aligned} \theta'_h(s, \theta, \Phi, k)p(s, \theta, \Phi, k) &= \frac{\theta'_h(s, \theta, \Phi, k)}{(1+g_s)\theta}\theta(1+g_s)p(s, \theta, \Phi, k) \\ &= \widehat{\theta}'_h(s, 1, \Phi, K)p(s, 1, \Phi, K) \end{aligned}$$

and also

$$\begin{aligned} &[p(s, \theta, \Phi, k) + d_f(s, \theta, \Phi, k)]\theta_h \\ &\quad = \Bigg[p(s, \theta, \Phi, k) + \frac{d(s, \theta, \Phi, k) + p(s, \theta, \Phi, k)(\theta(1+g_s) - \theta)}{\theta}\Bigg]\theta_h \end{aligned}$$

$$\times [p(s,\theta,\Phi,k)(1+g_s)\theta + d(s,\theta,\Phi,k)]\frac{\theta_h}{\theta}$$
$$= [p(s,1,\Phi,k) + d(s,\theta,\Phi,k)]\widehat{\theta}_h,$$

where $\sum_h(\theta_h/\theta) = 1$. Note that $d(s,\theta,\Phi,k) = d(s,\Phi,k)$ for all θ since production plans are unaffected by the firm's financial policy. □

PO allocations and the planner's problem

The set of PO allocations can be parameterized by welfare weights $\alpha \in \mathbb{R}^I_+$, where α_h denotes the welfare weight assigned to agent h. Since only the allocation of consumption across individuals is affected by α, first the planner solves (PPRN) and then distributes consumption across agent types following the allocation rule (26) given below. Specifically, the planner's problem solves

$$v(s,k) = \max_{(c,i,k')\geq 0}\left\{\xi(s)\frac{(c-\gamma_A)^{1-\sigma}}{1-\sigma} + \rho\sum_{s'}\pi(s,s')v(s',k')\right\} \tag{PPRN}$$

subject to

$$\sum_h c_h + \Psi(k,i) = F(s,k,H), \tag{24}$$

$$k'(1+g) = (1-\delta)k + i, \quad k' \in X, \tag{25}$$

where $\gamma_A = \gamma H$ is the aggregate minimum consumption requirement.

Denoting the set of continuous policy functions solving (PPRN) as $(c(s,k), k'(s,k), i(s,k))$, individual consumption is allocated according to

$$(c_h(s,k,\alpha) - \gamma) = \omega_h(c(s,k) - \gamma_A), \tag{26}$$
$$\omega_h = \frac{(\alpha_h)^{1/\sigma}}{\sum_j(\alpha_j)^{1/\sigma}} \quad \forall h.$$

Computing the RCE: Negishi's approach

Given the policy functions $(c(s,k), k'(s,k), i(s,k))$, the RCE is constructed as follows. State prices are given by the stochastic discount factor

$$q(s,k)(s') = \rho\pi(s,s')\left(\frac{s'}{s}\right)^{-\mu}\frac{(c(s',k'(s,k)) - \gamma_A)^{-\sigma}}{(c(s,k)-\gamma_A)^{-\sigma}}. \tag{27}$$

Regarding equity, let $v_F(s,k)$ denote the value of the representative firm given by the unique solution to

$$v_F(s,k) = d(s,k) + \sum_{s'} q(s,k)(s')v_F(s',k'(s,k)), \tag{RFP}$$

where

$$d(s,k) = F(s,k,H) - \Psi(k, i(s,k)) - w(s,k)H, \tag{28}$$
$$k'(s,k)(1+g) = (1-\delta)k + i(s,k), \quad k'(s,k) \geq 0,$$

and $w(s,k) = F_l(s,k,H)$ denotes the implicit wage. With outstanding equity shares normalized to 1, the ex-dividend price of equity $p(s,k)$ is given as

$$p(s,k) = v_F(s,k) - d(s,k). \tag{29}$$

Hereafter, $v_F(s,k)$ is interpreted as the aggregate value of financial wealth. Similarly, the value $v_M(s,k)$ of the minimum consumption requirement γ is given uniquely by

$$v_M(s,k) = \gamma + \sum_{s'} q(s,k)(s') v_M(s', k'(s,k)).$$

To compute the remaining ingredients needed to construct a RCE, consider any allocation parameterized by α such that $\sum_h (\alpha_h)^{1/\sigma} = 1$. Let $v_C^h(s,k,\alpha)$ denote the associated value of agent h's share of aggregate consumption $c_h(s,k,\alpha)$, as determined in (26); this value is given uniquely by

$$v_C^h(s,k,\alpha) = (\alpha_h)^{1/\sigma} c(s,k) + (\gamma - (\alpha_h)^{1/\sigma}\gamma_A) + \sum_{s'} q(s,k)(s') v_C^h(s', k'(s,k), \alpha).$$

Individual consumption is financed from two sources. The first is wages which provide an associated income stream valued uniquely as

$$v_W(s,k) = w(s,k) + \sum_{s'} q(s,k)(s') v_W(s', k'(s,k)).$$

Hereafter, we describe $v_W(s,k)$ as representing *individual human wealth*. The second source of financing is *individual financial wealth*, which is given by

$$\phi_h(s,k,\alpha) = v_C^h(s,k,\alpha) - v_W(s,k).$$

Given uniqueness, it follows from feasibility that

$$v_C^h(s,k,\alpha) = (\alpha_h)^{1/\sigma}(v_F(s,k) + v_W(s,k)H) + (1 - (\alpha_h)^{1/\sigma}H) v_M(s,k)$$

and, consequently,

$$\begin{aligned}\phi_h(s,k,\alpha) = {} & (\alpha_h)^{1/\sigma}(v_F(s,k) + v_W(s,k)H) \\ & + (1 - (\alpha_h)^{1/\sigma}H) v_M(s,k) - v_W(s,k).\end{aligned} \tag{30}$$

For each (k,α), let $[(a_h(k,\alpha), \theta_h(k,\alpha)]$ solve the 2×2 system

$$\phi_h(s,k,\alpha) = a_h(k,\alpha) + [p(s,k) + d(s,k)]\theta_h(k,\alpha) \quad \text{for all } s \in \{\underline{s}, \overline{s}\}. \tag{31}$$

As mentioned, this system (generically) has a unique solution for each h. Below we show how to use it to construct equilibrium portfolios.

The following proposition characterizes the unique PO allocation that can be decentralized as a RCE with zero initial transfers. Once this specific welfare vector has been identified, corresponding equilibrium prices and portfolios are constructed using the objects defined in (27)–(31).

PROPOSITION 2. *Given* (s_0, k_0), *there exists a unique welfare weight* α^0 *given by*

$$(\alpha_h^0)^{1/\sigma} = \frac{\theta_h^0 v_F(s_0, k_0) + v_W(s_0, k_0) - v_M(s_0, k_0)}{v_F(s_0, k_0) + (v_W(s_0, k_0) - v_M(s_0, k_0))H}, \tag{32}$$

such that the corresponding PO allocation can be decentralized as a RCE.

Equilibrium prices are given by

$$\begin{aligned} w(s, \Phi, k) &= F_l(s, k, H), \\ p(s, \Phi, k) &= p(s, k), \\ q^{\mathrm{RF}}(s, \Phi, k) &= \sum_{s'} q(s, k)(s'). \end{aligned}$$

Equilibrium portfolios for each h *are determined by* (31) *such that*

$$\begin{aligned} a_h'(s, \Phi, k) &= a_h(k'(s, k), \alpha^0), \\ \theta_h'(s, \Phi, k) &= \theta_h(k'(s, k), \alpha^0), \end{aligned} \tag{33}$$

where $\Phi = (\phi_h(s, k, \alpha^0))_{h \in \mathcal{H}}$ *represents the aggregate distribution of wealth determined by* (30) *at* α^0.

For the proof see Espino (2007).

DATA APPENDIX

A.1 *Definitions and sources*

Turnover is volume (total shares traded) as a percentage of shares outstanding on the NYSE. Volume data are from Yahoo (finance.yahoo.com); shares outstanding are from the NYSE fact book (www.nysedata.com/factbook).

Volume is reported as daily averages observed over the month; they are converted to a quarterly measure by averaging the (deseasonalized) monthly measures. Shares outstanding are reported as yearly averages; dividing by the number of trading days during the year (from the NYSE fact book) yields conversion to daily averages. Annual data are converted to a quarterly measure via log-linear interpolation. Letting $g(\tau)$ denote the growth in shares outstanding observed between years τ and $\tau + 1$, and letting so_τ denote shares outstanding reported in year τ, the quarterly measures $\mathrm{so}_{\tau,i}$, $i = (\mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV})$, are constructed as

$$\mathrm{so}_{\tau,i} = \mathrm{so}_\tau e^{0.25 g(\tau)(i-1)}.$$

Aggregate turnover T_{agg} is converted to a measure of turnover attributable to individual investors as described in the text: denoting by Vol_{agg} and SO_{agg} aggregate volume and shares outstanding, individual turnover T_{ind} is obtained via the adjustment

$$\begin{aligned} T_{\text{ind}} &= \frac{\text{Vol}_{\text{agg}} \cdot \%\text{Vol}_{\text{ind}}}{\text{SO}_{\text{agg}} \cdot \%\text{SO}_{\text{ind}}} \\ &= T_{\text{agg}} \cdot \frac{\%\text{Vol}_{\text{ind}}}{\%\text{SO}_{\text{ind}}}, \end{aligned}$$

where $\%\text{Vol}_{\text{ind}}$ and $\%\text{SO}_{\text{ind}}$ are the percentages of volume generated and shares held by individual investors. Snapshots of these measures are reported in the NYSE fact book. Given two observations (x_t, x_{t+j}), observations for intermediate dates are constructed via interpolation as

$$\begin{aligned} x_{t+i} &= x_t e^{i \cdot g_j}, \\ g_j &= \frac{\ln\left(\dfrac{x_{t+j}}{x_t}\right)}{j}. \end{aligned}$$

Interpolated observations beyond the final observation date x_{t+J} are constructed using g_J. Approximations are based on a total of 23 data points, the latest of which is 1980:IV.

Returns to equity r^e are annualized real returns accruing to the stocks included in the S&P 500 index. Both nominal and real S&P prices p and dividends d are reported on a monthly basis by Robert Shiller. Prices are monthly averages of daily closing prices; dividends are 12-month moving totals of dividends per share, adjusted to index. The real data are converted into monthly observations of annualized returns via geometric averaging:

$$r_t^e = \ln\left(\frac{p_{t+12} + d_t}{p_t}\right).$$

Quarterly returns are constructed by averaging over annualized monthly returns.

Price and dividend data are from www.econ.yale.edu/~shiller/data.htm. Conversion from nominal to real measures (as with the remaining nominal series described herein) is accomplished using the CPI-U (consumer price index, all urban consumers). This is available on a monthly basis from the Federal Reserve Bank of St. Louis' FRED data base (research.stlouisfed.org/fred2/). CPI-U is referenced under FRED as series CPIAUCSL.

Risk-free returns r^f are annualized real returns to 3-month Treasury bills. Nominal returns are available on a monthly basis from the FRED data base as series TB3MS. Quarterly returns are constructed by averaging over real monthly returns.

Consumption c is real personal consumption expenditures on nondurables (FRED series PCNDGC96) and services (FRED series PCESVC96). Investment i is real gross private domestic investment (FRED series GPDIC1). Output y is the sum of consumption and investment. The series are quarterly and in per capita terms, with population measured as the civilian noninstitutional population (FRED series CNP16OV).

The longest time span over which all series are available is 1950:I through 2004:II. The series are available for downloading at www.pitt.edu/~dejong/wp.htm.

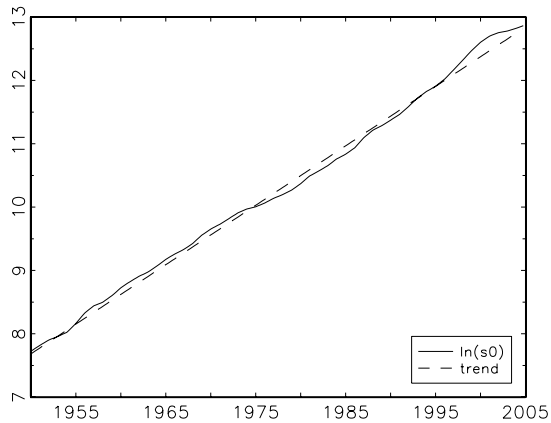

FIGURE A.1. Shares outstanding.

A.2 *Comparing volume and turnover*

Differences between volume and turnover amount to differences in trend behavior. Recall that turnover is defined as volume measured as a percentage of shares outstanding. The behavior of shares outstanding is depicted in Figure A.1. Note that the series conforms closely to its estimated log-linear trajectory, which grows at an annual average rate of 9.4% (the standard deviation of logged departures from trend is 0.093).

To illustrate the impact of normalizing volume by shares outstanding, Figure A.2 depicts logged trajectories of both turnover and volume in the upper diagrams, and their logged departures from estimated Hodrick–Prescott-filtered trends in the bottom diagram. The average growth rate of turnover over the sample period is 3.9%, compared with 13.3% for volume. Their logged departures from trend are virtually indiscernible: the correlation between measures is 0.992, and the standard deviations of these measures are 0.143 (turnover) and 0.144 (volume).

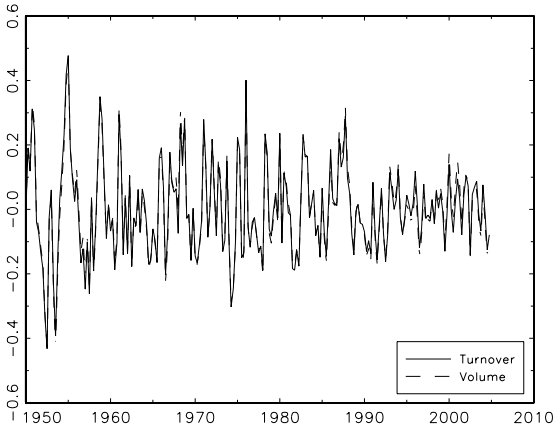

FIGURE A.2. Volume and turnover comparison.

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