Supplementary Material

Supplement to “Testing ambiguity theories with a mean-preserving design”
(Quantitative Economics, Vol. 8, No. 1, March 2017, 219–238)

CHUN-LEI YANG
Institute for Social and Economic Research, Nanjing Audit University and Research Center for Humanities and Social Sciences, Academia Sinica

LAN YAO
School of Economics, Shanghai University of Finance and Economics

APPENDIX A: INSTRUCTIONS (SLIDES TRANSLATED FROM CHINESE ORIGINAL) AND TREATMENT SUMMARY

TABLE A.1. Summary of treatments.

<table>
<thead>
<tr>
<th></th>
<th>Prob-1</th>
<th>Prob-2</th>
<th>Prob-3</th>
<th>Prob-4</th>
<th>Quiz 1</th>
<th>Quiz 2</th>
<th>Poll</th>
<th>Session</th>
<th>Obs./Subj.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FA1</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>2</td>
<td>72/75</td>
</tr>
<tr>
<td>FA2</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>–</td>
<td>X</td>
<td>2</td>
<td>84/86</td>
</tr>
<tr>
<td>PA</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>2</td>
<td>77/85</td>
</tr>
<tr>
<td>QUIZ</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>X</td>
<td>X</td>
<td>–</td>
<td>1</td>
<td>30/30</td>
</tr>
<tr>
<td>FAR</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>4</td>
<td>141/150</td>
</tr>
<tr>
<td>TOTAL</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>11</td>
<td>404/426</td>
</tr>
</tbody>
</table>

Note: X denotes the task implemented by a treatment. Each treatment starts from the left to the right with the applicable task modules. Observations do not include the dropped subjects because of Problem 1. One session in FAR is in the reversed order of Prob-3 and Prob-2.

General announcement

This is an economic decision experiment supported by the National Research Fund. Please listen to and read the instruction carefully, and make your choices seriously. Depending on your choice and luck, you will have the chance to earn different amounts of money in the experiment. Payments are confidential, and no other participant will be informed about the amount you make. From now on and until the end of the experiment, any communication with other participants is not permitted. If you have

Chun-Lei Yang: cly@gate.sinica.edu.tw
Lan Yao: yao.lan@mail.shufe.edu.cn

Copyright © 2017 The Authors. Quantitative Economics. The Econometric Society. Licensed under the Creative Commons Attribution-NonCommercial License 3.0. Available at http://www.qeconomics.org. DOI: 10.3982/QE460
a question, please raise your hand and one of us will come to your desk to answer it.

The experiment comprises $N$ decision problems.¹ At the end of the experiment, you will make a random draw to select one from the $N$ decision problems in today’s experiment. We will pay you fully based on the realization of your decision in that problem.²

**Problem 1.**

**Making a choice between option A and urn B.**

[For FA1, FA2, and FAR]

♦ Urn B contains 5 red balls and 5 white balls.

[For PA]

♦ Urn B contains 8 red balls and 8 white balls.

[For all treatments]

♦ **Payoff rule for urn B:** Two balls are to be drawn from urn B with replacement. You get 50 yuan if the first ball drawn is red and nothing if it is white. Conversely, you get 50 yuan if the second ball drawn is white and nothing if it is red. You get paid the sum of money earned in the two draws.

---

¹$N = 3$ in the treatments PA, FA1, and FAR and $N = 4$ in the treatment FA2. Subjects were not informed that there are quizzes and a poll after the problems.

²In the FAR treatment, additional information that only 12 students were randomly paid was also announced to students.
### Decision sheet for Problem 1

<table>
<thead>
<tr>
<th>Situation</th>
<th>Payoff of Option A</th>
<th>Option A</th>
<th>Urn B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5 yuan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10 yuan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>15 yuan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>45 yuan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>50 yuan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>95 yuan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>100 yuan</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At the end of the experiment, if your payoff is decided by Problem 1, the process of realizing the payment is as follows.

- You are asked to randomly draw 1 of the 20 situations in option A, and your choice (A or B) in this situation will decide how you are paid. For example, if you draw situation 1 and your choice in situation 1 is option A (to accept fixed payoff of 5 yuan and give up drawing balls from urn B), then you will be paid 5 yuan immediately. If you draw situation 1 and your choice in situation 1 is urn B (to draw balls from urn B and give up fixed payoff of 5 yuan), then we will let you draw balls from urn B to realize your payoffs. In another example, if you draw situation 20 and your choice in situation 20 is option A (to accept fixed payoff of 100 yuan and give up drawing balls from urn B), then you will be paid 100 yuan immediately. If you draw situation 20 and your choice in situation 20 is urn B (to draw balls from urn B and give up fixed payoff of 100 yuan), then we will let you draw balls from urn B to realize your payoffs. If you draw other situations, your payoff will be realized in a similar method.

### Problem 2.

**Make a choice between urn B and urn C.**

[For FA1, FA2, and FAR]

- **Urn C** contains a mixture of 10 red and white balls. The number of red and white balls is unknown; it could be any number between 0 red balls (and 10 white balls) to 10 red balls (and 0 white balls).
Payoff rule for urn C: same as payoff rule for urn B.

Urn C contains a mixture of 16 red and white balls. The number of red and white balls is unknown and satisfies the constraints that either there are at least 6 more white balls than red balls or there are at least 6 more red balls than white balls in the urn, in other words $|\text{number of the red} - \text{number of the white}| \geq 6$.

Payoff rule for urn C: same as payoff rule for urn B.

Quiz: Is there any possibility that urn C contains 9 red balls and 7 white balls?
The answer: There is NOT, because of $|9 - 7| < 6$.

[For all treatments]

Decision sheet for Problem 2
Question: If you are asked to make a choice between urn B and urn C, which urn will you choose?

□ Urn B  □ Urn C

Problem 3.
Make a choice between urn B and urn D.

[For FA1, FA2, and FAR]

Urn D contains a mixture of 10 red and white balls. The number of red and white balls is determined as follows: one ticket is drawn from a bag containing 11 tickets with the numbers 0 to 10 written on them. The number written on the drawn ticket will determine the number of red balls in the urn. For example, if the number 3 is drawn, then there will be 3 red balls and 7 black balls in the urn.
♦ Payoff rule for urn D: same as payoff rule for urn B.

For PA

♦ Urn D contains a mixture of 16 red and white balls. The number of red and white balls is determined as follows: one ticket is drawn from a bag containing 12 tickets with the numbers 0 to 5, and 11 to 16 written on them. The number written on the drawn ticket will determine the number of red balls in the urn. For example, if the number 3 is drawn, then there will be 3 red balls and 13 white balls in the urn.

♦ Payoff rule for urn D: same as payoff rule for urn B.

[For all treatments]

Decision sheet for Problem 3

Question: If you are asked to make a choice between urn B and urn D, which urn will you choose?

☐ Urn B  ☐ Urn D

Problem 4.

Make a choice between urn B and urn E.

♦ Urn E contains 9 red balls and 1 white ball.
The payoff rule for urn E is the same as for urn B.

Decision sheet for Problem 4

Question: If you are asked to make a choice between urn B and urn E, which urn will you choose?

- [ ] Urn B
- [ ] Urn E

Quiz 1. Urn A contains a mixture of 10 red and white balls. The number of red balls in urn A is denoted by \( n \), and accordingly, an urn containing \( n \) red balls and \( 10 - n \) white balls is denoted by \( A(n) \).

Suppose we play the following game of drawing balls from urn \( A(n) \). The payoff rule is as follows. Two balls are to be drawn consecutively from urn \( A(n) \) with replacement. You get 20 yuan if the first ball drawn is red and nothing if it is white. Conversely, you get 20 yuan if the second ball drawn is white and nothing if it is red. You get paid the sum of money earned in the two draws. Following the rule, for any urn like \( A(n) \), one of the payoffs 0, 20, or 40 will be realized under a certain probability.

Suppose there are 6 urns \( A(0), A(1), A(2), A(3), A(4), \) and \( A(5) \), as listed below, which contain \( n = 0, 1, 2, 3, 4, \) and 5 red balls in the 10 ball urn \( A(n) \), respectively. The 6 profiles of probabilities for respective payoffs are listed below. Please find the profile of probabilities that fits the urn correctly.
In Problem 1, you earn money by correctly matching the profile of probabilities with the fitting urn. You earn 1 yuan by making 1 correct match. Besides, you get a bonus of 4 yuan, in other words a total of 10 yuan, if you make all 6 matches correctly.

<table>
<thead>
<tr>
<th>Probability of 0 yuan</th>
<th>Probability of 40 yuan</th>
<th>Probability of 20 yuan</th>
<th>Your choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Question 2</td>
<td>0.25</td>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>Question 3</td>
<td>0.16</td>
<td>0.16</td>
<td>0.68</td>
</tr>
<tr>
<td>Question 4</td>
<td>0.24</td>
<td>0.24</td>
<td>0.52</td>
</tr>
<tr>
<td>Question 5</td>
<td>0.09</td>
<td>0.09</td>
<td>0.82</td>
</tr>
<tr>
<td>Question 6</td>
<td>0.21</td>
<td>0.21</td>
<td>0.58</td>
</tr>
</tbody>
</table>

**Quiz 2.** Suppose you have four urns A(1), A(3), A(5), and A(9), as described in Problem 1. The rule of payoffs for drawing balls is exactly the same as the one we used in Problem 1. Which urn would you prefer most to draw balls from? Please rank the four urns from the highest (the most preferred) to the lowest (the least preferred), and fill the four numbers 1, 3, 5, 9 into ( ), respectively. If your preference for any two urns is the same, please use “=” to connect the urns.

Because of the time constraint, three of you will be randomly selected for payment. Once selected, you will randomly draw two urns out of the four. Then we will let you draw balls from the urn that you revealed to like better. We will pay you fully based on the realized payoff resulting from your drawings. If you are indifferent between the randomly selected two urns, indicated with “=” on your decision sheet, the tie will be broken randomly for you. Please rank the four urns:

The most preferred: A( ) A( ) A( ) A( )

The least preferred

**Poll.** The following sheet was given to those who chose **Urn B:**

The reason(s) that you have chosen Urn B in Problem 2 is (are) as follows:

1. I think Urn C is associated with more uncertain information than Urn B is.
2. I think that I have more chances to get 100 yuan by choosing Urn B, and I do not care about the increased chances of getting zero.
3. Neither of options 1 and 2. I think that the monetary values of both urns are the same. I randomly chose one of the two.
4. None of the above.

If you chose option 4, please write down the reasons you had for choosing Urn B.
The following sheet was given to those who chose **Urn C**:
The reason(s) that you have chosen “Urn C” in Problem 2 is (are) as follows:

1. I do not care that Urn C is associated with more uncertain information than Urn B is.
2. I think that I have more chances to get a sure payoff at 50 yuan by choosing Urn C.
3. Neither of options 1 and 2. I think that the monetary values of both urns are the same. I randomly chose one of the two.
4. None of the above.

If you chose option 4, please write down the reasons you held for choosing Urn C.

---

**Gender**

- [ ] Male
- [ ] Female

### APPENDIX B: FURTHER DATA

**B.1 Additional data for treatment FAR**

As a pilot study, we ran three sessions with a total of 109 subjects for FA1, with only about 10% of subjects randomly chosen for payment, called FAR treatment. Because of non-monotone choices in Problem 1, nine subjects were excluded from the data analyses below.

Note that we also ran an additional session with 41 subjects with the alternate order of Problems 1, 3, and 2 to control for potential order effects. A chi-square test confirms no existence of order effects, with $p = 0.831$, $0.640$, and $0.759$ for Problems 2, 3, and both combined, in comparison with the order used in FA1. Note that Arló-Costa and Helzner (2009) have an order-independence finding similar to ours.

Table B.1 presents the violation rate in Problems 2 and 3 separately by risk attitude. Significant difference between FA and FAR is observed in Problem 2 over risk-seeking

<table>
<thead>
<tr>
<th></th>
<th>Problem 2</th>
<th>Problem 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obs.</td>
<td>Violation (%)</td>
</tr>
<tr>
<td>FAR Risk averse$^a$</td>
<td>38</td>
<td>55.26</td>
</tr>
<tr>
<td>Risk seeking$^b$</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>Risk neutral$^c$</td>
<td>32</td>
<td>0</td>
</tr>
<tr>
<td>All pooled</td>
<td>100</td>
<td>39</td>
</tr>
</tbody>
</table>

*Note:* Pairs of numbers in square brackets [−, −] refer to 95% confidence intervals defined by percentage. All risk-neutral DMs count as nonviolation in the “all pooled” category.

$^a$The numbers of choice B are 21 and 14 in Problems 2 and 3, respectively.

$^b$The numbers of choice B are 15 and 11 in Problems 2 and 3, respectively.

$^c$The number of choice B is 25 and 18 in Problems 2 and 3, respectively.
Supplementary Material  

Testing ambiguity theories  

DMs \((p = 0.079;\) two-sample test of proportions) and in Problem 3 over risk-averse DMs \((p = 0.0433;\) two-sample test of proportions).

**B.2 Other data**

![Figure B.1. Distribution of CE values in Problem 1.](image)

**Table B.2. Raw summary data by risk attitude in Problems 2 and 3.**

<table>
<thead>
<tr>
<th>Treatment</th>
<th>N</th>
<th>BB</th>
<th>CD</th>
<th>BD</th>
<th>CB</th>
<th>BB</th>
<th>CD</th>
<th>BD</th>
<th>CB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PA</td>
<td>22</td>
<td>3</td>
<td>13</td>
<td>3</td>
<td>3</td>
<td>13.64</td>
<td>59.09</td>
<td>13.64</td>
<td>13.64</td>
</tr>
<tr>
<td>FA1</td>
<td>25</td>
<td>5</td>
<td>10</td>
<td>6</td>
<td>4</td>
<td>20.00</td>
<td>40.00</td>
<td>24.00</td>
<td>16.00</td>
</tr>
<tr>
<td>FA2</td>
<td>40</td>
<td>6</td>
<td>18</td>
<td>13</td>
<td>3</td>
<td>15.00</td>
<td>45.00</td>
<td>32.50</td>
<td>7.50</td>
</tr>
<tr>
<td>FAR</td>
<td>38</td>
<td>11</td>
<td>10</td>
<td>10</td>
<td>7</td>
<td>28.95</td>
<td>26.32</td>
<td>26.32</td>
<td>18.42</td>
</tr>
<tr>
<td>Risk neutral</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PA</td>
<td>33</td>
<td>9</td>
<td>16</td>
<td>6</td>
<td>2</td>
<td>27.27</td>
<td>48.48</td>
<td>18.18</td>
<td>6.06</td>
</tr>
<tr>
<td>FA1</td>
<td>25</td>
<td>8</td>
<td>9</td>
<td>6</td>
<td>2</td>
<td>32.00</td>
<td>36.00</td>
<td>24.00</td>
<td>8.00</td>
</tr>
<tr>
<td>FA2*</td>
<td>18</td>
<td>4</td>
<td>11</td>
<td>3</td>
<td>0</td>
<td>22.22</td>
<td>61.11</td>
<td>16.67</td>
<td>0.00</td>
</tr>
<tr>
<td>FAR</td>
<td>32</td>
<td>9</td>
<td>7</td>
<td>10</td>
<td>6</td>
<td>28.13</td>
<td>21.88</td>
<td>31.25</td>
<td>18.75</td>
</tr>
<tr>
<td>Risk seeking</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PA*</td>
<td>22</td>
<td>9</td>
<td>9</td>
<td>2</td>
<td>2</td>
<td>40.91</td>
<td>40.91</td>
<td>9.10</td>
<td>9.10</td>
</tr>
<tr>
<td>FA1</td>
<td>22</td>
<td>9</td>
<td>7</td>
<td>4</td>
<td>2</td>
<td>40.91</td>
<td>31.82</td>
<td>18.18</td>
<td>9.10</td>
</tr>
<tr>
<td>FA2</td>
<td>26</td>
<td>10</td>
<td>8</td>
<td>6</td>
<td>2</td>
<td>38.46</td>
<td>30.77</td>
<td>23.08</td>
<td>7.69</td>
</tr>
<tr>
<td>FAR</td>
<td>30</td>
<td>7</td>
<td>13</td>
<td>5</td>
<td>5</td>
<td>23.33</td>
<td>43.33</td>
<td>16.67</td>
<td>16.67</td>
</tr>
<tr>
<td>Total</td>
<td>333</td>
<td>90</td>
<td>131</td>
<td>74</td>
<td>38</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

**Note:** The formula for 95% confidence interval (CI) is \([\bar{x} - 1.96 \times \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \times \frac{\sigma}{\sqrt{n}}]\), where \(n\) denotes the observations in one of the categories BB/CD/BD/CB in the table, \(\bar{x} = n/N\), and \(\sigma\) is the standard deviation. The 95% CI lower bound for inconsistent types (BD and CB) as stated in Hypothesis 2 is at least 10% for all but the risk-seeking class in PA (4%) and risk-neutral class in FA2 (4%), indicated by *.
B.3 Problem 4 in treatment FA2

As illustrated in Table A.1, FA2 is exactly FA1 with three further parts attached, to collect further relevant information. In Problem 4, we found that 92.5% (37 out of 40), 94.44% (17 out of 18), and 50% (13 out of 26) of risk-averse, risk-neutral, and risk-seeking subjects, respectively, chose E. Risk-averse subjects have significantly stronger preference for E than risk-seeking ones (\( p = 0.000 \), \( \chi^2 \) test). This suggests that our Problem 1 as a device to elicit the attitude of risk aversion is generally consistent with Problem 4, but no clear consistency is found for risk-seeking subjects in Problem 1. Regardless of risk attitude, we find that the switch rates are \( \frac{BE}{BE + BB} = 78.57\% \) ([63.19, 89.70]) and \( \frac{CB}{CB + CE} = 19.05\% \) ([8.60, 34.12]), significantly different (\( p = 0.0000 \); two-sample test of proportions). This is consistent with our new poll results that many chose B to avoid information uncertainty, while preference for C was more likely linked to statistical calculation.

B.4 Poll in treatment FA2

After subjects finished all four problems and the comprehension test, they were reminded of their choices in Problem 2, and were required to indicate their reason for making the choice B or C. Table B.3 lists the distribution of the reasons by risk attitudes.

B.5 The comprehension tests

B.5.1 Motivation and design

At the core of our study here is the double-draw, alternate-color-win lottery design associated with choices B, C, and D. Since it is novel in the literature, it is legitimate to question whether subjects may not have fully understood its statistical implications and whether they would behave consistently when comparing simple objective lotteries such as choice B.

To test for comprehension, we conducted an additional treatment Quiz with only the instructions Quiz 1 and Quiz 2, and also tested Quiz 1 in FA2 for exploring the relation between decisions and comprehension. In the treatment Quiz, they first faced Quiz 1, where they were asked to match outcome distributions (those in Table 1) with urns of 6
Table B.3. Raw summary data in FA2 by risk attitude for joint decisions in Problems 2 and 4.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>BB</th>
<th>CE</th>
<th>BE</th>
<th>CB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk averse</td>
<td>40</td>
<td>2</td>
<td>20</td>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>Risk neutral</td>
<td>18</td>
<td>0</td>
<td>10</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>Risk seeking</td>
<td>26</td>
<td>7</td>
<td>4</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

Table B.4. Summary of reasoning for choice B in questionnaire.

<table>
<thead>
<tr>
<th>Options:</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>1/2</th>
<th>1/3</th>
<th>1/4</th>
<th>2/3</th>
<th>Sum</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>1/2</th>
<th>1/3</th>
<th>1/4</th>
<th>2/3</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>CE &lt; 50</td>
<td>10</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>19</td>
<td>3</td>
<td>13</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>CE = 50</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>1</td>
<td>7</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>CE &gt; 50</td>
<td>6</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>16</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

Note: --/-- denote subject’s choosing two options.

different color compositions in the \( N = 5 \) condition. Correct answers are rewarded with money payments. After handing in their Quiz 1 decisions, they faced Quiz 2, the task of revealing their preferences over four different objective urns with our novel double-draw rule. And 3 out of a total of 30 participants were randomly chosen to receive monetary payment according to their decisions. The session lasted 15 min excluding time for payment. Average payoff was 12 yuan. In the treatment FA2, Quiz 1 was implemented right after Problem 4. A total of 84 subjects were paid with an average payoff of 9.8 yuan.

B.5.2 The results

In treatment Quiz In Quiz 1, 28 out of 30 subjects answered all six questions correctly. The remaining two subjects made two mistakes each. The observed 6.67% (= 2/30, [0.82, 22.07]) failure ratio is similar to 6.88% (= 11/160, [3.48, 11.97]), which is the ratio of people with anomalous decisions in Problem 1 of our PA and FA treatments. The two-sample test of proportions yields \( p = 0.4835 \). Thus, we conclude that aside from regular noise there is no reason to believe that subjects in our study had an abnormal comprehension issue jeopardizing our main conclusions in this paper.

In Quiz 2, 24 out of 30 subjects ranked the four urns in a standard manner that made them clearly identifiable as either risk averse (12 obs.) with \( A(1) = A(9) > A(3) > A(5) \), risk seeking (10 obs.) with \( A(5) > A(3) > A(1) = A(9) \), or risk neutral (2 obs.) with \( A(5) = A(3) = A(1) = A(9) \). Two had the ranking \( A(5) > A(1) = A(9) > A(3) \), which can be viewed as consistent with prospect theory with an unusual reference point. The remaining four displayed the rankings of \( A(5) > A(9) > A(3) > A(1) \), \( A(5) > A(3) > A(1) > A(9) \), and \( A(9) > A(5) > A(1) > A(3) \). They failed to realize that statistically \( A(1) = A(9) \). All in all, 86.67% (= 26/30, [69.28, 96.24]) of subjects could be considered consistent with standard theories on decision over simple lotteries.
In treatment FA2 In Quiz 1, 79 out of 84 subjects answered all 6 questions correctly. Among the 5 with mistakes, 3 subjects made 3 and 2 subjects made 2 mistakes, respectively. The observed 5.95% failure ratio is similar to our quiz treatment.

Appendix C: Miscellaneous

C.1 Proof of Lemma 1

Lemma 1. For any combination of decisions in Problems 1 and 2, there is a weighting function \( w \) under the CEU model that rationalizes them.

Proof. Note that CEU reduces to standard expected utility when there is no ambiguity, in the same manner SEU/MEU do. Recall that with \( x_i \) \( \leq x_i \) \( \in S \) denoting the event with payoffs no higher than \( x_i \), CEU has the general form

\[
\text{CEU}(f) = \sum_{i=1}^{n} [w([x \geq x_i]) - w([x \geq x_{i-1}])]u(x_i).
\]  

(A.1)

Now, letting \( w_{100} = w([x = 100]) = w([x = 100]) = w(rw), w_0 = w([x = 0]) = w(ur), \) and \( w_5 = w([x = 100] \cup [x = 50]) = w(rw \cup ww \cup rr) \), we obtain the expression for the ambiguous choice C:

\[
\text{CEU}(C) = w_{100}u(100) + [w_5^c - w_{100}]u(50) + [1 - w_5^c]u(0).
\]  

(A.2)

Use the normalization \( u(0) = 0 \). Recalling that CEU\((B) = 0.25u(100) + 0.5u(50) \), with straightforward resorting of terms we obtain

\[
\text{CEU}(B) - \text{CEU}(C) = [0.25 - w_{100}] [u(100) - u(50)] + [0.75 - w_5^c]u(50).
\]  

(A.3)

Let \( \bar{w} := \frac{0.25 - w_{100}}{w_{100} - 0.75} \) and \( \eta := \frac{u(50)}{u(100) - u(50)} \). Then we have CEU\((B) - \text{CEU}(C) > 0 \ iff\ \) \( (\bar{w} - \eta)(0.75 - w_5^c) > 0 \). Note that \( \eta \) is an alternative definition of risk attitudes as \( \eta \leq 1 \Leftrightarrow \text{CE} \geq 50 \). It is easy to show that \( \bar{w} \) can take any value in \([0, \infty]\), while ensuring \([0.75 - w_5^c] > 0 \), by manipulating \( w_{100} \) and \( w_5^c \) accordingly under the restriction of \( 0.25 \leq w_{100} \leq w_5^c < 0.75 \). Thus, for any \( \eta > 0 \) and any decision in Problem 2, there is an admissible weighting function \( w \) that justifies this decision under CEU.

\( \square \)

Discussion

Note that an alternative extension is to integrate the idea of CEU for ambiguity with that of RDU by Quiggin (1982) for decision under risks, as for example, by Tversky and Kahneman (1992) and Wakker (2008). Unlike CEU, the weighting takes the form of an increasing function \( w : [0, 1] \rightarrow [0, 1] \) that transforms probabilities, instead of a capacity defined for events. Parallel to (A.1), we have the form

\[
\text{RDU}(f) = \sum_{i=1}^{n} [w(p(x \geq x_i)) - w(p(x \geq x_{i-1}))]u(x_i).
\]  

(A.4)
Supplementary Material

Testing ambiguity theories 13

We assume that the DM uses the same weighting function \( w : [0, 1] \rightarrow [0, 1] \) to evaluate all three prospects of B, C, and D. Note that it is not necessary or natural to assume the exact same probability weighting function for both objective risk and ambiguity, as conceptually they reflect different sources of uncertainty.\(^3\) However, it is obviously technically more challenging to prove Lemma 1A below under this assumption.

**Lemma 1A.** For any combination of decisions in Problems 1, 2, and 3, there is a weighting function \( w \) under the RDU model that rationalizes them.

**Proof.** In our design, (A.4) takes the specific form

\[
\text{RDU}(C) = w(\pi(100))u(100) + \left[w(\pi(100) + \pi(50)) - w(\pi(100))\right]u(50) + \left[1 - w(\pi(100) + \pi(50))\right]u(0).
\]

(A.5)

Note that, given our design, any feasible lottery \( \pi \) is uniquely represented by some \( \theta = \pi(0) = \pi(100) \in [0, 0.25] \). Let us first consider the FA case. By normalization \( u(0) = 0 \), (7b) can be reformulated as

\[
\text{RDU}(C) = w(\theta)u(100) + \left[w(1 - \theta) - w(\theta)\right]u(50)
\]

(A.6)

for some \( \theta \in [0, 0.25] \). Consequently, we have

\[
\text{RDU}(B) - \text{RDU}(C) = \left[w(0.25) - w(\theta)\right]\left[u(100) - u(50)\right] + \left[w(0.75) - w(1 - \theta)\right]u(50).
\]

(A.7)

Let \( W_\theta := \frac{w(0.25) - w(\theta)}{w(1 - \theta) - w(0.75)} \), \( \eta := \frac{u(50)}{u(100) - u(50)} \). The term \( W_\theta \) measures the relative curvature steepness of \( w \) from low to high probabilities, and \( \eta \) is an alternative definition of risk attitudes as \( \eta \leq 1 \Leftrightarrow \text{CE} \leq 50. \) Then \( \text{RDU}(B) - \text{RDU}(C) > 0 \) iff \( W_\theta > \eta \). Similarly in Problem 3, \( \text{RDU}(B) - \text{RDU}(D) > 0 \) iff \( W_{0.15} > \eta \), as \( P^{\text{PA}}_D(x = 0) = 0.15 \) under ROCL, where \( P^{\text{FA}}_D \in \Delta(X) \) denotes the reduced lottery for choice D in FA. Table C.1 lists all possible choice combinations in Problems 2 and 3 given these two parameters.\(^4\) It is straightforward to check that proper parameters for \( u \) and \( w \) can be found for each behavior profile to make it consistent under RDU.

For example, suppose \( \eta > 1 \), which is equivalent to \( \text{CE} < 50 \) in Problem 1. Then the subject prefers B to D in Problem 3 if \( W_{0.15} \) is sufficiently greater than 1, which means he is sufficiently more sensitive to changes in small-probability events than those in large-probability ones. If \( W_{0.15} \leq 1 \), however, he prefers D to B. For Problem 2, CEU has the additional maneuver room in the form of picking any \( \theta \in [0, 0.25] \). Note that the proof is also valid for PA by replacing \( \theta = 0.15 \) with \( \theta = P^{\text{PA}}_D(x = 0) \) in the term \( W_\theta \) in the proof.

\(^3\)See Abdellaoui, Baillon, Placido, and Wakker (2011) for such arguments.

\(^4\)Note that the case \( \eta = 1 \) implies risk neutrality and any decision combination for Problems 2 and 3 is permitted; therefore, this is omitted from Table C.1.
Table C.1. Choice combinations given decision weight and risk attitude.

<table>
<thead>
<tr>
<th></th>
<th>$\eta &gt; 1$</th>
<th>$\eta &lt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B &gt; C</td>
<td>C &gt; B</td>
</tr>
<tr>
<td>B &gt; D</td>
<td>$W_\theta &gt; \eta$</td>
<td>$W_\theta &lt; \eta$</td>
</tr>
<tr>
<td></td>
<td>$W_{0.15} &gt; \eta$</td>
<td>$W_{0.15} &gt; \eta$</td>
</tr>
<tr>
<td>D &gt; B</td>
<td>$W_\theta &gt; \eta$</td>
<td>$W_\theta &lt; \eta$</td>
</tr>
<tr>
<td></td>
<td>$W_{0.15} &lt; \eta$</td>
<td>$W_{0.15} &lt; \eta$</td>
</tr>
</tbody>
</table>

C.2 Elicitation of risk attitudes: BDM versus MPL

As a methodological note, most experiments on the Ellsberg paradox used to use the standard BDM mechanism in which the subject is asked to state a minimum certainty-equivalent selling price to give up the lottery he has been endowed with. This auction procedure provides a formal incentive for the subject to truthfully reveal their CE of the lottery. However, in its original form it appears hard for some subjects to comprehend. In a pilot study where subjects were to make binary decisions first and to reveal a BDM price for their preferred choices second, 26 out of 89 subjects (29.2%) displayed inconsistent evaluations. More specifically, aside from the Problems 2 and 3 binary decisions as in this paper, subjects in the pilot faced another choice between urn B and an urn with equal likelihood of either 3 or 7 red balls. After the binary decision is made, the subjects have to announce their selling price for their preferred prospect. The inconsistency comes from the fact that they evaluate the same choice with different values in different problems. Additionally, Stecher, Shields, and Dickhaut (2011) also studied an Ellsberg-type problem by making a choice between risk and ambiguity accompanied by the standard BDM mechanism for both prospects. Among the 60 subjects, only 40% (24 subjects) had clear, consistent decisions on choice and price, in other words, to choose the prospect with a higher BDM price. About 23% had a clear conflict between choice and price, and 37% of subjects priced both prospects the same but preferred one of them.5

Thus, we choose to use a modified version of the BDM mechanism—multiple price list (MPL)—to elicit subjects’ risk attitude. The MPL is a relatively simple procedure for eliciting values from a subject and has been widely used in experimental economics. First, instead of asking subjects to reveal a single selling price, we ask them to make 20 simple binary decisions, where a randomizing device determines which of them is realized. Compared with the standard BDM, the attraction is not only how easy it is to explain to the subjects, but also the fact that if the subject believes that his responses have no effect on which row is chosen, then the task collapses to a binary choice in which the subject gets what he wants if he answers truthfully. Andersen, Harrison, Lau, and Rutström (2006) studied the properties of the MPL method by a series of experimental

5Stecher, Shields, and Dickhaut (2011), for example, made their subjects take a quiz on the procedure and reviewed them with the experimenter before being admitted into the experiment, to minimize the problem associated with difficulties comprehending the experimental procedure.
Supplementary Material  Testing ambiguity theories  15
designs. Also, Sapienza, Zingales, and Maestripieri (2009) use a similarly modified BDM method, which they consider an adaptation from the mechanism used in Holt and Laury (2002). The elicitation process of the certainty equivalent associated with a bet is also one of the basic steps in Abdellaoui et al. (2011) for elicitation of risk and ambiguity attitudes.6

In addition, the binary decision in our modified BDM is similar in shape to the subsequent parts of the experiments, which facilitates the comparison to ambiguity attitudes. Weber and Johnson (2008) argue that when measuring levels of risk taking with the objective of predicting risk taking in other situations, it is important to use a decision task that is as similar as possible to the situation for which behavior is being predicted. To quote Harrison and Rutström (2008), “For the instrument to elicit truthful responses, the experimenter must ensure that the subject realizes that the choice of a buying price does not depend on the stated selling price. If there is reason to suspect that subjects do not understand this independence, the use of physical randomizing devices (e.g., a die or bingo cages) may mitigate such strategic thinking.” And the 29.2% inconsistency rate encountered in a pilot to the present study using the original BDM fittingly echoes this reasoning.

When comparing the distribution of risk attitudes in the present study to some related studies in the literature, we find quite consistent results. In our experiments for PA (FA), there are 28.57% (34.72%) risk-averse, 42.86% (34.72%) risk-neutral, and 28.57% (30.56%) risk-seeking subjects, respectively. In comparison, using standard BDM, Halevy (2007) has for the small (big) incentive treatment 31.73% (44.74%) risk-averse, 30.77% (44.74%) risk-neutral, and 37.5% (10.52%) risk-seeking subjects, respectively.

References


6Also see Trautmann, Vieider, and Wakker (2011) for further comparisons between BDM and certainty equivalent measurements under risk and ambiguity.


Co-editor Rosa L. Matzkin handled this manuscript.

Manuscript received 28 May, 2014; final version accepted 5 February, 2016; available online 15 March, 2016.