Supplement to "The cyclical dynamics of illiquid housing, debt, and foreclosures"

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A. Comparison of boom–bust dynamics

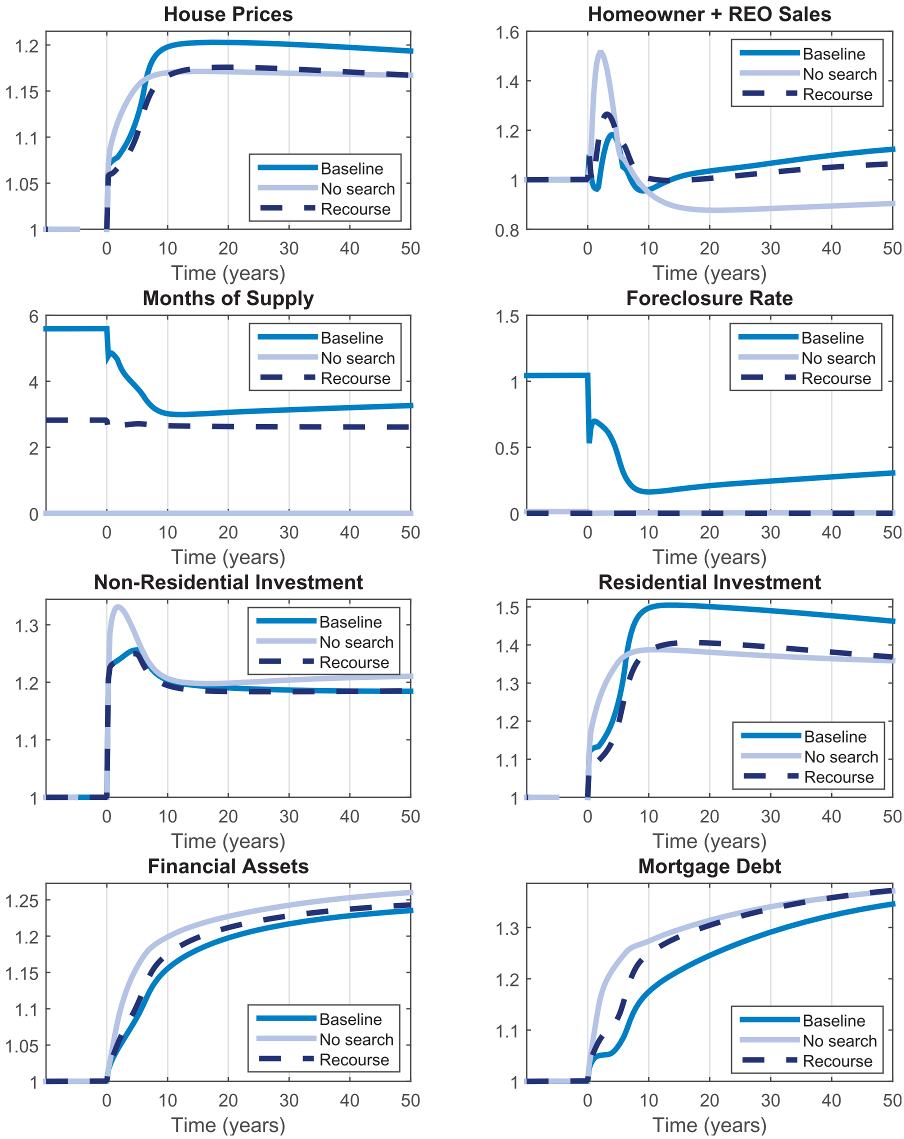

Figure S.1. Economic dynamics in a *bust-to-boom* transition caused by a permanent, *positive* z_c shock. All variables except months supply and the foreclosure rate are initially normalized to 1.

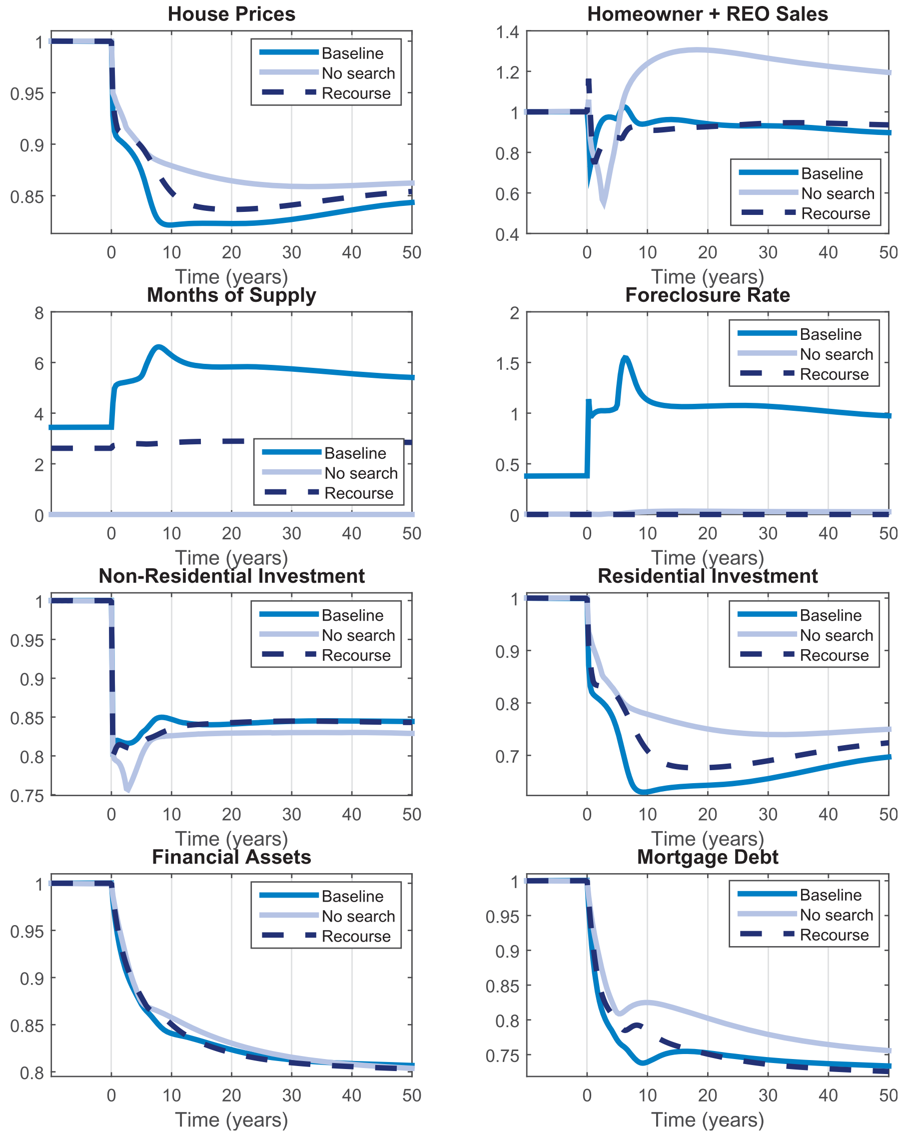

FIGURE S.2. Economic dynamics in a *boom-to-bust* transition caused by a permanent, *negative* z_c shock. All variables except months supply and the foreclosure rate are initially normalized to 1.

B. CALIBRATION DETAILS

B.1 *Calibrating the labor efficiency process*

As explained in the calibration section, it is not possible to estimate quarterly income processes from the Panel Study of Income Dynamics (PSID) data because the PSID is only conducted annually. Instead, I start by specifying a labor process like that in Storesletten et al. (2004), except without life cycle effects or a permanent shock at birth.

I adopt their values for the annual autocorrelation of the persistent shock and for the variances of the persistent and transitory shocks, transforming them to quarterly values.

B.1.1 *Persistent shocks* I assume that households play a lottery each period in which, with probability 3/4, they receive the same persistent shock as they did in the previous period, and with probability 1/4, they draw a new shock from a transition matrix calibrated to the persistent process in Storesletten et al. (2004) (in which case they still might receive the same persistent labor shock). This is equivalent to choosing transition probabilities that match the expected amount of time that households expect to keep their current shock. Storesletten et al. (2004) report an annual autocorrelation coefficient of 0.952 and a frequency-weighted average standard deviation over expansions and recessions of 0.17. I use the Rouwenhorst method to calibrate this process, which gives the transition matrix

$$\tilde{\pi}_s(\cdot,\cdot) = \begin{pmatrix} 0.9526 & 0.0234 & 0.0006 \\ 0.0469 & 0.9532 & 0.0469 \\ 0.0006 & 0.0234 & 0.9526 \end{pmatrix}.$$

As a result, the final transition matrix is

$$\pi_s(\cdot,\cdot) = 0.75 I_3 + 0.25\tilde{\pi}_s(\cdot,\cdot) = \begin{pmatrix} 0.9881 & 0.0059 & 0.0001 \\ 0.0171 & 0.9883 & 0.0171 \\ 0.0001 & 0.0059 & 0.9881 \end{pmatrix}.$$

B.1.2 *Transitory shocks* Storesletten et al. (2004) report a standard deviation of the transitory shock of 0.255. To replicate this, I assume that the annual transitory shock is actually the sum of four, independent quarterly transitory shocks. I make use of the same identifying assumption that Storesletten et al. (2004) use, namely, that all households receive the same initial persistent shock. Any variance in initial labor income is then due to different draws of the transitory shock. Recall that the labor productivity process is given by

$$\ln(e \cdot s) = \ln(s) + \ln(e).$$

Therefore, total labor productivity (which, when multiplied by the wage w, is total wage income) over a year in which s stays constant is

$$(e \cdot s)_{\text{year 1}} = \exp(s_0)\big[\exp(e_1) + \exp(e_2) + \exp(e_3) + \exp(e_4)\big].$$

For different variances of the transitory shock, I simulate total annual labor productivity for many individuals, take logs, and compute the variance of the annual transitory shock. It turns out that quarterly transitory shocks with a standard deviation of 0.49 give the desired standard deviation of annual transitory shocks of 0.255.

B.2 *Calculating submarket trading probabilities*

Using (8) to solve for buyers' trading probabilities gives

$$p_b\big(\theta_b(x_b, h; p_h)\big) = \begin{cases} 0 & \text{if } x_b < \underline{x}_b(p_h), \\ \dfrac{\left[\left(\dfrac{A_b(x_b - p_h h)}{\kappa_b h}\right)^{1/\gamma_b} - 1\right]\kappa_b h}{x_b - p_h h} & \text{if } \underline{x}_b(p_h) \le x_b \le \overline{x}_b(p_h), \\ 1 & \text{if } x_b > \overline{x}_b(p_h), \end{cases}$$

where $\underline{x}_b(p_h) = (p_h + \frac{\kappa_b}{A_b})h$ and $\overline{x}_b(p_h) = (p_h + \kappa_b(A_b^{\gamma_b} - 1)^{-1/\gamma_b})h$.

Similarly, sellers' trading probabilities come from solving (9), giving

$$p_s\big(\theta_s(x_s, h; p_h)\big) = \begin{cases} 0 & \text{if } x_s > \overline{x}_s(p_h), \\ \dfrac{\left[\left(\dfrac{A_s(p_h h - x_s)}{\kappa_s h}\right)^{1/\gamma_s} - 1\right]\kappa_s h}{p_h h - x_s} & \text{if } \underline{x}_s(p_h) \le x_s \le \overline{x}_s(p_h), \\ 1 & \text{if } x_s < \underline{x}_s(p_h), \end{cases}$$

where $\overline{x}_s(p_h) = (p_h - \frac{\kappa_s}{A_s})h$ and $\underline{x}_s(p_h) = (p_h - \kappa_s(A_s^{\gamma_s} - 1)^{-1/\gamma_s})h$.

As this characterization of trading probabilities shows, search frictions give rise to endogenous adjustment costs. Matching fails if households try for too small of an adjustment cost, matching succeeds if they accept the maximum adjustment cost, and trade occurs probabilistically for intermediate values.

C. The intermediary's balance sheet

Period-t cash flows are given by

$$\begin{aligned}
\pi_t = {} & (1-\delta_c + r_t)K_t - B_t + \sum_{n_{m,t}}\sum_{s_t}\int \Big\{ p_s\big(\theta_{s,t}(x^*_{s,t}, h_{t-1})\big) + \big[1 - p_s\big(\theta_{s,t}(x^*_{s,t}, h_{t-1})\big)\big] \\
& \times (1-d^*_t)\underbrace{\big[m_t - (1+\phi)m^*_{t+1}/(1+r_{m,t}) + \Pi_t\big(m^*_{t+1}, b^*_{t+1}, h_t = h_{t-1}, s_t\big)\big]}_{\text{mortgage payments+value of selling continuation mortgages}}\Big\} F(de)\pi_s(s_t|s_{t-1})n_{m,t} \\
& + \sum_{n_{m,t}}\sum_{s_t}\int \underbrace{\big[1 - p_s\big(\theta_{s,t}(x^*_{s,t}, h_{t-1})\big)\big]d^*_t J_{\text{REO},t}(h_{t-1})F(de)\pi_s(s_t|s_{t-1})n_{m,t}}_{\text{value of new REOs prior to selling}} + \underbrace{\sum_{h\in H} J_{\text{REO},t}(h)H_{\text{REO},t}(h)}_{\text{value of REO inventories prior to selling}} \\
& - \sum_{n_{m,t}}\sum_{s_t}\int \underbrace{\big[1 - p_s\big(\theta_{s,t}(x^*_{s,t}, h_{t-1})\big)\big]d^*_t\big[J_{\text{REO},t}(h_{t-1}) - p_s\big(\theta_{s,t}(x^{\text{REO}}_{s,t}, h_{t-1})\big)x^{\text{REO}}_{s,t}(h_{t-1})\big]F(de)\pi_s(s_t|s_{t-1})n_{m,t}}_{\text{cost of purchasing unsold new REOs}} \\
& - \underbrace{\sum_{h\in H}\big[J_{\text{REO},t}(h) - p_s\big(\theta_{s,t}(x^{\text{REO}}_{s,t}, h)\big)x^{\text{REO}}_{s,t}(h)\big]H_{\text{REO},t}(h)}_{\text{cost of purchasing unsold REO inventories}} + \underbrace{q_{b,t}B_{t+1}}_{\text{new bond issuances}} \\
& - \sum_{n_{m,t+1}} \underbrace{q^0_{m,t}(m_{t+1}, b_{t+1}, h_t, s_t)(1+\zeta)m_{t+1}n_{m,t+1}}_{\text{cost of new and vintage mortgages}} - K_{t+1}.
\end{aligned}$$

The first three lines represent the intermediary's beginning-of-period profits resulting from capital (+), households' redemption of bonds (−), revenues from mortgage payments (+), and the sale of continuation mortgages and REO inventories to other intermediaries (+). The last three lines represent intermediary expenses at the end of the period. The intermediary purchases unsold REO inventories (−), sells bonds (+), issues new mortgages (−), purchases vintage mortgages (−), and chooses next period's capital (−).

When prices are in equilibrium, next period's capital stock equals the sum of the other subperiod 3 expenses. In other words, higher bond issuances B_{t+1} increase next period's capital stock, while higher mortgage originations lower the capital stock. Furthermore, the mortgage pricing condition (12) and REO value function (13) ensure zero ex ante beginning-of-period profits.

D. Computing the model

In the spirit of Krusell and Smith (1998), I compute a bounded rationality equilibrium where households approximate the aggregate state space when forming expectations. As in Favilukis et al. (2013), I use capital K and the shadow housing price p_h as the approximating variables. Therefore, the aggregate state in this bounded rationality economy is $\hat{\mathbf{Z}} = (z_c, p_h, K)$. I posit the forecasting functions

$$p_h'(z_c, p_h, K, z_c') = a_0^p(z_c, z_c') + a_1^p(z_c, z_c')p_h + a_2^p(z_c, z_c')K, \tag{1}$$

$$K'(z_c, p_h, K, z_c') = a_0^K(z_c) + a_1^K(z_c)p_h + a_2^K(z_c)K, \tag{2}$$

$$\tau'(z_c, p_h, K, z_c') = a_0^\tau(z_c, z_c') + a_1^\tau(z_c, z_c')p_h + a_2^\tau(z_c, z_c')K. \tag{3}$$

In an approximating equilibrium, the forecasting function coefficients maximize predictive accuracy relative to simulated time series of p_h', K', and τ'. The entire computational algorithm is outlined below:

1. Solve for equilibrium submarket tightnesses $\{\theta_b(x_b, h; p_h)\}$ and $\{\theta_s(x_s, h; p_h)\}$ for each value of the housing shadow price, p_h, using (8) and (9).

2. *Loop 1.* Make an initial guess of coefficients $\mathbf{a}^{p,0}$, $\mathbf{a}^{K,0}$, and $\mathbf{a}^{\tau,0}$.

(a) Solve for $w(\mathbf{Z}')$, $r(\mathbf{Z}')$, $q_b(\mathbf{Z})$, and $q_m(\mathbf{Z})$ using the aggregate laws of motion implied by the coefficients above, the equilibrium conditions for the firm's problem, (1), (2), (5), and (6), the intermediary conditions (10) and (11), and the market clearing conditions for land/permits and labor. In practice, this procedure involves solving a simple fixed point problem in the amount of labor employed in the consumption good sector and then substituting to calculate the remaining quantities and factor prices.

(b) *Loop 2a.* Make an initial guess for the mortgage company's REO value function $J_{\text{REO}}^0(h; \mathbf{Z})$:

i. Substitute J_{REO}^0 into the right hand side of (13) and solve for $J_{\text{REO}}(h, Z)$.

ii. If $\sup(|J_{\text{REO}} - J^0_{\text{REO}}|) < \varepsilon_J$, then exit the loop. Otherwise, set $J^0_{\text{REO}} = J_{\text{REO}}$ and return to (i).

(c) *Loop 2b*. Make an initial guess of mortgage prices $q^{0,n}_m(m', b', h, s; \mathbf{Z})$ for $n = 0$.

i. Calculate the lower bound of the budget set for homeowners with good credit entering subperiod 3, $\underline{y}(m, h, s; \mathbf{Z})$, by solving

$$\underline{y}(m, h, s; \mathbf{Z}) = \min_{m',b'}\left[q_b(\mathbf{Z})b' + m - q_m\left(m', b', h, s, \mathbf{Z}\right)m'\right], \quad \text{where}$$

$$q_m\left(m', b', h, s; \mathbf{Z}\right) = \begin{cases} q^0_m\left(m', b', h, s; \mathbf{Z}\right), & \text{if } m' > m, \\ \dfrac{1}{1 + r_m(\mathbf{Z})}, & \text{if } m' \leq m. \end{cases}$$

ii. *Loop 3*. Make an initial guess for $V^0_{\text{rent}}(y, s, f; \mathbf{Z})$ and $V^0_{\text{own}}(y, m, h, s, f; \mathbf{Z})$.

A. Substitute V^0_{rent} and V^0_{own} into the right hand side of (18) and (19) and solve for R_{buy}.

B. Substitute V^0_{rent}, V^0_{own}, and R_{buy} into the right hand side of (20) and (21) and solve for W_{own}.

C. Substitute W_{own}, V^0_{rent}, and R_{buy} into the right hand side of (22) and (23) and solve for R_{sell}.

D. Substitute W_{own}, V^0_{rent}, R_{sell}, and R_{buy} into the right hand side of (15) and (16) and solve for V_{rent} and V_{own}.

E. If $\sup(|V_{\text{rent}} - V^0_{\text{rent}}|) + \sup(|V_{\text{own}} - V^0_{\text{own}}|) < \varepsilon_V$, then exit the loop. Otherwise, set $V^0_{\text{rent}} = V_{\text{rent}}$ and $V^0_{\text{own}} = V_{\text{own}}$ and return to A.

iii. Substitute $q^{0,n}_m$, J_{REO}, and the household's policy functions for bonds, mortgage choice, and selling and default decisions into the right hand side of (12) and solve for q^0_m.

iv. If $\sup(q^0_m - q^{0,n}_m) < \varepsilon_q$, then exit the loop. Otherwise, set $q^{0,n+1}_m = (1-\lambda_q)q^{0,n}_m + \lambda_q q^0_m$ and return to (i).

(d) *Loop 4*. Initialize the distribution Φ of homeowners and renters, the capital stock K, and the stock H^0_{REO} of REO houses.

i. Draw an initial shock $z_{c,0}$ from the stationary distribution Π_z followed by a sequence of aggregate shocks $\{z_{c,t}\}_1^T$ using the Markov transition matrix $\pi_z(z'_c|z_c)$.

ii. Simulate the economy for T periods[1] using the decision rules of the households and intermediaries. In each period, compute the tax rate that properly distributes the intermediary's ex post profits/losses, calculate tomorrow's capital stock (equal to bond issuances minus the value of new and vintage mortgages and unsold REO inventories), and solve for the equilibrium housing shadow price $p_{h,t}$ that satisfies (26).

[1]I simulate the economy for 5600 periods and ignore the first 600 periods for the regressions.

(e) Run the regressions

$$p_{h,t+1} = a_0^p(z_{c,t}, z_{c,t+1}) + a_1^p(z_{c,t}, z_{c,t+1})p_{h,t} + a_2^p(z_{c,t}, z_{c,t+1})K_t,$$
$$K_{t+1} = a_0^K(z_{c,t}) + a_1^K(z_{c,t})p_{h,t} + a_2^K(z_{c,t})K_t,$$
$$\tau_{t+1} = a_0^\tau(z_{c,t}, z_{c,t+1}) + a_1^\tau(z_{c,t}, z_{c,t+1})p_{h,t} + a_2^\tau(z_{c,t}, z_{c,t+1})K_t,$$

which gives new coefficients $\mathbf{a}^p$, $\mathbf{a}^K$, and $\mathbf{a}^\tau$.

(f) If $\sup(|\mathbf{a}^p - \mathbf{a}^{p,n}|) + \sup(|\mathbf{a}^K - \mathbf{a}^{K,n}|) + \sup(|\mathbf{a}^\tau - \mathbf{a}^{\tau,n}|) < \varepsilon_a$, then the algorithm is complete. Otherwise, set $\mathbf{a}^{p,n+1} = (1-\lambda_a)\mathbf{a}^{p,n} + \lambda_a \mathbf{a}^p$, $\mathbf{a}^{K,n+1} = (1-\lambda_a)\mathbf{a}^{K,n} + \lambda_a \mathbf{a}^K$, $\mathbf{a}^{\tau,n+1} = (1-\lambda_a)\mathbf{a}^{\tau,n} + \lambda_a \mathbf{a}^\tau$, and go to (a).

E. Accuracy of approximate equilibria

I try several modifications of the regressions explained in the approximating equilibrium section. In particular, I impose various identifying restrictions on the coefficients to see if doing so affects the R^2 of the regressions or the stability of the computational algorithm. Eliminating the house price terms in the capital stock regression and combining terms for large and small shocks (e.g., lumping transitions $(z_1, z' = z_2)$ and $(z_1, z' = z_3)$ into one term, $(z_1, z' > z_1)$) gives the most accurate and stable solution. The converged results are reported below for the baseline, no-search, and recourse economies.

Table S.1. Baseline equilibrium.

	(z_1, z_1)	$(z_1, z' > z_1)$	(z_2, z_1)	(z_2, z_2)	(z_2, z_3)	$(z_3, z' < z_3)$	(z_3, z_3)
a_0^p	0.0308	0.0590	0.0256	0.0465	0.0813	0.0099	0.0471
a_1^p	0.9646	0.9646	0.9480	0.9480	0.9480	0.9628	0.9628
a_2^p	−0.0007	−0.0007	0.0001	0.0001	0.0001	−0.0007	−0.0036
							$R^2 = 0.998$
a_0^K	0.0613	0.0613	0.0652	0.0652	0.0652	0.1072	0.1072
a_1^K	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
a_2^K	0.9728	0.9728	0.9739	0.9739	0.9739	0.9600	0.9600
							$R^2 = 0.99997$
a_0^τ	0.0001	−0.0018	0.0026	0.0009	−0.0007	0.0020	0.0005
a_1^τ	−0.0009	−0.0009	−0.0015	−0.0015	−0.0015	−0.0005	−0.0005
a_2^τ	0.0003	0.0003	0.0002	0.0002	0.0002	−0.0000	−0.0000
							$R^2 = 0.988$*

Note: *Simulated taxes/subsidies usually $\sim 0.00x\%$ and never exceed $0.x\%$ (when z switches values).

Table S.2. No-search equilibrium.

	(z_1, z_1)	$(z_1, z' > z_1)$	(z_2, z_1)	(z_2, z_2)	(z_2, z_3)	$(z_3, z' < z_3)$	(z_3, z_3)
a_0^p	0.0229	0.0418	0.0037	0.0246	0.0583	0.0291	0.0518
a_1^p	0.9822	0.9822	0.9853	0.9853	0.9853	0.9594	0.9594
a_2^p	−0.0039	−0.0039	−0.0049	−0.0049	−0.0049	−0.0048	−0.0048
							$R^2 = 0.998$
a_0^K	0.0686	0.0686	0.0467	0.0467	0.0467	0.1238	0.1238
a_1^K	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
a_2^K	0.9676	0.9676	0.9799	0.9799	0.9799	0.9515	0.9515
							$R^2 = 0.99997$
a_0^τ	0.0001	−0.0007	0.0008	−0.0001	−0.0008	0.0010	0.0003
a_1^τ	−0.0005	−0.0005	−0.0001	−0.0001	−0.0001	−0.0003	−0.0003
a_2^τ	0.0001	0.0001	0.0001	0.0001	0.0001	−0.0000	−0.0000
							$R^2 = 0.993^*$

Note: *Simulated taxes/subsidies never exceed $0.0x\%$ (when z switches values).

Table S.3. Full recourse equilibrium.

	(z_1, z_1)	$(z_1, z' > z_1)$	(z_2, z_1)	(z_2, z_2)	(z_2, z_3)	$(z_3, z' < z_3)$	(z_3, z_3)
a_0^p	0.0344	0.0583	0.0290	0.0473	0.0800	0.0104	0.0440
a_1^p	0.9558	0.9558	0.9431	0.9431	0.9431	0.9697	0.9697
a_2^p	0.0013	0.0013	0.0016	0.0016	0.0016	−0.0050	−0.0050
							$R^2 = 0.999$
a_0^K	0.0651	0.0651	0.0701	0.0701	0.0701	0.1006	0.1006
a_1^K	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
a_2^K	0.9715	0.9715	0.9723	0.9723	0.9723	0.9630	0.9630
							$R^2 = 0.99997$
a_0^τ	−0.0002	−0.0013	0.0016	0.0005	−0.0005	0.0011	0.0001
a_1^τ	−0.0000	−0.0000	−0.0009	−0.0009	−0.0009	−0.0004	−0.0004
a_2^τ	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
							$R^2 = 0.993^*$

Note: *Simulated taxes/subsidies never exceed $0.0x\%$ (when z switches values).

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