REGIME-DEPENDENT EFFECTS OF UNCERTAINTY SHOCKS: A STRUCTURAL INTERPRETATION

STÉPHANE LHUISSIER* AND FABIEN TRIPIER**

Abstract. Using a Markov-switching VAR, we show that the effects of uncertainty shocks on output are four times higher in a regime of economic distress than in a tranquil regime. We then provide a structural interpretation of these facts. To do so, we develop a business cycle model in which agents are aware of the possibility of regime changes when forming expectations. The model is estimated using a Bayesian minimum distance estimator that minimizes, over the set of structural parameters, the distance between the regime-switching VAR-based impulse response functions and those implied by the model. Our results point to worsening credit-market conditions that amplify shocks during distress periods. Finally, we show that the expectation effect of regime switching in financial conditions is an important component of the financial accelerator mechanism. If agents are more pessimistic about future financial conditions, then the output effects of shocks are amplified.

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*Banque de France, 31, Rue Croix des Petits Champs, DGSEI-DEMFI-POMONE 41-1422, 75049 Paris Cedex 01, FRANCE (Email: stephane.lhuissier@hotmail.com; URL: http://www.stephanelhuissier.eu).
The views expressed in this paper are those of the authors and should under no circumstances be interpreted as reflecting those of the Banque de France or the Eurosystem. **Université Paris-Saclay, Univ Evry, EPEE, 91025, Evry-Courcouronnes, France & CEPII. (Email: fabien.tripier@univ-evry.fr). Fabien Tripier acknowledges the financial support of the Labex MME-DII's grant ANR-11-LABX-0023-01.

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I. Introduction

It has been well documented that greater uncertainty reduces aggregate activity, leading to higher unemployment and lower investment and output.[1] Recent empirical studies have also emphasized highly nonlinear effects, depending on the state of the economy; the adverse effects of uncertainty shocks are greater in periods of economic distress than in tranquil periods.[2] However, little is known about the structural factors that account for these changes, as inference with nonlinear relationships presents econometric challenges within a quantitative general equilibrium framework.

The objective of this paper is to close part of this gap by exploring, through a novel econometric estimation, potential changes in the underlying structure of the economy that could explain such a nonlinearity. Disentangling these causes is important for understanding the extent to which economic activity responds to future uncertainty shocks as well as the role that policy can play in mitigating those adverse effects.

We first reproduce the empirical evidence of highly nonlinear effects within a Markov-switching structural vector autoregression (MS-SVAR) framework. We use U.S. quarterly data and include GDP growth, a measure of uncertainty (i.e., the VIX index), and a credit spread. The model identifies two distinct regimes. The first was seen in nearly all the years during episodes of high inflationary pressure in the 1970s and 1980s, during the serious turbulence that marked the 2001-2003 period (including the 9/11 terrorist attacks, the dot-com bubble, and the corporate scandals), and during the global financial crisis. The second covers periods of tranquility. We show that under the first regime, the adverse output effects of an increase in uncertainty appear to be four times higher than those under the second regime.

We then propose a potential explanation for these differential responses based on agency problems associated with financial intermediation. To do so, we construct and estimate a Markov-switching dynamic stochastic general equilibrium (MS-DSGE) model with financial

[1]See, among others, Bloom (2009), Stock and Watson (2012), Glover and Levine (2015), Leduc and Liu (2016), Basu and Bundick (2017), and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018).

[2]See Caggiano, Castelnuovo, and Groshenny (2014), Caggiano, Castelnuovo, and Nodari (2017), and Alessandri and Mumtaz (2019).

frictions and uncertainty shocks. Our framework is an extension of the model with asymmetric information between borrowers and lenders and costly monitoring proposed by Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2014) that allows key macroeconomic and financial parameters of the model to evolve over time according to a Markov-switching process. Our empirical approach is analogous to the impulse response matching approach used by Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005), except that we are estimating the parameters to fit our regime-dependent impulse responses from a MS-SVAR, as opposed to impulse responses from a constant-parameter SVAR. To the best of our knowledge, our paper represents the first attempt to estimate a medium-scale MS-DSGE model by matching the MS-SVAR-implied impulse responses to those produced by the MS-DSGE model. We believe our MS-SVAR-implied impulse responses approach is a promising tool for inferring MS-DSGE models and can be seen as an alternative to the full Bayesian approach notably implemented by Liu, Waggoner, and Zha (2011) and Bianchi (2013).

Our estimates point out greater problems of asymmetric information in the distress regime, which manifest themselves through a higher cost of monitoring defaulting borrowers than in the normal regime. As a result, the optimal financial contract typically implies greater deadweight losses and an external finance premium that becomes more (negatively) sensitive to the borrower's net worth. In other words, the financial accelerator mechanism is stronger during the distress regime. It then becomes straightforward to understand why the response of the economy to uncertainty shocks[3] differs across regimes. Under both regimes, when uncertainty increases, banks protect themselves by raising the interest rate charged on loans to firms (i.e., external finance premium), as there are more low-productivity firms—and more high-productivity firms, but this does not benefit banks—and thus more default risks. From this follows a decline in demand for capital, and then in net worth, investment spending and economic activity. In distress periods, higher monitoring costs cause banks to charge higher interest rates and firms to make larger cuts to their investment projects, implying a larger and longer-lasting decline in economic activity than in normal times.

[3]Christiano, Motto, and Rostagno (2014) refer to risk shocks rather than uncertainty shocks. As stated in Bloom (2014), uncertainty is "a stand-in for a mixture of risk and uncertainty" in this literature. In our model, as in Christiano, Motto, and Rostagno (2014), uncertainty shocks shift the variance of the cross-sectional distribution of idiosyncratic productivity shocks.

The key insight of our MS-DSGE model is that variations in the MS-SVAR dynamics of the effects of uncertainty shocks have important effects on rational agents' expectation formation in the MS-DSGE model. Our estimates are based on the fact that agents are aware of the possibility of regime switches in the dynamics. That is, our MS-SVAR-based impulse response matching approach takes into account the fact that all agents in the MS-DSGE model know the transition probabilities and use them when forming expectations.

Under these circumstances, in any given regime, agents anticipate that uncertainty shocks may be accompanied by a switch to the other regime, considerably altering the macroeconomic outcomes. We consider how these expectation effects, using the terminology of Liu, Waggoner, and Zha (2009)[4], on a particular regime affect the equilibrium in the other regime. During tranquil periods, characterized by a small degree of agency problems, agents may expect that the economy will move to the distress regime. If they are overly pessimistic, that is, they overestimate the probability of the regime switching to more severe financial conditions in the future, the contractionary effects of uncertainty shocks on aggregate activity will be amplified. Conversely, overly optimistic behavior dampens these negative effects. As a result, the expectation effects of regime shifts related to financial conditions are part of the financial accelerator mechanism.

This paper proceeds as follows. Section II describes our contributions to the literature. To illustrate the possibility of nonlinearity between uncertainty and the macroeconomy, Section III provides empirical insights into how different the impact of uncertainty on aggregate activity is between distress and non-distress periods. Section IV interprets these differences in terms of an estimated DSGE model with financial frictions, in which agents form expectations regarding possible changes in the economy and investigates the expectation effects of regime switching on the degree of financial frictions. Section V concludes.

II. Literature review

This paper is related to an increasing literature that examines how uncertainty manifests itself and what its effects are on the rest of the economy.

[4]Liu, Waggoner, and Zha (2009) originally defined the expectation effects for monetary policy as "the difference between equilibrium outcomes from a model that ignores probabilistic shifts in the future policy regime and those from a model that takes into account such expected changes in the regime".

Focusing on the United States, Bloom (2009), Stock and Watson (2012), Bekaert, Hoerova, and Duca (2013), Glover and Levine (2015), Leduc and Liu (2016), Basu and Bundick (2017), Creal and Wu (2017) and Ferrara and Guérin (2018) employ the "constant parameters" approach to quantify the role of uncertainty in business cycle fluctuations. In particular, all of these studies adopt linear SVARs and find a significant and long-lasting decrease in aggregate activity after a positive uncertainty shock. Empirical studies have been increasingly interested in the time-varying effects of these shocks, as events of high uncertainty have not always seemed to spill over into the economy.[5]

Mumtaz and Theodoridis (2018) extend the standard approach by allowing time-varying parameters in SVARs. They emphasize the importance of taking shifts in the generation of uncertainty shocks into account. They show that in particular, the impact of uncertainty shocks on aggregate activity has declined over time. However, the limited ability of this paper to study episodes of distress, as considered herein, lies in the methodology itself—a model with smooth and drifting coefficients seems to be less well suited to capturing the rapid shifts in the behavior of the data observed during distress periods. Economic or financial crises are well known for hitting the economy instantaneously, which favors models that allow for abrupt changes such as Markov-switching models. Therefore, we follow Sims and Zha (2006) and estimate an MS-SVAR with Bayesian methods. Hubrich and Tetlow (2015) and Lhuissier (2017) also consider a MS-SVAR framework to capture regime switching in macroeconomic time series during distress periods.

Employing an alternative regime-switching method (i.e., a threshold VAR model), Caggiano, Castelnuovo, and Groshenny (2014), Caggiano, Castelnuovo, and Nodari (2017), and Alessandri and Mumtaz (2019) show that the real effects of uncertainty shocks strongly depend on the state of the economy. In particular, Alessandri and Mumtaz (2019) show that the effects depend on the state of financial markets and estimate that the impact on output is five times larger in periods of financial stress than in tranquil periods, while Caggiano, Castelnuovo, and Groshenny (2014) and Caggiano, Castelnuovo, and Nodari (2017) capture the recession and expansion phases and show that uncertainty shocks are substantially more costly during recessions than during expansions. Our approach clearly differs since we assign probabilities

[5]Bloom (2009) documents a variety of events that generate significant uncertainty about the future, but they are not always associated with a large decline in output.

to events; therefore, we avoid assuming that the probability of a regime switch is either one or zero. Moreover, estimating these probabilities is essential for analyzing the importance of the expectation effects of regime shifts in the equilibrium dynamics of our MS-DSGE model, and therefore, in the transmission mechanism of uncertainty shocks to the aggregate economy.

Our analysis is related to a growing body of evidence that documents the interactions between uncertainty and financial conditions within an equilibrium business cycle framework—notable examples are Christiano, Motto, and Rostagno (2014), Gilchrist, Sim, and Zakrajšek (2014), Bloom, Alfaro, and Lin (2019), Brand, Isoré, and Tripier (2019), and Arellano, Bai, and Kehoe (2019). More specifically, our framework closely follows Christiano, Motto, and Rostagno (2014), who investigate the real role of uncertainty shocks in the context of the financial accelerator model initially developed by Bernanke, Gertler, and Gilchrist (1999). Note, however, that the severity of the agency problems (i.e., monitoring costs) remains unchanged over time within their framework. Levin, Natalucci, and Zakrajšek (2004), and more recently, Lindé, Smets, and Wouters (2016) and Fuentes-Albero (2019) make it time-varying without, however, investigating the macroeconomic implications of uncertainty shocks or the role of expectation effects of regime shifts in financial frictions in shaping the macroeconomic outcomes.

A few other papers in the literature study the origins and effects of uncertainty shocks in empirical medium-scale DSGE models. In particular, Bianchi, Kung, and Tirskikh (2018) examine the effects of demand-side and supply-side uncertainty through multiple endogenous risk propagation channels. We differ from this paper in that we (i) focus our structural analysis on the nonlinear effects of uncertainty shocks, (ii) emphasize the key role of financial conditions,(iii) propose a novel econometric estimation based on the impulse response matching approach, and (iv) do not allow for various risk channels.

Our paper is also related to an increasing body of literature investigating the importance of expectation effects in regime shifts with a Markov-switching framework. This concept was originally defined by Liu, Waggoner, and Zha (2009) in the context of regime changes in monetary policy and has since been extensively studied. Bianchi (2013) considers "belief counterfactuals" to quantify the importance of expectation effects for business cycle fluctuations. Foerster (2016) distinguishes the expectation effects of regime switching in the inflation target from those in the inflation response. Bianchi and Melosi (2016) develop a

Bayesian learning process for regime shifts that influences the expectations formation mechanism. Bianchi and Ilut (2017) consider expectation effects in monetary/fiscal policy mix changes. We extend this concept and apply it to regime shifts in the degree of financial frictions. Interestingly, the expectation effects embedded in our model share some features with the anticipation effects described by He and Krishnamurthy (2019) in the context of a model with occasionally binding financial constraints. In their model, financial constraints have effects on the equilibrium even when they are not binding (which corresponds to the tranquil regime in our model) because agents anticipate that they may bind in the future (which corresponds to the realization of the stress regime in our model).

From a methodological standpoint, this paper is related to a growing literature dealing with the estimation and simulation of DSGE models in which stochastic volatilities and structural parameters are allowed to follow Markov-switching processes. This literature includes, among others, Liu, Waggoner, and Zha (2011), Bianchi (2013), Davig and Doh (2014), Lhuissier and Zabelina (2015), Bianchi and Melosi (2017), and Lhuissier (2018). The standard approach for inference in MS-DSGE models employed by all of these papers is to build the state-space representation of the MS-DSGE models adapted from the standard Kim and Nelson (1999) filter. In contrast, our approach dispenses with such a filter, as inference is directly done by minimizing the gap between the theoretical and empirical impulse response functions.

III. Evidence of time variation in the effects of uncertainty shocks

This section documents changes in the effects of uncertainty shocks on aggregate activity over time by employing a Markov-switching framework.

III.1. Markov-switching structural Bayesian VARs.

Following Hamilton (1989), Sims and Zha (2006), and Sims, Waggoner, and Zha (2008), we employ a Markov-switching Bayesian structural VAR model of the following form:

$$y_t' A(s_t^{\mathrm{c}}) = \sum_{i=1}^{\rho} y_{t-i}' A_i(s_t^{\mathrm{c}}) + C(s_t^{\mathrm{c}}) + \varepsilon_t' \Xi^{-1}(s_t^{\mathrm{v}}), \qquad t = 1, \ldots, T, \tag{1}$$

where y_t is defined as $y_t \equiv [gdp_t, vix_t, sp_t]'$; gdp_t is the logarithm of U.S. real gross domestic product (GDP); vix_t is the VIX index, a proxy for uncertainty; and sp_t is the BAA-AAA credit spread. Data sources are presented in the Online Appendix. The overall sample period is 1962:Q3 to 2018:Q2. We set the lag order to $\rho = 2$. Our parsimonious specification is justified by the fact that it becomes quickly challenging to estimate Bayesian MS-SVAR models as the number of observables and of lags grows. Note also that this is in line with the literature that allows for time-varying parameters in VARs (e.g., Primiceri, 2005; Cogley and Sargent, 2005; Bianchi and Melosi, 2017).

We assume a two-regime process governing the equation coefficients and constants (s_t^{c}) and a three-regime process governing the disturbance variances (s_t^{v}). The regimes evolve according to two transition matrices as follows:

$$Q^{\mathrm{c}} = \begin{bmatrix} q_{1,1}^{\mathrm{c}} & q_{1,2}^{\mathrm{c}} \\ q_{2,1}^{\mathrm{c}} & q_{2,2}^{\mathrm{c}} \end{bmatrix}, \qquad \text{and} \qquad Q^{\mathrm{v}} = \begin{bmatrix} q_{1,1}^{\mathrm{v}} & (1-q_{2,2}^{\mathrm{v}})/2 & 0 \\ 1-q_{1,1}^{\mathrm{v}} & q_{2,2}^{\mathrm{v}} & 1-q_{3,3}^{\mathrm{v}} \\ 0 & (1-q_{2,2}^{\mathrm{v}})/2 & q_{3,3}^{\mathrm{v}} \end{bmatrix}. \tag{2}$$

The restricted transition matrix Q^{v} implies that when we are in regime j, we can only move to regime $j-1$ or $j+1$. Sims, Waggoner, and Zha (2008) argue that such a restriction tends to fit the macroeconomic data better.

We assume that ε_t follows the following distribution:

$$p(\varepsilon_t) = \mathrm{normal}(\varepsilon_t | 0_n, I_n), \tag{3}$$

where 0_n denotes an $n \times 1$ vector of zeros, I_n denotes the $n \times n$ identity matrix, and $\mathrm{normal}(x|\mu, \Sigma)$ denotes the multivariate normal distribution of x with mean μ and variance Σ. Finally, T is the sample size; $A(s_t)$ is an n-dimensional invertible matrix under regime s_t; $A_i(s_t)$ is an n-dimensional matrix that contains the coefficients for lag i and regime s_t; $C(s_t)$ contains the constant terms; and $\Xi(s_t)$ is an n-dimensional diagonal matrix.

Following Sims and Zha (1998), we exploit the idea of a Litterman random walk prior for the structural-form parameters.[6] The Online Appendix provides the detailed techniques for the Sims and Zha (1998) prior.

[6] Regarding the Sims and Zha (1998) prior, the hyperparameters are defined as follows: $\mu_1 = 1.00$ (overall tightness of the random walk prior); $\mu_2 = 1.00$ (relative tightness of the random walk prior on the lagged parameters); $\mu_3 = 0.1$ (relative tightness of the random walk prior on the constant term); $\mu_4 = 1.0$ (erratic sampling effects on the lag coefficients); $\mu_5 = 0.0$ (belief about unit roots); and $\mu_6 = 0.0$ (belief in cointegration relationships). To match the usual interpretation of the Litterman prior for the reduced form, we drop the two true dummy observations (μ_5 and μ_6) introduced by Sims and Zha (1998). See also Doan, Litterman, and Sims (1984) and Sims (1993).

Finally, the prior duration of each regime is approximately five quarters. We also used other prior durations, and the main conclusions remain unchanged.

III.2. **Identification.** We identify uncertainty shocks by combining two kinds of restrictions. The first is based on traditional sign restrictions on the impulse response functions, as developed by Faust (1998), Canova and Nicolo (2002), and Uhlig (2005). We impose that an uncertainty shock induces a simultaneous increase in the VIX index and in the credit spread. The argument for this restriction is based on the idea that increases in financial uncertainty are frequently associated with significant increases in credit spreads, as shown in Stock and Watson (2012). We also assume that innovations to uncertainty cause an immediate drop in output. This restriction is motivated by the large theoretical literature that views uncertainty as having recessionary effects. See Bloom (2014) for a survey of this literature.

The above restriction is not sufficient to guarantee *pure* uncertainty shocks due to the high degree of comovement between the uncertainty proxy and the credit spread (e.g., Stock and Watson, 2012). It might be possible that shocks originating from the financial sector are present as uncertainty shocks. The second kind of restriction allows us to completely disentangle these two types of shock. We use a criterion that imposes a restriction on the variance in the one-step-ahead prediction error of our uncertainty variable. We impose the restriction that the uncertainty shock is the overwhelming driver of the unexpected movement in the VIX index, i.e., it explains at least 50 percent of variation in the VIX index. This kind of restriction is in line with Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016), who identify uncertainty shocks as innovations explaining the maximum amount of variability in an uncertainty indicator in order to disentangle them from financial shocks.

By combining the appeal of the forecast error variance restrictions approach with the advantages of sign restrictions, we are able to isolate fluctuations in uncertainty and its effects on economic activity.

III.3. **Empirical results.** In this section, we report the main empirical results produced by our MS-SVAR model. First, we present, in Section III.3.1, the posterior distribution of the estimated model. We then report, in Section III.3.2, the historical evolution of uncertainty for each variable. Finally, the impulse responses of endogenous variables to an uncertainty shock are reported in Section III.3.3. The results shown are based on 10 million draws with the Gibbs sampling procedure (see the Online Appendix for details). We discard the first $1,000,000$ draws as burn-in and then keep every 100th draw.

III.3.1. *Posterior distribution.* In this section, we present the key results produced by the model. Figure 1 shows the probabilities of being in a specific regime for each process (s_t^v and s_t^c) over time. The probabilities are smoothed in the sense of Kim (1994); i.e., full sample information is used to obtain the regime probabilities for each date.

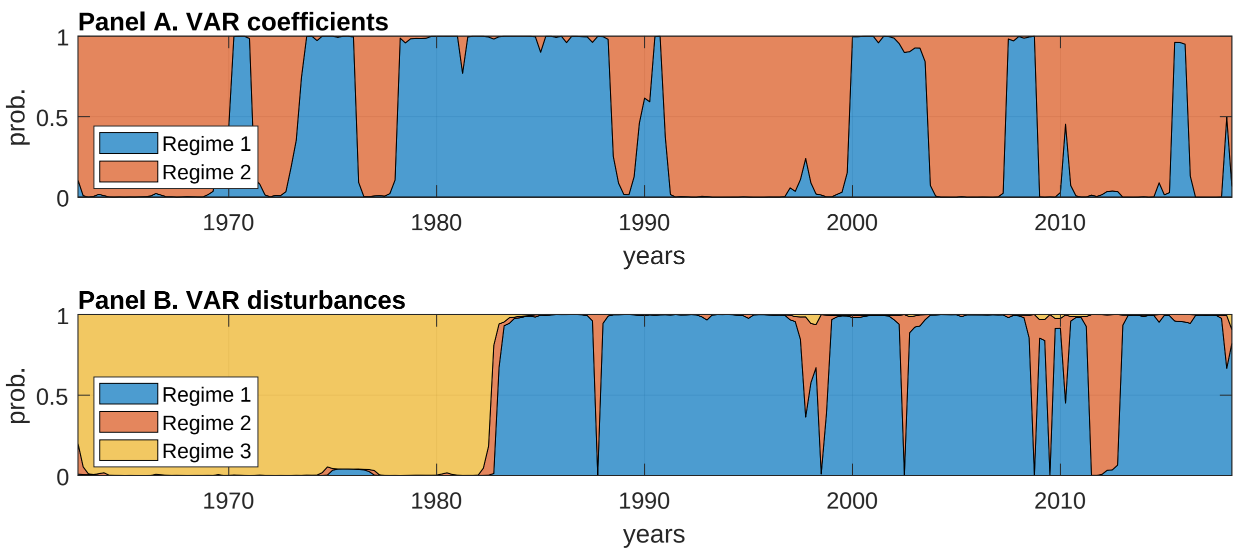

Figure 1. Sample period: 1962.Q4-2018.Q2. Regime probabilities (Panel A: Two-regime structural coefficients; Panel B: Three-regime disturbance variances).

When looking at the process in which equation coefficients are allowed to change (see s_t^c shown in Panel A of Figure 1), it is apparent that Regime 1 ($s_t^c = 1$, blue areas) prevailed during episodes of high inflationary pressure in the 1970s and 1980s and was dominant during the age of the 9/11 attacks, the dot-com bubble, and the corporate scandals. This regime was also in place during the financial crisis originating from subprime mortgages, as well as during the European debt crisis. We thus label this regime as the *distress* regime. All of the abovementioned subperiods captured by this regime contain the same characteristics, namely, major disruption in financial markets, macroeconomic imbalances, and heightened uncertainty. Regime 2 ($s_t^c = 2$, red areas) has prevailed over the remaining years of the sample and is characterized by episodes of tranquility. We label it as the *tranquil* regime.

Table 1. Relative shock (standard deviations) across regimes

	Production (GDP)	Uncertainty (VIX)	Financial (SP)
$s_t^{\mathrm{v}} = 1$	1.0000 [1.0000;1.0000]	1.0000 [1.0000;1.0000]	1.0000 [1.0000;1.0000]
$s_t^{\mathrm{v}} = 2$	1.4270 [1.2750;2.2982]	5.6380 [3.7603;6.4075]	3.3247 [1.9506;3.7224]
$s_t^{\mathrm{v}} = 3$	2.0527 [1.8723;2.4634]	1.5571 [1.2646;1.7623]	1.1065 [0.9246;1.2625]

Note: Posterior modes and [16th , 84th] percentiles are reported.

Regarding the process governing the structural disturbance variances, s_t^v, the model clearly captures three distinct regimes of volatility: a *low*-, *high*-, and *extreme*-volatility regime, as shown in Table 1.[7] Looking at Panel B of Figure 1, the high-volatility regime (i.e., Regime 3, yellow areas) corresponds clearly to the *pre*-Great Moderation period, where the size of the shock variance in output is relatively twice as large as that experienced in the low-volatility regime (i.e., Regime 1, blue areas). The higher degree of volatility in the pre-1980s period coincides, for example, with Kim and Nelson (1999). Finally, the extreme-volatility regime (i.e., Regime 2, red areas) identifies exceptional events, such as the beginning of the Great Recession in 2008.

Table 2 reports the estimated transition matrices for both Markov-switching processes at the posterior mode with 68% probability intervals in brackets. Looking at the s_t^c process, the distress regime ($q_{11}^{\mathrm{c}} = 0.8969$) is slightly less persistent (an average duration of approximately 9 quarters) than the tranquil regime ($q_{22}^{\mathrm{c}} = 0.9325$), with an average duration of over 15 quarters. Looking at the s_t^v process, Regimes 1 and 3 are unsurprisingly the most persistent, with $q_{11}^{\mathrm{v}} = 0.9432$ and $q_{33}^{\mathrm{v}} = 0.9846$, respectively. Regime 2 has a very short-lived duration of approximately 3 quarters. The tight interval probabilities reinforce the credibility of the estimated mode values.

In summary, our results suggest that the economy has experienced shocks whose size changes over time. Interestingly, the behavior of the economy—characterized by the systematic part of the model, i.e., the equation coefficients—is different during distress periods than during tranquil periods. The objective of the next sections is therefore to investigate the extent to which economic dynamics differ across regimes.

[7]Following Sims and Zha (2006), we normalize the size of the shock variances to unity in Regime 1, $s_t^v = 1$.

Table 2. Estimated transition matrices

	VAR disturbances				VAR coefficients	
	$s_t^{\mathrm{v}} = 1$	$s_t^{\mathrm{v}} = 2$	$s_t^{\mathrm{v}} = 3$		$s_t^{\mathrm{c}} = 1$	$s_t^{\mathrm{c}} = 2$
$s_t^{\mathrm{v}} = 1$	0.9432 [0.9178;0.9717]	0.1828 [0.0869;0.2066]	0.0000 [0.0000;0.0000]	$s_t^{\mathrm{c}} = 1$	0.8969 [0.7754;0.8980]	0.0675 [0.0536;0.1215]
$s_t^{\mathrm{v}} = 2$	0.0568 [0.0283;0.0822]	0.6345 [0.5869;0.8261]	0.0154 [0.0157;0.0660]	$s_t^{\mathrm{c}} = 2$	0.1031 [0.1020;0.2246]	0.9325 [0.8785;0.9464]
$s_t^{\mathrm{v}} = 3$	0.0000 [0.0000;0.0000]	0.1828 [0.0869;0.2066]	0.9846 [0.9340;0.9843]			

Note: Posterior modes and [16th , 84th] percentiles are reported.

III.3.2. *Historical evolution of uncertainty.* Given the importance we attach to uncertainty fluctuations in this paper, we characterize agents' uncertainty using our MS-SVAR model. Bianchi (2016) describes how to characterize uncertainty in the presence of regime changes in multivariate models. Uncertainty computed in this way reflects all sources of uncertainty faced by an agent: the possibility of regime changes, uncertainty regarding the state of the economy, uncertainty about the regime currently in place, and the possibility of Gaussian shocks. Following Bianchi (2016)'s methodology, uncertainty is measured for each endogenous variable as the h-step conditional standard deviation: $\sqrt{\mathbb{V}_t(y_{t+h})}$. Interestingly, since the level of VIX represents a measure of uncertainty and the standard deviation is fundamentally a measure of volatility in and of itself, then the standard deviation of the VIX variable reflects "the uncertainty of uncertainty", a measure that until now has been largely neglected in the uncertainty literature.

Figure 2 reports the evolution of uncertainty at each point in time. The time horizon goes from one quarter (blue line) to five years (green line). Not surprisingly, the volatility regime that is in place affects the evolution of uncertainty. For example, prior to the mid-1980s, the level of uncertainty for the VIX index turns out to be lower than that after the mid-1980s. While short-run uncertainty for output is larger than its long-run uncertainty, this is not always the case for the VIX index or the premium. Periods of high volatility imply relatively higher short-run uncertainty. This is because as the time horizon increases, the probability of still being in the high volatility regime decreases. Finally, times of recession are remarkably well associated with high levels of output uncertainty. This is in line with the literature; see, for example, Bloom (2014).

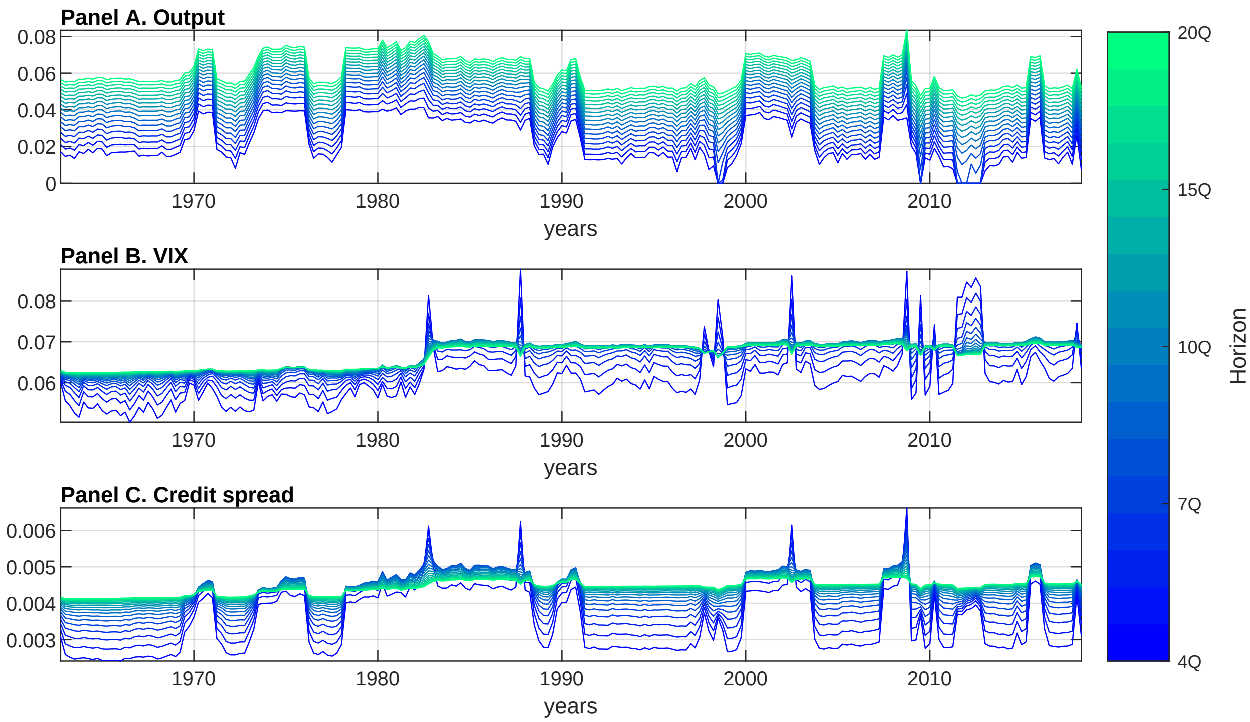

FIGURE 2. Historical evolution of volatility for output, VIX, and credit spread for horizons ranging from 1 year to 5 years. The blue lines correspond to the short horizons, while the green lines correspond to the long horizons.

III.3.3. *Regime-dependent dynamic effects of uncertainty shocks.* We illustrate possible differences in dynamics across the two regimes of the process governing the equation coefficients, s_t^c, by examining the conditional response of the rest of the economy to a pure disturbance in uncertainty ("one-time uncertainty shock").[8]

Figure 3 reports the impulse responses of the endogenous variables across the two regimes. The first column shows the responses during the tranquil regime, while the responses in the stress regime are displayed in the second column. All of these panels display the deviation in percent for the series entered in log-levels (output), whereas it displays the deviation

[8]Here, we assume that a particular regime will last in the wake of the shock, although agents take into account the possibility of regime shifts. Alternatively, we could have employed the generalized impulse-response function (GIRF) developed by Koop, Pesaran, and Potter (1996) and transposed it to the MS-SVAR models of Karamé (2015) and Bianchi (2016). GIRF makes allowances for dependence on initial conditions, future shocks, and future regimes. Our choice for conditional impulse responses is justified by the fact that we want to highlight the regime-specific features and, more specifically, to understand how the economy behaves if a specific regime prevails over the relevant horizon.

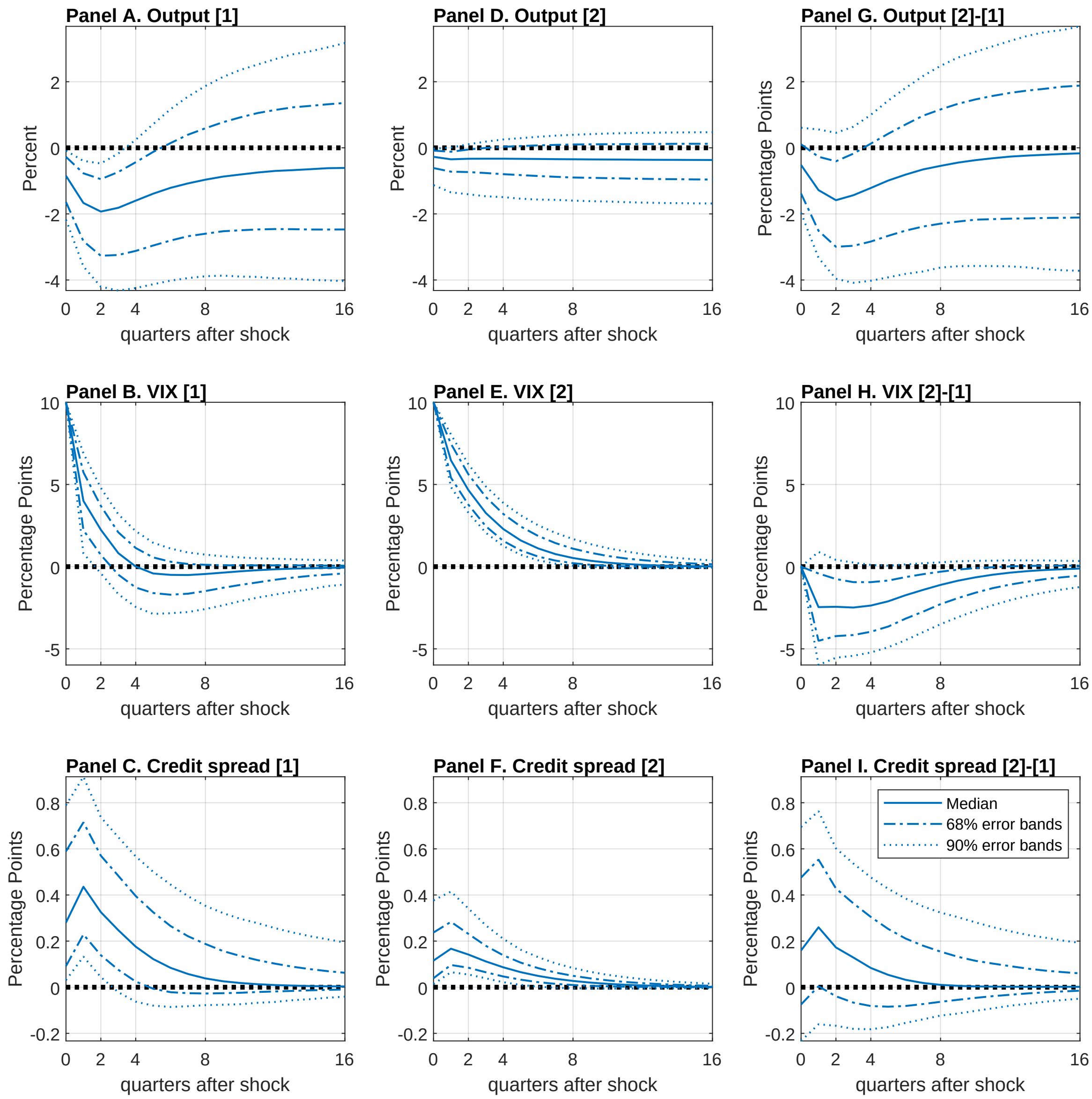

FIGURE 3. Impulse-response functions for uncertainty shocks under both regimes obtained from the identified MS-SVAR model. The first and second columns report the impulse responses of the endogenous variables under the distress and tranquil regimes, respectively. The last column displays the difference between the two regimes. In each case, the median is reported as a solid line and the 68% and 90% error bands as dotted lines.

in percentage points for the VIX index and credit spread. The third column shows the differences between the impulse responses under the two regimes. In any column, the solid lines represent the median, with the 68% and 90% probability intervals displayed as dotted lines. For comparability across regimes, our uncertainty shock is scaled to induce a 10-percentage-point immediate increase in the VIX index.

Looking at this figure, the responses of our measure of aggregate activity vary greatly over time, indicating that the differences between the two regimes in terms of the coefficients of the system of equations are very large. After a positive shock to our uncertainty measure that causes a 10-percentage-point increase in the VIX index, output falls slowly and moderately during the tranquil regime but falls quickly and considerably during the distress regime until reaching its minimum after 3 quarters (at this point, the impact is four times higher in the distress regime). These differences appear to be robust when taking into account the 68 percent probability intervals (right-top panel); the error bands for the differences lie exclusively within the negative region over the first 5 quarters.

Interestingly, the response of credit spread appears to be much larger during the distress regime. Indeed, the maximum response is approximately 0.40 percentage points, which is twice as large asthe response during the tranquil regime. When stress is high, credit costs for firms tend to range near higher levels, which produces noticeably more adverse effects on the economy. This result is thus consistent with the notion that the amplification effects of uncertainty shocks on output occur primarily through changes in credit spreads.

We investigate this intuition in Section IV through inference of a MS-DSGE model by using the regime-dependent impulse responses obtained from the identified MS-SVAR model.

III.3.4. *Robustness.* To assess the robustness of the results, we study a number of alternative identification schemes. First, we change the threshold level for the minimum contribution of the shock to the prediction error variance in the VIX index. Second, we adjust the assumption that restrictions are only imposed within quarters. Third, we employ an alternative restriction on the forecast error variance. The results for this section are available in the Online Appendix.

(1) The minimum contribution threshold. In our benchmark identification, we impose the restriction that the uncertainty shock is the overwhelming driver of the unexpected movement in the VIX index, i.e., it explains at least 50% of the uncertainty variation. Several other thresholds were examined to determine if they deliver different outcomes. The levels of the alternative threshold are (1) 60%; (2) 70%; and (3) 80%. Clearly, these changes in the threshold level do not affect the main conclusions. The impulse responses are close to those reported in the previous section.

(2) Alternative restriction on the effect horizon. Both types of restrictions are imposed only within quarter in the baseline identification. In this section, we reinforce this assumption and assume now that those restrictions are also imposed over the next two quarters. In so doing, we guarantee that uncertainty shocks generate a persistent increase in the VIX index rather than just a one-off spike in volatility. We find that the economic implications produced from this scheme remain unchanged relative to our main findings.

(3) Alternative restriction on the forecast error variance. The restriction imposed on the forecast error variance is similar in some ways to the identification strategy used by Uhlig (2003), who identifies a structural shock by searching for the shock that has a contribution larger than the largest contribution of any other shock. By contrast, we identify a shock by searching for the one with a contribution that is larger than the sum of the contributions of all other shocks. One may ask how sensitive these results are relative to the traditional Uhlig (2003) method. We have performed this exercise, and clearly, the regime-dependent impulse responses remain similar.

IV. A structural interpretation

The empirical evidence that the amplification effects of uncertainty shocks on output occur primarily through changes in credit spreads provides an intuitively appealing explanation for the state-dependent response of the economy to uncertainty shocks. In this section, we investigate whether this mechanism is also quantitatively relevant within a general equilibrium model. In particular, we ask whether a variant of the model proposed by Christiano, Motto, and Rostagno (2014) that allows for regime changes in the degree of agency problems between borrowers and lenders matches the regime-dependent impulse responses in the data. Sections IV.1 and IV.2 present the microfounded model and the solution method, respectively. The estimation results are reported in Section IV.3.

IV.1. **Model.** We use a medium-scale DSGE model along the lines of Christiano, Motto, and Rostagno (2014) but with two important differences. First, we assume only one source of perturbations in the economy, namely, uncertainty shocks, defined in the model as fluctuations in the volatility of cross-sectional idiosyncratic uncertainty. Second, key parameters governing the financial contract between borrowers and lenders are allowed to vary over time according to a two-state, first-order Markov-switching process, χ_t, with transition matrix $P = (p_{ij})_{(i,j \in \{1,2\})}$, where p_{ij} denotes the transition probabilities. We also allow the size of investment adjustment costs to change over time given their prominent role in the literature on the real effects of uncertainty (e.g., Bernanke, 1983; Bloom, 2009) and their potential links with financial frictions.[9]

The following sections present a complete description of the optimization problems solved by firms, households, and entrepreneurs.

IV.1.1. *Producers.* The final goods sector is perfectly competitive. The representative final goods producer combines intermediate goods $Y_{j,t}$ to produce a homogeneous good Y_t using the following technology:

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{1}{\lambda_f}} dj \right]^{\lambda_f}, \qquad 1 \leq \lambda_f < \infty, \tag{4}$$

where λ_f is the elasticity of substitution among intermediate goods. Monopolistic producers, indexed by j, demand capital and labor to maximize their cost of production subject to the demand function for their good using the following production technology:

$$Y_{j,t} = (u_t K_{j,t})^{\alpha} (z_t l_{j,t})^{1-\alpha} - \varphi z_t^*, \qquad 0 < \alpha < 1, \tag{5}$$

where u_t is the utilization rate of capital, φ is a fixed cost of production, and z_t^* is a combination of the labor productivity trend, z_t, and the investment-specific technology trend, Υ^t, as follows: $z_t^* = z_t \Upsilon^{(\frac{\alpha}{1-\alpha})t}$. Additionally, $K_{j,t}$ represents the services of effective capital, and $l_{j,t}$ is the quantity of homogeneous labor hired by the jth intermediate good producer.

The monopoly supplier of $Y_{j,t}$ sets its price, $P_{j,t}$, subject to nominal rigidity. In each period, a randomly selected fraction of intermediate goods firms, $1 - \xi_p$, can reset their price, while the complementary fraction follows a simple rule of thumb, $P_{j,t} = \tilde{\pi}_t P_{j,t-1}$, where $\tilde{\pi}_t \equiv (\pi^{target})^{\iota_p} (\pi_{t-1})^{1-\iota_p}$, with $\pi_{t-1} \equiv P_{t-1}/P_{t-2}$, and π^{target} is the target inflation rate of the monetary authority. Note that the pricing process is allowed for regime changes.

[9]Bolton, Chen, and Wang (2013) show that a high level of capital adjustment costs may reflect tough financial conditions in the economy.

IV.1.2. *Households.* There are a large number of identical and competitive households. We assume that each household contains every type of differentiated labor, $h_{i,t}, i \in [0,1]$. Each household has a large number of entrepreneurs, but we defer the description of these agents to the next subsection. The preferences of the representative household maximize the expected discounted sum of utilities given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log\left(C_t - bC_{t-1}\right) - \psi_L \int_0^1 \frac{h_{i,t}^{1+\sigma_L}}{1+\sigma_L} di \right\}, \tag{6}$$

subject to the law of capital accumulation

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}, \chi_t\right)\right] I_t, \tag{7}$$

and the budget constraint

$$R_t B_t + (1-\tau^l) \int_0^1 W_{i,t} h_{i,t} di + Q_{\bar{K},t}\bar{K}_{t+1} + \Pi_t = B_{t+1} + (1+\tau^c) P_t C_t + Q_{\bar{K},t}(1-\delta)\bar{K}_t + \frac{P_t}{\Upsilon^t} I_t, \tag{8}$$

where C_t stands for consumption, P_t for the price of consumption, $h_{i,t}$ for hours worked, $\bar{K}_t$ for raw capital, $Q_{\bar{K},t}$ for the price of capital, I_t for investment, B_t for one-period bonds, R_t for the nominal interest rate on these bonds, $W_{i,t}$ for the wages of the differentiated labor, and Π_t for the firms' profits. Household preferences are determined by the discount factor β, the degree of habit formation b, and ψ_L and σ_L, which determine the labor supply elasticity. The parameters τ^c and τ^l are consumption and labor tax rates. Physical capital depreciates at rate δ. The investment adjustment cost function is subject to the Markov-switching process according to the following specification:

$$S\left(x_t, \chi_t\right) = \frac{1}{2}\left[\exp\left[\sqrt{S''(\chi_t)}(x_t - \bar{x})\right] + \exp\left[-\sqrt{S''(\chi_t)}(x_t - \bar{x})\right] - 2\right], \tag{9}$$

where $x_t \equiv (I_t/I_{t-1})$ and in which the steady-state level is x. Note that $S(x, \chi_t) = S'(x, \chi_t) = 0$ and the curvature parameter $S''(x, \chi_t) = S''(\chi_t)$ change across regimes.

Households' differentiated labor services are aggregated by a "labor contractor" into a homogeneous labor supply, l_t, as

$$l_t = \left[\int_0^1 (h_{i,t})^{\frac{1}{\lambda_w}} di\right]^{\lambda_w}, \tag{10}$$

with $\lambda_w \geq 1$ being the elasticity of substitution across the i labor types. This homogeneous labor is sold to monopolistic producers at wage W_t, while each worker's type i is paid a wage

$W_{i,t}$. Therefore, the contractor chooses the quantity of each labor type i, $h_{i,t}$, such that

$$\max_{h_{i,t}} W_t l_t - \int_0^1 W_{i,t} h_{i,t} di. \tag{11}$$

The wage rate for each labor type $W_{i,t}$ is subject to nominal rigidity. In each period, a randomly selected subset of $1-\xi_w$ contractors can change their wage optimally, while the other follows an indexation rule based on the productivity growth rate, wage inflation, and their past wage as follows: $W_{i,t} = \mu_z^* \tilde{\pi}_{w,t} W_{i,t-1}$. Here, μ_z^* denotes the growth rate of z_t^* in the nonstochastic steady state, and $\tilde{\pi}_{w,t} \equiv (\pi^{target})^{\iota_w} (\pi_{t-1})^{1-\iota_w}$. Note that the wage pricing process is allowed for regime changes.

IV.1.3. *Financial contracts.* In each period of time, an exogenous fraction $(1-\gamma)$ of entrepreneurs dies and is born. Each N-type entrepreneur is endowed with personal wealth N, and aggregate wealth is $N_{t+1} = \int_0^\infty N f_t(N)\, dN$, where $f_t(N)$ is the density function of entrepreneurs with wealth N.

At the end of period t, each N-type entrepreneur chooses individual capital holding $\bar{K}_{t+1}^N$ for the next period, bought at market price $Q_{\bar{K},t}$, which is taken as given, from households. This capital purchase is made using personal wealth N and a one-period debt amount B_{t+1}^N contracted optimally with the bank at the end of time t. Thus, we obtain the following constraint

$$Q_{\bar{K},t} \bar{K}_{t+1}^N = N + B_{t+1}^N. \tag{12}$$

This $\bar{K}_{t+1}^N$ "raw" capital is then transformed into $\omega \bar{K}_{t+1}^N$ "efficiency" units, where ω has a unit-mean log-normal distribution that is drawn independently across time and across entrepreneurs. The standard deviation of $\log \omega$, denoted σ_t, captures the idiosyncratic uncertainty in actual business activities and follows an exogenous stochastic process as

$$\log \sigma_t = (1-\rho_\sigma(\chi_t)) \log \sigma + \rho_\sigma(\chi_t) \log \sigma_{t-1} + \varepsilon_{\sigma,t}, \tag{13}$$

with σ denoting its steady-state value and

$$\varepsilon_{\sigma,t} = \text{normal}(\varepsilon_{\sigma,t} | 0, \sigma_\sigma(\chi_t)). \tag{14}$$

The uncertainty shock, σ_t, captures the extent of cross-sectional dispersion in ω. The persistence and variance of the shock, $\rho_\sigma(\chi_t)$ and $\sigma_\sigma(\chi_t)$, respectively, are subject to the Markov-switching process.

Once the idiosyncratic productivity shock is realized, each N-type entrepreneur determines the utilization rate, u_t^N, of its effective capital and then supplies its capital services, $u_t^N \omega \bar{K}_t^N$, at a market rental rate, τ^k. Then, each N-type entrepreneur is left, after depreciation, with $(1-\delta)\omega K_t^N$ units of capital, which are sold to households at price $Q_{\bar{K},t}$. Consequently, an N-type entrepreneur who draws idiosyncratic productivity ω at the end of period t enjoys the rate of return ωR_{t+1} in $t+1$, where

$$R_{t+1}^k \equiv \frac{\left(1-\tau^k\right)\left(u_{t+1} r_{t+1}^k - a(u_{t+1})\right) P_{t+1}/\Upsilon^{t+1} + (1-\delta) Q_{K,t+1} + \tau^k \delta Q_{\bar{K},t}}{Q_{\bar{K},t}}, \tag{15}$$

where τ^k denotes the tax rate on capital income, $a(u_t)$ is the capital utilization cost defined as $a\left(u_t\right) = \frac{r^k \left(\exp(\sigma_a(u_t-1))-1\right)}{\sigma_a}$, with $\sigma_a > 0$, and r^k is the steady-state rental rate of capital.

The financial intermediary borrows the amount B_{t+1} from households at the short-term risk-free rate R_t, and that amount is provided to the entrepreneur as a one-period loan B_{t+1} at interest rate Z_{t+1}. According to a costly-state verification loan contract, the N-type entrepreneur can either (i) repay loan B_{t+1}^N with state-contingent (gross) interest rate Z_{t+1} or (ii) default on the loan, in which case the bank seizes a fraction $(1-\mu(\chi_t))$ of the entrepreneur's assets, where $\mu(\chi_t)$ denotes the monitoring costs. As emphasized in Carlstrom and Fuerst (1997), monitoring technology can be viewed as bankruptcy costs and, more broadly, as liquidation costs since the firm is closed and its assets are liquidated. This technology is allow to vary across regimes.

There is a threshold value $\overline{\omega}$ such that an N-type entrepreneur pays back the loan if $\omega > \overline{\omega}_{t+1}$ and defaults otherwise, i.e., such that

$$R_{t+1}^k \overline{\omega}_{t+1} Q_{\bar{K},t} \bar{K}_{t+1}^N = B_{t+1}^N Z_{t+1}. \tag{16}$$

An N-type entrepreneur values a particular debt contract according to his or her expected net worth in period $t+1$:

$$\max_{\overline{\omega}_{t+1}, L_t} \mathrm{E}_t \left\{\left[1-\Gamma_t\left(\overline{\omega}_{t+1}\right)\right] R_{t+1}^k L_t N\right\}, \tag{17}$$

where $\Gamma_t(\overline{\omega}_{t+1})$ is the bank's share of entrepreneurial earnings, defined as

$$\Gamma_t\left(\overline{\omega}_{t+1}\right) \equiv \left[1-F_t\left(\overline{\omega}_{t+1}\right)\right] \overline{\omega}_{t+1} + G_t\left(\overline{\omega}_{t+1}\right), \tag{18}$$

with $G_t\left(\overline{\omega}_{t+1}\right) \equiv \int_0^{\overline{\omega}_{t+1}} \omega dF_t\left(\omega\right)$, where $F_t(\omega) \equiv F(\omega_{t+1}, \sigma_t)$ is the cumulative distribution function of ω, and the leverage ratio is defined as $L_t = Q_{\bar{K},t} K_{t+1}^N / N$, subject to the following bank participation constraint:

$$[1 - F_t(\overline{\omega}_{t+1})] Z_{t+1} B_{t+1} + [1 - \mu(\chi_t)] \int_0^{\overline{\omega}_{t+1}} \omega dF_t(\omega) R^k_{t+1} Q_{\bar{K},t} \bar{K}_{t+1} \geq B_{t+1} R_t, \tag{19}$$

and the default condition (16).

IV.1.4. *Aggregation.* We aggregate the quantity of raw capital purchased by the entrepreneurs and the quantity of debt extended to the entrepreneurs in period t as follows:

$$\bar{K}_{t+1} = \int_0^\infty \bar{K}^N_{t+1} f_t(N) dN, \qquad \text{and} \qquad B_{t+1} = \int_0^\infty B^N_{t+1} f_t(N) dN. \tag{20}$$

Additionally, the aggregate supply of capital services, $K_t = u_t \bar{K}_t$, must equal the corresponding demand, $\int_0^1 K_{j,t} dj$, of the intermediate goods producers.

Finally, at the aggregate level, net worth at the end period t is given by

$$N_{t+1} = \gamma [1 - \Gamma_{t-1}(\overline{\omega}_t)] R^k_t Q_{K,t-1} K_t + W^e_t, \tag{21}$$

where W^e_t is a transfer from households to entrepreneurs.

IV.1.5. *Monetary authority and resource constraint.* The monetary policy rule is defined as follows:

$$R_t - R = \rho_p (R_{t-1} - R) + (1 - \rho_p) \left[\phi_\pi (\pi_{t+1} - \pi^*_t) + \frac{\phi_y}{4} (g_{y,t} - \mu^*_z) \right], \tag{22}$$

where $g_{y,t}$ denotes the quarterly growth in GDP, GDP_t, with $GDP_t = G_t + C_t + I_t \Upsilon^{-t}$, where G_t is government consumption and η_g denotes its share in GDP. Note that the elasticities of the nominal rate to the inflation gap and the output growth gap, ϕ_π and ϕ_y, respectively, are subject to the Markov-switching process. Finally, the resource constraint for the economy is

$$Y_t = C_t + \frac{I_t}{\Upsilon^t} + G_t + a(u_t) \frac{\bar{K}_t}{\Upsilon^t} + \frac{\mu(\chi_t) G(\overline{\omega}_t) R^k_t Q_{\bar{K},t-1} \bar{K}_t}{P_t}, \tag{23}$$

taking into account the costs of capital utilization and the resources used to monitor defaulting entrepreneurs.

IV.2. **Solving the model.** Because the economy exhibits a trend, we stationarize variables by their corresponding trend. We then rescale and linearize the model around the steady state equilibrium.[10] A detailed derivation of the steady-state equilibrium for the stationary variables is provided in the Online Appendix. The model is then solved using a solution algorithm based on the mean square stable (MSS) concept proposed by Farmer, Waggoner, and Zha (2009), Farmer, Waggoner, and Zha (2011), and Cho (2016). Such an algorithm allows agents to take into account the possibility of future regime shifts when forming expectations. For efficiency and speed reasons, we use Cho (2016)'s algorithm, which uses a forward method.

The solution can be characterized as follows

$$f_t = c(\chi_t, \theta, P) + T(\chi_t, \theta, P) f_{t-1} + R(\chi_t, \theta, P)\varepsilon_t, \tag{24}$$

where c is the constant term, f_t is the vector of endogenous components, ε_t is a vector of exogenous shocks, and θ is a vector containing all structural parameters. The law of motion for the model depends on the structural parameters (θ), the prevailing regime (χ_t), and the probability of switching across regimes (P).

IV.3. **Empirical Results.** This section provides the main quantitative results from the estimated MS-DSGE model. First, we present our estimation strategy in Section IV.3.1. Second, we report the estimates of the structural parameters in Section IV.3.2, and we highlight the key role of monitoring costs. Third, in Section IV.3.3, we present the impulse response functions for the uncertainty shocks.

IV.3.1. *Estimation strategy.* Our estimation strategy is analogous to the impulse response matching approach used by Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005), except that we are estimating the parameters to fit our regime-dependent

[10]As in Schorfheide (2005), Liu, Waggoner, and Zha (2011), and Bianchi, Ilut, and Schneider (2018), regime changes affect the steady state. To define the steady state equilibrium, we take the ergodic mean of each regime switching parameter, $\bar{x}$, as follows: $\bar{x} = \bar{p}_1 x(1) + \bar{p}_2 x(2)$, where $\bar{p}_i$ stands for the ergodic probability of being in regime i and $x(i)$ is a parameter in regime i. We then refer to $\bar{y}$ as the value of the endogenous variable y_t at the ergodic steady state given by $\bar{y} = f(\bar{x})$, where $f(.)$ is the steady-state function that maps the values of endogenous variables to the ergodic values of structural parameters.

impulse responses from a MS-SVAR, as opposed to impulse responses from a constant-parameters SVAR.[11] Our empirical analysis matches the estimated conditional impulse response functions for each endogenous variable obtained from the identified MS-SVAR model.

While output and credit spread[12] are directly observable from the theoretical model, this is not the case for our uncertainty measure. To the best of our knowledge, Basu and Bundick (2017) are the first to define the VIX index in a DSGE model, but this requires a third-order approximation of the model policy functions. At this stage, there is no efficient estimation algorithm to allow high-order approximations for MS-DSGE models. Nevertheless, it should be stressed that Foerster, Rubio-Ramírez, Waggoner, and Zha (2016) attempt to fill part of this gap using perturbation methods. However, their solution method is not sufficiently quick and accurate to be used in an estimation algorithm.

That being said, Leahy and Whited (1996), Bloom, Bond, and Reenen (2007), and Bloom (2009) document that a number of cross-sectional measures of uncertainty are highly correlated with time-series stock-market volatility. In particular, Bloom (2009) presents evidence that the cross-sectional standard deviation of firm-level stock returns can be used as a proxy for time-series stock-market volatility since they are strongly correlated. Motivated by this evidence, we compute the analog of Bloom's cross-sectional uncertainty measure in our model and use it as a proxy for the VIX index. Following Christiano, Motto, and Rostagno (2014), we compute such a measure from the standard deviation, *std*, of the entrepreneurial return on equity in a cross-section and including only nonbankrupt entrepreneurs (i.e., those with $\omega > \bar{\omega}$) as $std\left(R^e_t(\omega)|\omega > \bar{\omega}_t\right) = R^k_t L_{t-1}\sqrt{\mathbb{VAR}\left(\omega - \bar{\omega}_t|\omega > \bar{\omega}_t\right)}$, where L_{t-1} denotes leverage, R^k_t is the cross-sectional average return on capital, and $\mathbb{VAR}(x|D)$ denotes the variance of x conditional on event D.[13]

[11]We thank Mathias Trabandt for sharing the computer code used in Christiano, Trabandt, and Walentin (2010) for inference of constant DSGE models with the standard impulse response matching approach. We adapt their code for a Markov-switching environment.

[12]For the credit spread, we consider Christiano, Motto, and Rostagno (2014)'s definition of the risk premium in the model, which takes into account net expected losses in the case of default:

$$spread_t = Z_{t+1} - R_t - F_t(\bar{\omega}_{t+1})Z_{t+1} + \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) R^k_{t+1} \frac{Q_{\bar{K},t}\bar{K}_{t+1}}{Q_{\bar{K},t}\bar{K}_{t+1} - N_{t+1}}.$$

[13]Conditional on the period t aggregate shocks, an entrepreneur with idiosyncratic shock ω earns $R_t = \max\left\{0, [\omega - \bar{\omega}_t]\right\} \times R^k_t L_{t-1}$. Following Christiano, Motto, and Rostagno (2014), $\mathbb{VAR}\left(\omega - \bar{\omega}_t|\omega > \bar{\omega}_t\right)$ is given

We link the two measures through the following measurement equation:

$$VIX_t = \kappa_0 + \kappa E_t \left\{ std \left(R^e_{t+1}(\omega) | \omega \geq \bar{\omega}_{t+1} \right) \right\}, \tag{25}$$

where κ_0 is an intercept and κ is a constant of proportionality. Equation (25) determines the VIX index, VIX_t, as a function of the expectation of the standard deviation of the entrepreneurial return on equity. In the rest of the paper, we assume that the intercept κ_0 is equal to zero since we are interested only in impulse responses based on the log-linearized model.

Like the MS-SVAR model, we compute DSGE impulse responses by assuming that a particular regime is in place over the entire sample. Let $\tilde{\xi}$ be an $N \times 1$ vector, which stacks the contemporaneous and 16 lagged responses to each of three endogenous variables to the uncertainty shock. The number of elements in $\tilde{\xi}$ is equal to 2 (i.e., the number of regimes) times 3 (i.e., the number of variables) times 17 (i.e., the horizon) $=$ 102 elements. Let $\xi(\theta)$ denote the mapping from θ to the MS-DSGE model impulse response functions, with θ being a vector containing all estimated parameters. The likelihood function for the data, $\tilde{\xi}$, is defined as a function of θ:

$$f(\tilde{\xi}|\theta, \bar{V}) = \left(\frac{1}{2\pi} \right)^{\frac{N}{2}} |\bar{V}^{-\frac{1}{2}}| \times \left[-\frac{1}{2} (\tilde{\xi} - \xi(\theta)') \bar{V}^{-1} (\tilde{\xi} - \xi(\theta)) \right], \tag{26}$$

where $\bar{V}$ is a diagonal matrix with the sample variances of the $\tilde{\xi}$s along the diagonal. Conditional on $\tilde{\xi}$ and $\bar{V}$, the Bayesian posterior of θ is as follows:

$$f\left(\theta, \bar{V}\right) \propto f(\tilde{\xi}|\theta, \bar{V}) \times f(\theta), \tag{27}$$

where $f(\theta)$ denotes the priors on θ.

The estimation strategy begins by maximizing the logarithm of equation (27) using the `CSMINWEL` program, the optimization routine developed by Christopher A. Sims. Once in the posterior mode, we can start a Markov chain Monte Carlo method to sample the posterior distribution. More specifically, we employ the random-walk Metropolis-Hasting procedure to generate draws from the joint posterior distribution of the MS-DSGE model. The results

as follows

$$\mathbb{VAR}\left(\omega - \bar{\omega}_t | \omega > \bar{\omega}_t\right) = \frac{1}{1 - F(\bar{\omega}_t)} e^{\sigma_t^2} \left[1 - \Phi\left(\frac{\log \bar{\omega}_t}{\sigma_t} - \frac{3}{2}\sigma_t \right) \right] - \left(\frac{1 - G(\bar{\omega}_t)}{1 - F(\bar{\omega}_t)} \right)^2,$$

where $\Phi(.)$ denotes the cumulative density function of a standard normal distribution.

shown in the paper are based on 50,000 draws. We discard the first 10 percent of draws as burn-in, and every 10th draw is retained.

IV.3.2. *Estimates of key parameters.* To keep the estimation procedure tractable and consistent with our MS-SVAR model, we calibrate several parameters. Each of them is set along the lines of those estimated (at the mode) in Christiano, Motto, and Rostagno (2014). Additionally, in this respect, it may be worth mentioning that the parameters that determine the degree of nominal rigidities and Taylor rules are calibrated to be consistent with our MS-SVAR model, which was estimated on the real data only. Table 3 summarizes the calibration.

TABLE 3. Calibration of structural parameters.

α	Capital share	0.40	π^{target}	SS inflation rate	2.43
ξ_p	Calvo prices	0.74	ξ_w	Calvo wages	0.81
ι_p	Price indexation	0.89	ι_w	Wage indexation	0.49
ϕ_π	Taylor rule: inflation	2.40	ϕ_y	Taylor rule: output	0.36
δ	Depreciation rate	0.025	σ_c	Elasticity: utility	1.5073
λ_w	SS markup: labor	1.05	ρ_R	Taylor rule smoothing	0.8519
σ_L	Curvature: disutility of labor	1.00	$F(\bar{\omega})$	SS probability of default	0.0056
h	Consumption habit	0.74	λ_w	SS markup: suppliers of labor	1.05
ψ_L	Disutility weight on labor	0.7705	W^e	Transfer received by entrepreneurs	0.005
β	Discount factor	0.9987	$1-\gamma$	Net worth transfer	$1-0.985$
σ_a	Elas.: capital utilization costs	2.54	λ_f	SS markup: intermediate good firms	1.20
τ^c	Tax rate: consumption	0.05	η_g	SS g/GDP ratio	0.20
τ^k	Tax rate: capital	0.32	μ_z	Growth rate of the economy	0.41
τ^l	Tax rate: labor	0.24	Υ	Trend rate for investment-specific technological change	0.42

Note: Calibration is based on the calibrated and estimated parameters (at the mode) in Christiano, Motto, and Rostagno (2014).

Table 4 reports the specific distribution, the mean and the standard deviation for each estimated parameter. Most of the prior distributions for the parameters closely follow those in Christiano, Motto, and Rostagno (2014). Additionally, all priors remain unchanged across regimes so that the data, through the likelihood, dominate the posterior distribution. In other words, our results are not driven by asymmetric priors.

Table 4. Prior and posterior distributions.

		Prior			Posterior		
Coefficient	Description	Density	Mean	Std. Dev.	Mode	[5;	95]
p_{11}	Transition matrix	D	0.89	0.05	0.9170	0.7971	0.9600
p_{22}	Transition matrix	D	0.93	0.05	0.9684	0.8385	0.9868
κ	Measurement VIX	N	1.00	2.00	0.9142	0.5849	1.5301
$S''(\chi_t = 1)$	Investment adjustment costs	G	0.75	0.50	0.5993	0.2820	1.2952
$S''(\chi_t = 2)$	Investment adjustment costs	G	0.75	0.50	1.3437	0.7985	1.9746
$\mu(\chi_t = 1)$	Monitoring costs	N	0.275	0.10	0.2090	0.1125	0.3231
$\mu(\chi_t = 2)$	Monitoring costs	N	0.275	0.10	0.0615	0.0326	0.1290
$\rho_\sigma(\chi_t = 1)$	Persistence shock	B	0.60	0.20	0.5509	0.2962	0.6603
$\rho_\sigma(\chi_t = 2)$	Persistence shock	B	0.60	0.20	0.7689	0.6902	0.8103
$\sigma_\sigma(\chi_t = 1)$	Std. Dev. shock	Inv-G	1.00	1.00	0.4096	0.3197	0.5682
$\sigma_\sigma(\chi_t = 2)$	Std. Dev. shock	Inv-G	1.00	1.00	0.4204	0.3045	0.5588

Note: N stands for normal, B for beta, D for Dirichlet, G for gamma, and Inv-G for inverted gamma. The 5 percent and 95 percent values demarcate the bounds of the 90 percent probability interval.

We begin with the financial contract parameters. The mean and the standard deviation of the prior distribution for monitoring costs, μ, are 0.275 and 0.10, respectively, thus covering the parameter space suggested by Carlstrom and Fuerst (1997). The prior distributions of the uncertainty shock process are weakly informative. We use a beta distribution for the persistence of the shock with a mean equal to 0.60 and a standard deviation of 0.20. Regarding the priors for shock variances, we impose an inverted gamma distribution, with the mean and the standard deviation equal to 1.00.[14] We assume that the priors for the investment adjustment cost parameter S'' follow the gamma distribution, with the mean and standard deviation equal to 0.75 and 0.50, respectively. Finally, the priors on the transition matrix, p_{ij}, are chosen to reflect the frequency of regime changes reported by the MS-VAR (at the mode). The parameters p_{11} and p_{22} each follow a Dirichlet distribution, with means

[14]The inverted gamma distribution is as follows: $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$, where ν and s are hyperparameters.

equal to 0.89 and 0.93, respectively, and a standard deviation of 0.05. In other words, the prior duration is approximately 9 quarters and 14 quarters for Regimes 1 and 2, respectively.

The group of estimated parameters is stacked as follows:

$$\theta = \left[p_{11}, p_{22}, \kappa, S''(k), \mu(k), \rho_\sigma(k), \sigma_\sigma(k)\right], \tag{28}$$

with $k = \{1, 2\}$.

The last three columns of Table 4 report the posterior mode with the 90 percent probability interval for each structural parameter.

The role of financial frictions in explaining dynamics is very much in evidence. Indeed, the estimates for the monitoring costs, μ, differ considerably between the two regimes. While monitoring costs represent approximately 6.15% of the firm's value prior to bankruptcy in the tranquil regime, they are relatively higher in the distress regime, with a value of 20.90%. The fact that the 90 percent probability intervals barely overlap reinforces the evidence that the macroeconomic impact of uncertainty shocks depends crucially on the degree of financial frictions in the economy. The persistence of shocks, ρ_σ, is lower in the distress regime than in the tranquil regime, and the estimated standard deviations for shocks are almost identical across regimes, with $\sigma_\sigma(1) = 0.41$ and $\sigma_\sigma(2) = 0.42$. Finally, the persistence of each regime is fairly similar, with $p_{11} = 0.92$ and $p_{22} = 0.97$.

Regarding the investment adjustment cost parameter, $S''(k)$, the estimates are approximately 0.60 and 1.34 in the distress and tranquil regimes, respectively, slightly lower than the values obtained, for example, by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). The 90 percent probability intervals overlap, meaning that there are no important differences across regimes.

IV.3.3. *Impulse responses.* Figure 4 reports, as the black line, the impulse responses of the endogenous variables to the uncertainty shock obtained from the MS-DSGE model. Each column represents the responses of a particular variable under each regime. For comparison purposes, we also present the median and the 68 and 90 percent probability intervals for the MS-SVAR model-implied responses. A number of results are worth emphasizing here. First, the model performs well in accounting for the dynamic responses of the economy to an uncertainty shock. All the DSGE model-implied responses lie within the 68 percent probability intervals computed from the MS-SVAR model. From a qualitative point of view, the

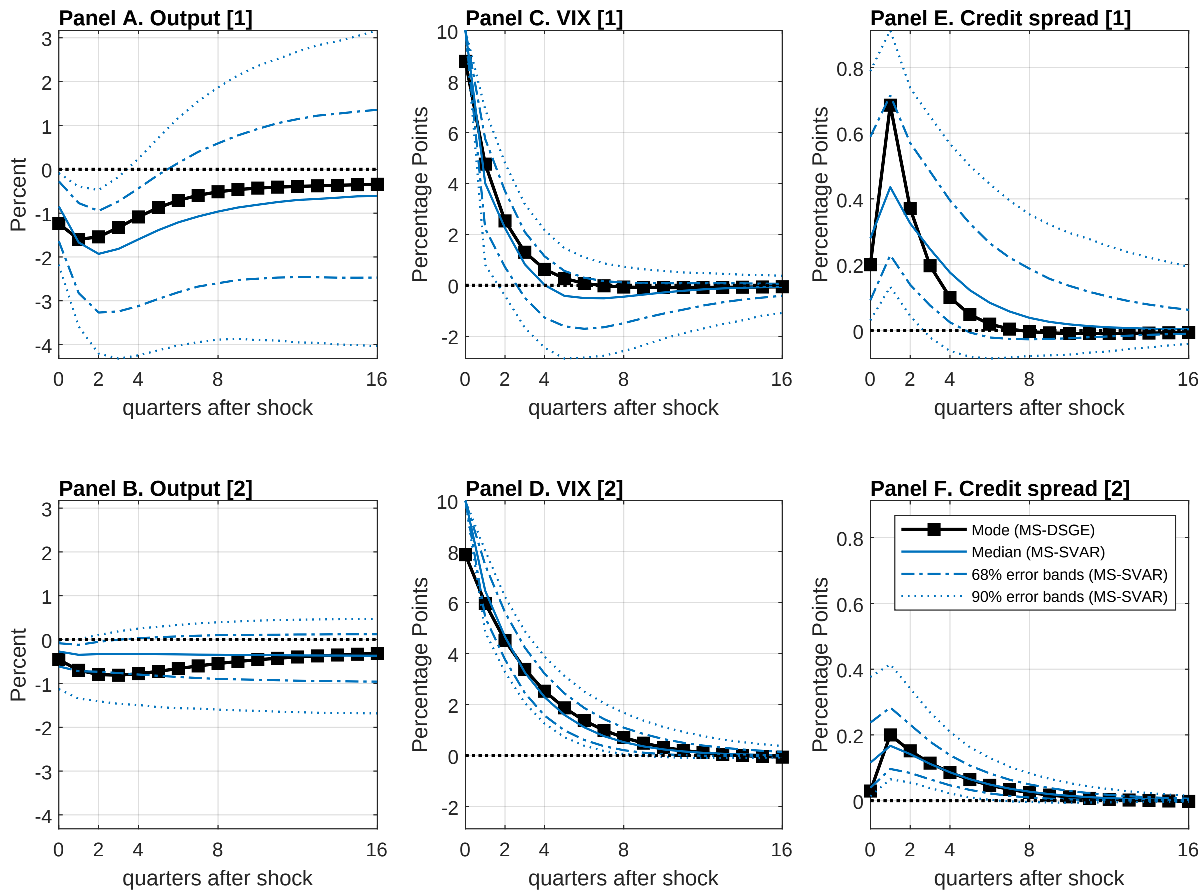

FIGURE 4. Impulse-response functions for an uncertainty shock. The median responses from the identified MS-SVAR model are reported as solid blue lines, and the 68% and 90% error bands are reported as dotted blue lines. The black lines report the responses (at the mode) from the MS-DSGE model.

responses of output and credit spread in the tranquil regime share some common features with the responses in the distress regime. Credit spread and output move in opposite directions; output declines progressively, while credit spread rises immediately and then steadily returns to its pre-shock level.

The transmission mechanism is straightforward. The uncertainty shock directly alters the degree of risk associated with the asymmetric information between lenders and entrepreneurs who borrow external funds to produce physical capital goods. It shifts the dispersion of entrepreneurs' idiosyncratic productivity. With imperfect financial markets, this shock implies higher external finance costs since more entrepreneurs draw low levels of productivity and are then unable to repay their debts. Therefore, a positive uncertainty shock increases both the risk of default and the cost of external funds, which leads to the drop in the economic activity of entrepreneurs being transmitted to the overall economy in general equilibrium through an increase in the credit spread and a decrease in investment and production. In other words, financial frictions act as the main mechanism through which changes in uncertainty affect macroeconomic variables.

Furthermore, the model succeeds in accounting for the differences in the responses of endogenous variables across the two regimes. Indeed, there is a notable change in the way both output and credit spread respond to the shock. Concerning the changes in the impulse responses between the two regimes, the responses under the distress regime are remarkably amplified compared to those in the tranquil regime. Under these circumstances, financial frictions act as an amplification mechanism.

This stronger effect of uncertainty in distress periods can be explained as follows. Because of the greater cost of information asymmetry in periods of distress, financial intermediaries charge higher premiums than they do during normal periods. In our baseline model, this manifests itself as a greater sensitivity of premium to the firm's net worth in distress periods. In this context, an uncertainty shock causes larger credit spread increases and therefore larger and longer-lasting negative effects on economic activity. In contrast, when stress is low, the economy is more capable of absorbing the coming economic shocks. As a result, the macroeconomic effects are less pronounced.

IV.4. **Further discussions of μ.** A remarkable feature of our evidence is an increase in the asymmetry of information between borrowers and lenders in periods of distress, which manifests itself in higher monitoring costs. Our evidence is closely related to the estimates of monitoring costs in Lindé, Smets, and Wouters (2016). The authors estimate, with full information methods, a DSGE model with a Bernanke-Gertler-Gilchrist financial accelerator mechanism in which the monitoring costs are allowed to change according to a Markov-switching process. Interestingly, they capture changes in the degree of financial frictions, with repeated changes in the monitoring costs between a low (2.90%) and high (8.40%) value over time. These estimated values appear to be lower than ours. This difference can be explained in two main ways. First, they estimate an MS-DSGE with full information methods—i.e., key macroeconomic and financial variables are directly observable in their model—while we estimate our MS-DSGE with the impulse-response matching approach. Second, our MS-SVAR model properly takes into account the heteroskedasticity in U.S. macroeconomic disturbances, while their model does not. Indeed, Sims (2001), and more recently Lhuissier and Zabelina (2015), have shown the importance of capturing heteroskedasticity before allowing changes in economic dynamics in order to avoid misleading results. In Lindé, Smets, and Wouters (2016), only the monitoring cost parameter is allowed to change over time while shock variances remain constant. Our paper overcomes this issue by allowing both the equation coefficients and the shock variances to change over time independently.

Our results also corroborate the existing literature that introduces time-varying monitoring costs into partial or general equilibrium models. For example, Levin, Natalucci, and Zakrajšek (2004) and Fuentes-Albero (2019) show that the estimated value for μ varies between 0% and 50% over the period 1954.Q4 to 2006.Q4, with peaks in periods of financial distress. This wide range covers the estimates for our monitoring costs, $\mu(\chi_t)$, in both regimes.

Our evidence of regime-dependent monitoring costs appears to be consistent with the financial literature that reports higher bankruptcy costs in periods of distress. Frye (2000a,b) pleads for a consideration of the *double misfortune* in credit risk models: during crises "many obligators default, and the value of collateral is damaged". Indeed, as emphasized by Altman, Brady, Resti, and Sironi (2005), there is a strong negative correlation between the recovery rate and the rate of default. Acharya, Bharath, and Srinivasan (2007) supplement this evidence by showing that the recovery rate of creditors from default or bankruptcy is $10\% - 15\%$ lower in distressed industries than in healthy industries.

The common explanation for these empirical facts, and by consequence, the rationale for time-varying monitoring costs in DSGE models, tracks back to the Shleifer and Vishny (1992) model of fire sales, which explains that assets are sold at a discounted price during periods of financial stress. Discounted prices for assets imply higher costs of bankruptcy in distress periods, e.g., the higher μ assumed in our model. Interestingly, Candian and Dmitriev (2019) recently included the fire sales mechanism in the Bernanke, Gertler, and Gilchrist (1999) setup, assuming that the monitoring cost parameter μ is a function of market liquidity, which is defined as the ratio of net sales over net purchases of capital in the capital goods markets.

IV.5. **Alternative specifications.** In our baseline model, nominal rigidities and monetary policy parameters are calibrated along the lines of Christiano, Motto, and Rostagno (2014) since our MS-SVAR was estimated using real data only. It would be useful to consider some alternative specications in which those parameters are fixed along the lines suggested by other studies. For this purpose, we re-estimate several modified versions of the baseline model, calibrating one category of parameters at a time, and study the resulting posterior distributions.

The modified models under consideration are the following: first, a model with Taylor rule parameters fixed at those values estimated (at the mode) in Liu, Waggoner, and Zha (2011), which correspond to $\phi_\pi = 1.655$ and $\phi_y = 0.006$; second, a model in which nominal rigidities (Calvo prices and wages, the partial indexation of prices and wages to past inflation) are calibrated with those estimated (at the mode) in Liu, Waggoner, and Zha (2011), i.e., $\xi_p = 0.41$, $\xi_w = 0.21$, $\iota_p = 0.17$, and $\iota_w = 1.00$; and third, a model with flexible prices and wages, which corresponds to $\xi_p = \xi_w = 0.00$. The choice to use Liu, Waggoner, and Zha (2011)'s estimates appears to be relevant for at least two reasons. First, their DSGE model allows for regime changes in a similar way to ours. Second, their estimates contain large differences relative to those estimated by Christiano, Motto, and Rostagno (2014).

The results of this exercise are reported in the Online Appendix. The main result of note is that variations in specifications do not alter the findings reported by the baseline model. There are still large differences in monitoring costs across regimes, the estimated magnitudes of which are remarkably similar to those reported in the baseline specification.

IV.6. **Expectation effects of regime shifts in financial conditions.** In the previous section, we illustrated the role of financial frictions in propagating uncertainty shocks by comparing the economic outcomes of two possible regimes: one regime with high monitoring costs and another regime with low monitoring costs. The results were based not only on the estimated structural parameters of each regime but also on the transition matrix used by agents when forming their expectations. In this section, we run counterfactual exercises to gauge what would have happened if agents had considered different probabilities of moving across regimes. Such counterfactuals are interesting to execute because they allow the role of expectation effects of regime switching in financial conditions to be assessed.

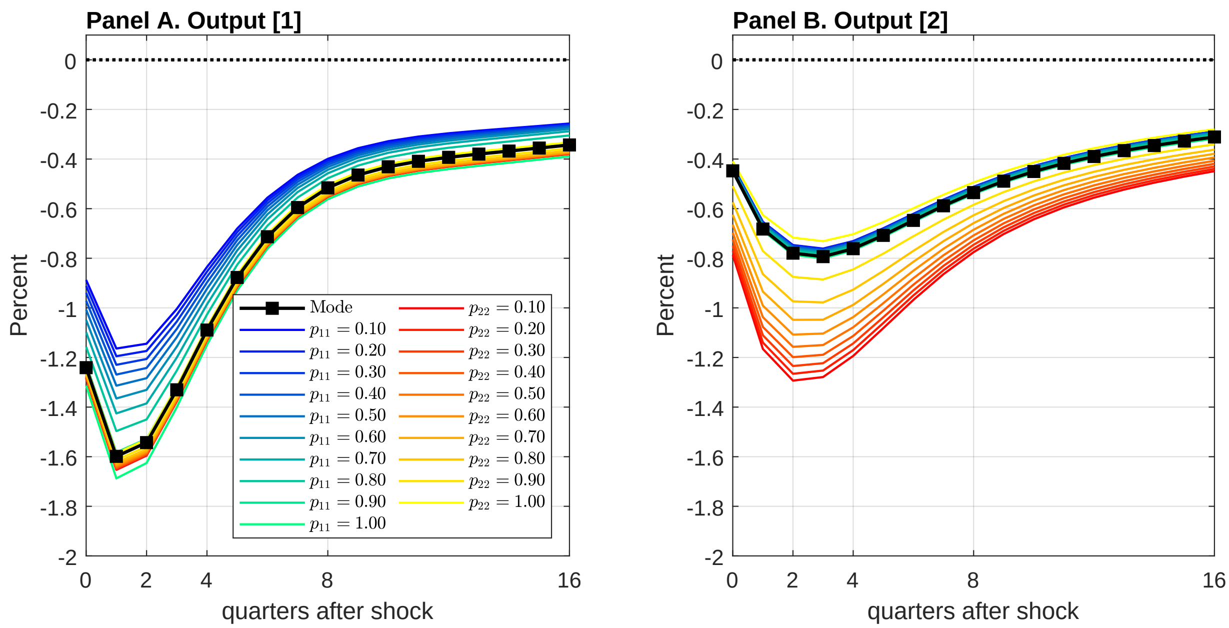

FIGURE 5. Impulse-response functions for uncertainty shocks as a function of the transition matrix.

Figure 5 displays the conditional impulse responses of output following an uncertainty shock when agents' beliefs about the probability of staying in a regime vary between 0.00 and 1.00. When considering $p_{ii} = 1$, agents believe that the regime which they are currently in will last indefinitely. Conversely, the more p_{ii} declines, the more the agents believe that the economy will move to the other regime in the next period. Clearly, expectation effects play an important role in shaping the dynamic behavior of economic activity. As one can see, if agents take into account the effects of possible changes in future financial conditions, macroeconomic outcomes are remarkably altered. For example, the more agents are optimistic about future financial conditions (i.e., gradual moves from the mode toward $p_{11} = 0$ or $p_{22} = 1$), the more the macroeconomic effects are dampened. This is particularly true for the distress regime, in which output effects are 0.40 percentage points smaller when agents believe that the economy will likely switch to the tranquil regime in the next period (small values of p_{11}). By contrast, the tranquil regime experiences relatively smaller effects. This is because the persistence of this regime (at the mode) is already very high ($p_{22} = 0.93$), implying modest additional expectation effects. Reciprocally, the pessimism of agents about financial conditions (i.e., gradual moves from the mode toward $p_{11} = 1$ or $p_{22} = 0$) amplifies the effects of uncertainty shocks, especially in the tranquil regime. The response of output to the shock is substantially amplified when agents believe the distress regime is around the corner (small values of p_{22}). The impulse response in the distress regime is mildly altered since the economy is already well anchored in that regime. Interestingly, it appears that the impulse responses of a particular regime are mildly affected by the persistence of the alternative regime. This is because the persistence of each particular regime is very high, implying that agents form their expectations by looking mostly at the current regime even if the persistence of the alternative regime is modified.

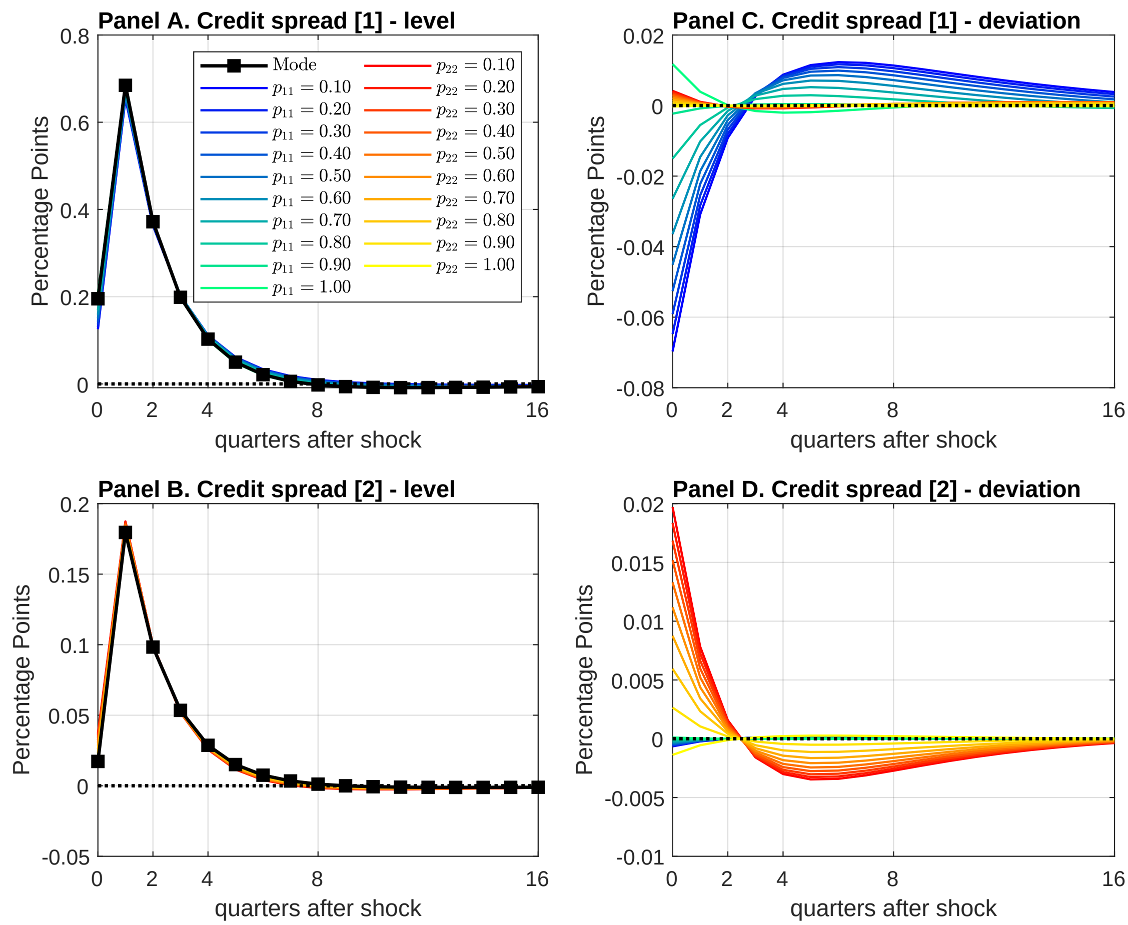

FIGURE 6. Impulse-response functions for uncertainty shocks as a function of the transition matrix (panels A and B) and in deviations relative to the benchmark case (panels C and D).

Figure 6 repeats our counterfactual exercises for the credit spread. Since the expectation effects are weakly visible (panels A and B), we also report the differences between the impulse responses at the mode and those from the counterfactuals (panels C and D). The response of credit spread is consistent with the previously shown output responses. Although numerically small, there are expectation effects. For example, the increase in the credit spread is diminished when agents are more optimistic, while it is amplified when agents become more pessimistic.

Overall, the role of expectation effects of regime switching in the degree of financial frictions appears to be important for amplifying or mitigating the propagation of uncertainty shocks throughout the economy. Therefore, these expectation effects are an important component of the financial accelerator mechanism.

V. Conclusion

Why are the real effects of uncertainty shocks so different over time? Our results point to a key role for changes in the degree of financial frictions; the financial accelerator is strengthened during distress periods. Under these circumstances, agents' expectations around the level of frictions can alter macroeconomic outcomes. Optimistic expectations about future financial conditions dampen the contractionary effects of uncertainty shocks on aggregate activity. Conversely, pessimistic expectations amplify their effects.

These conclusions have important implications for the conduct of monetary and macroprudential policies. For example, the bulk of the evidence suggests that these policies can reduce the frequency and severity of financial disruptions and thus the likelihood of observing a regime characterized by a high degree of financial frictions. In this context, if policymakers communicate to and persuade agents, in a clear way, that such policies are around the corner, then they can, even before implementing them, dampen the adverse effects of uncertainty shocks. The ability of policymakers to manage agents' expectations is crucial in shaping business cycle fluctuations.

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ONLINE APPENDIX: REGIME-DEPENDENT EFFECTS OF UNCERTAINTY SHOCKS: A STRUCTURAL INTERPRETATION

Not for Publication

STÉPHANE LHUISSIER AND FABIEN TRIPIER

This Appendix consists of the following sections:

A. Data
B. Bayesian inference for MS-SVAR model
C. Robustness results for MS-SVAR model
D. Equilibrium conditions of MS-DSGE model
E. Results for alternative specifications of MS-DSGE model

Appendix A. Data

All data are organized quarterly from the second Quarter of 1962 to the second Quarter of 2018. Most data comes from Federal Reserve Economic Database (FRED).

- gdp_t: output is the real GDP (GDPC1).
- vix_t: uncertainty is the Chicago Board of Options Exchange Market Volatility Index. From 1963 to 2009, we use the constructed index by Bloom (2009). Then, from 2009, we follow Stock and Watson (2012) and take a quarterly average of daily VIX.
- sp_t: credit spread is constructed as the difference between BAA corporate bond yields (BAA) and AAA corporate bond yields (AAA).

For inference, we use the natural log of output. Our spread and uncertainty variables remain unchanged.

Appendix B. Bayesian inference for MS-SVAR model

This section provides a detailed description of the Bayesian inference employed in this paper. More specifically, we closely follow Sims, Waggoner, and Zha (2008).

B.1. **The posterior.** Before describing the posterior distribution, we introduce the following notation: θ and q are vectors of parameters where θ contains all the parameters of the model (except those of the transition matrix) and $q = (q_{i,j}) \in \mathbb{R}^{h^2}$. $Y_t = (y_1, \ldots, y_t) \in (\mathbb{R}^n)^t$ are observed data with n denoting the number of endogenous variables and $S_t = (s_0, \ldots, s_t) \in H^{t+1}$ with $H \in \{1, \ldots, h\}$.

The log-likelihood function, $p(Y_T|\theta, q)$, is combined with the prior density functions, $p(\theta, q)$, to obtain the posterior density, $p(\theta, q|Y_T) = p(\theta, q)p(Y_T|\theta, q)$.

B.1.1. *The likelihood.* Following Hamilton (1989), Sims and Zha (2006), and Sims, Waggoner, and Zha (2008), we employ a class of Markov-switching structural VAR models of the following form:

$$y_t' A(s_t) = x_t' F(s_t) + \varepsilon_t' \Xi^{-1}(s_t), \tag{B.1}$$

with $x_t' = \begin{bmatrix} y_{t-1}' & \cdots & y_{t-\rho}' & 1 \end{bmatrix}$ and $F(s_t) = \begin{bmatrix} A_1(s_t) & \cdots & A_\rho(s_t) & C(s_t) \end{bmatrix}'$.

Let $a_j(k)$ be the jth column of $A(k)$, $f_j(k)$ be the jth column of $F(k)$, and $\xi_j(k)$ be the jth diagonal element of $\Xi(k)$. The conditional likelihood function is as follows:

$$p(y_t|s_t, Y_{t-1}) = |A(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| \exp\left(-\frac{\xi^2(s_t)}{2}\left(y_t' a_j(s_t) - x_t' f_j(s_t)\right)^2\right). \tag{B.2}$$

To simplify the Gibbs-sampling procedure described in the next section, it is preferable to rewrite the condition likelihood function with respect to free parameters from matrix $A(s_t)$ and $F(s_t)$:

$$|A(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| \exp\left(-\frac{\xi^2(s_t)}{2}\left((y_t' + x_t' W_j) U_j b_j(s_t) - x_t' V_j g_j(s_t)\right)^2\right), \tag{B.3}$$

where $a_j(s_t) = U_j b_j(k)$ and $f_j(s_t) = V_j g_j - W_j U_j b_j(k)$ is a result from the linear restrictions $R_j \begin{bmatrix} a_j & f_j \end{bmatrix}' = 0$; and U_j and V_j are matrices with orthonormal columns and W_j is a matrix. See Waggoner and Zha (2003) for further details.

The log likelihood function is given by

$$p(Y_T|\theta, q) = \sum_{t}^{T} \ln\left\{\sum_{s_t=1}^{h} p(y_t|s_t, Y_{t-1}) \Pr\left[s_t|Y_{t-1}\right]\right\}, \tag{B.4}$$

where

$$\Pr\left[s_t = i|Y_{t-1}\right] = \sum_{j=1}^{h} \Pr\left[s_t = i, s_{t-1} = j|Y_{t-1}\right] \tag{B.5}$$

$$= \sum_{j=1}^{h} \Pr\left[s_t = i|s_{t-1} = j\right] \Pr\left[s_{t-1} = j|Y_{t-1}\right]. \tag{B.6}$$

with $q_{i,j} = \Pr\left[s_t = i|s_{t-1} = j\right]$ are the transition probabilities from the $h \times h$ matrix Q

$$Q = \begin{bmatrix} q_{1,1} & \cdots & q_{1,j} \\ \vdots & \ddots & \vdots \\ q_{i,1} & \cdots & q_{i,j} \end{bmatrix} \tag{B.7}$$

The probability terms are updated as follows:

$$\Pr\left[s_t = j|Y_t\right] = \Pr\left[s_t = j|Y_{t-1}, y_t\right] = \frac{p(s_t = j, y_t|Y_{t-1})}{p(y_t|Y_{t-1})} \tag{B.8}$$

$$= \frac{p(y_t|s_t = j, Y_{t-1}) \Pr[s_t = j|Y_{t-1}]}{\sum_{j=1}^{h} p(y_t|s_t = j, Y_{t-1}) \Pr[s_t = j|Y_{t-1}]}. \tag{B.9}$$

B.1.2. *The prior.* Following Sims and Zha (1998), we exploit the idea of a Litterman's random-walk prior from structural-form parameters. Note that dummy observations are not introduced as a component of the prior to keep in line with the original Litterman's prior. Using linear restrictions, the overall prior, $p(\theta, q)$, is given in the following way:

$$p(b_j(k)) = \text{normal}(b_j(k)|0, \bar{\Sigma}_{b_j}), \tag{B.10}$$

$$p(g_j(k)) = \text{normal}(g_j(k)|0, \bar{\Sigma}_{g_j}), \tag{B.11}$$

$$p(\xi_j^2(k)) = \text{gamma}(\xi_j^2(k)|\bar{\alpha}_j, \bar{\beta}_j), \tag{B.12}$$

$$p(q_j) = \text{dirichlet}(q_{i,j}|\alpha_{1,j}, \dots, \alpha_{k,j}), \tag{B.13}$$

where $\bar{\Sigma}_{b_j}$, $\bar{\Sigma}_{\psi_j}$, and $\bar{\Sigma}_{\delta_j}$ denotes the prior covariance matrices and $\bar{\alpha}_j$ and $\bar{\beta}_j$ are set to one, allowing the standard deviations of shocks to have large values for some regimes.

The Gamma distribution is defined as follows:

$$\text{gamma}(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)} \beta^{\alpha} x^{\alpha-1} e^{-\beta x}. \tag{B.14}$$

Regarding the transition matrix, Q, suppose that $q_j = [q_{1,j}, \dots, q_{h,j}]'$. The prior, denoted $p(q_j)$, follows a Dirichlet form as follows:

$$p(q_j) = \left(\frac{\Gamma\left(\sum_{i \in H} \alpha_{i,j}\right)}{\prod_{i \in H} \Gamma(\alpha_{i,j})} \right) \times \prod_{i \in H} (q_{i,j})^{\alpha_{i,j}-1}, \tag{B.15}$$

where Γ denotes the standard gamma function.

B.2. **Gibbs-sampling.** Following Kim and Nelson (1999) and Sims, Waggoner, and Zha (2008), a Markov Chain Monte Carlo (MCMC) simulation method is employed to approximate the joint posterior density, $p(\theta, q, S_T|Y_T)$. The advantage of using VARs is that conditional distributions like $p(S_T|Y_T, \theta, q)$, $p(q|Y_T, S_T, \theta)$, and $p(\theta|Y_T, q, S_T)$ can be obtained in order to exploit the idea of Gibbs-sampling by sampling alternatively from these conditional posterior distributions.

B.2.1. *Conditional posterior densities,* $p(\theta|Y_T, q, S_T)$. To simulate draws of $\theta \in \{b_j(k), g_j(k), \xi_j^2\}$ from $p(\theta|Y_T, S_t, q)$, one can start to sample from the conditional posterior

$$\begin{aligned} p(b_j(k)|y_t, S_t, b_i(k)) = & \\ & \exp\left(-\frac{1}{2}b_j'(k)\bar{\Sigma}_{b_j}^{-1}b_j(k)\right) \\ & \times \prod_{t\in\{t:s_t=k\}}\left[|A(k)|\exp\left(-\frac{\xi^2(s_t)}{2}(y_t'a_j(k)-x_t'f_j(k))^2\right)\right], \end{aligned} \tag{B.16}$$

using the Metropolis-Hastings (MH) algorithm. Then a multivariate normal distribution is employed to draw $g_j(k)$:

$$p(g_j(k)|y_t, S_t) = \text{normal}(g_j(k)|\tilde{\mu}_{g_j(k)}, \tilde{\Sigma}_{g_j(k)}). \tag{B.17}$$

The computational details of the posterior mean vectors and covariance matrices are given in Sims, Waggoner, and Zha (2008).

Disturbance variances ξ_j^2 are simulated from a gamma distribution

$$p(\xi_j^2(k)|y_t, S_t) = \text{gamma}(\xi_j^2(k)|\tilde{\alpha}_j(k), \tilde{\beta}_j(k)), \tag{B.18}$$

where $\tilde{\alpha}_j(k) = \bar{\alpha}_j + \frac{T_{2,k}}{2}$ and

$$\tilde{\beta}_j(k) = \bar{\beta}_j + \frac{1}{2}\sum_{t\in\{t:s_{2t}=k\}}(y_t'a_j(s_t) - x_t'f_j(s_t))^2, \tag{B.19}$$

with $T_{2,k}$ denoting the number of elements in $\{t : s_{2t} = k\}$.

B.2.2. *Conditional posterior densities,* $p(S_T|Y_T, \theta, q)$. A multi-move Gibbs-sampling is employed to simulate $S_t, t = 1, 2, ..., T$. First, draw s_t according to

$$p(s_t|y_t, S_t) = \sum_{s_{t+1}\in H} p(s_t|Y_T, \theta, q, s_{t+1})p(s_{t+1}|Y_T, \theta, q), \tag{B.20}$$

where

$$p(s_t|Y_t, \theta, q, s_{t+1}) = \frac{q_{s_{t+1},s_t}p(s_t|Y_t, \theta, q)}{p(s_{t+1}|Y_t, \theta, q)}. \tag{B.21}$$

Then, in order to generate s_t, one can use a uniform distribution between 0 and 1. If the generated number is less than or equal to the calculated value of $p(s_t|y_t, S_t)$, we set $s_t = 1$. Otherwise, s_t is set equal to 0.

B.2.3. *Conditional posterior densities,* $p(q|Y_T, S_T, \theta)$. The conditional posterior distribution of q_j is as follows:

$$p(q_j|Y_t, S_t) = \prod_{i=1}^{h} (q_{i,j})^{n_{i,j}+\beta_{i,j}-1}, \tag{B.22}$$

where $n_{i,j}$ is the number of transitions from $s_{t-1} = j$ to $s_t = i$.

Appendix C. Robustness results for MS-SVAR model

C.1. The threshold of minimum contribution. 60%, 70%, and 80%.

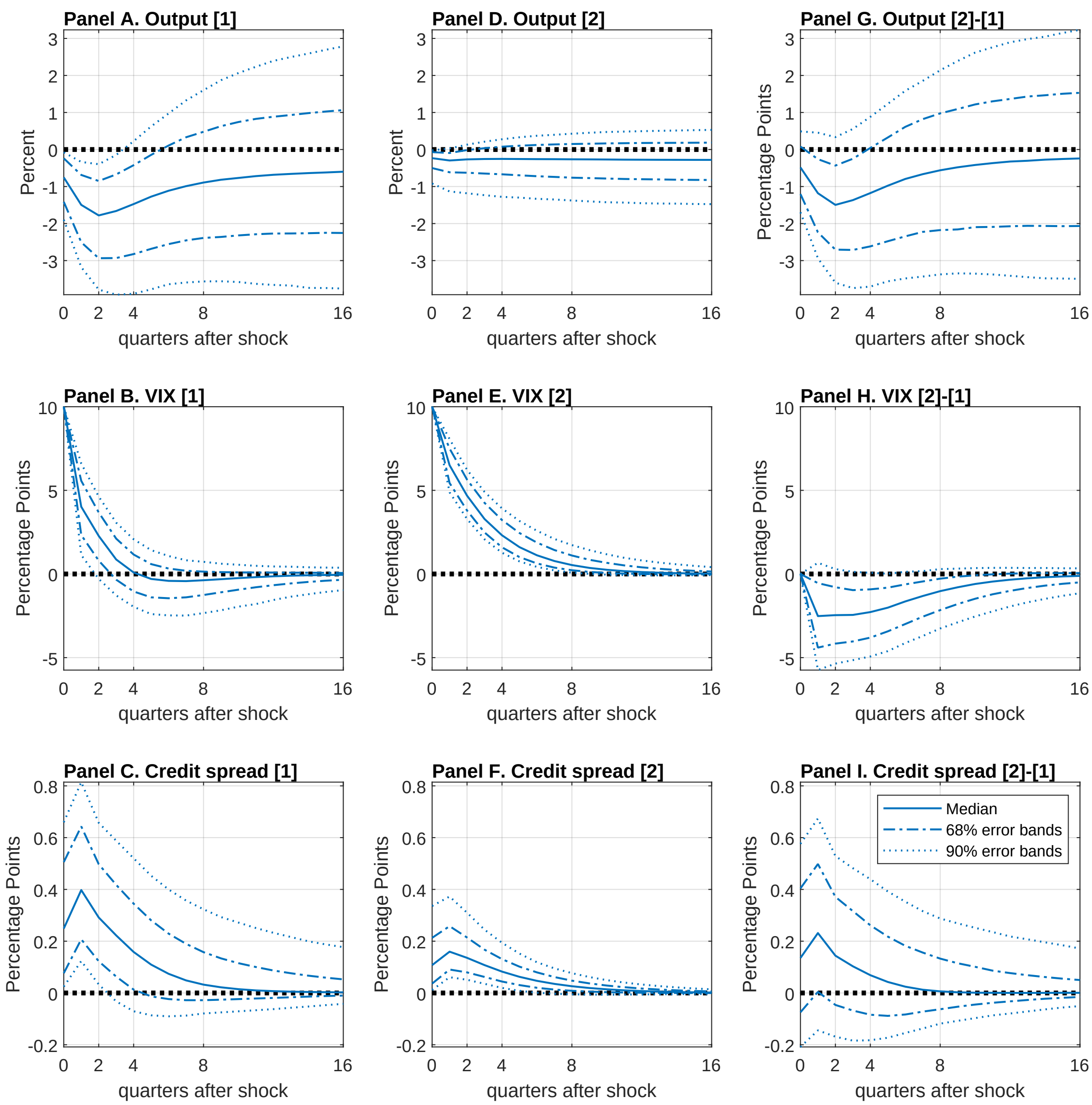

Figure 7. Impulse-response functions to uncertainty shock. Threshold of 60%.

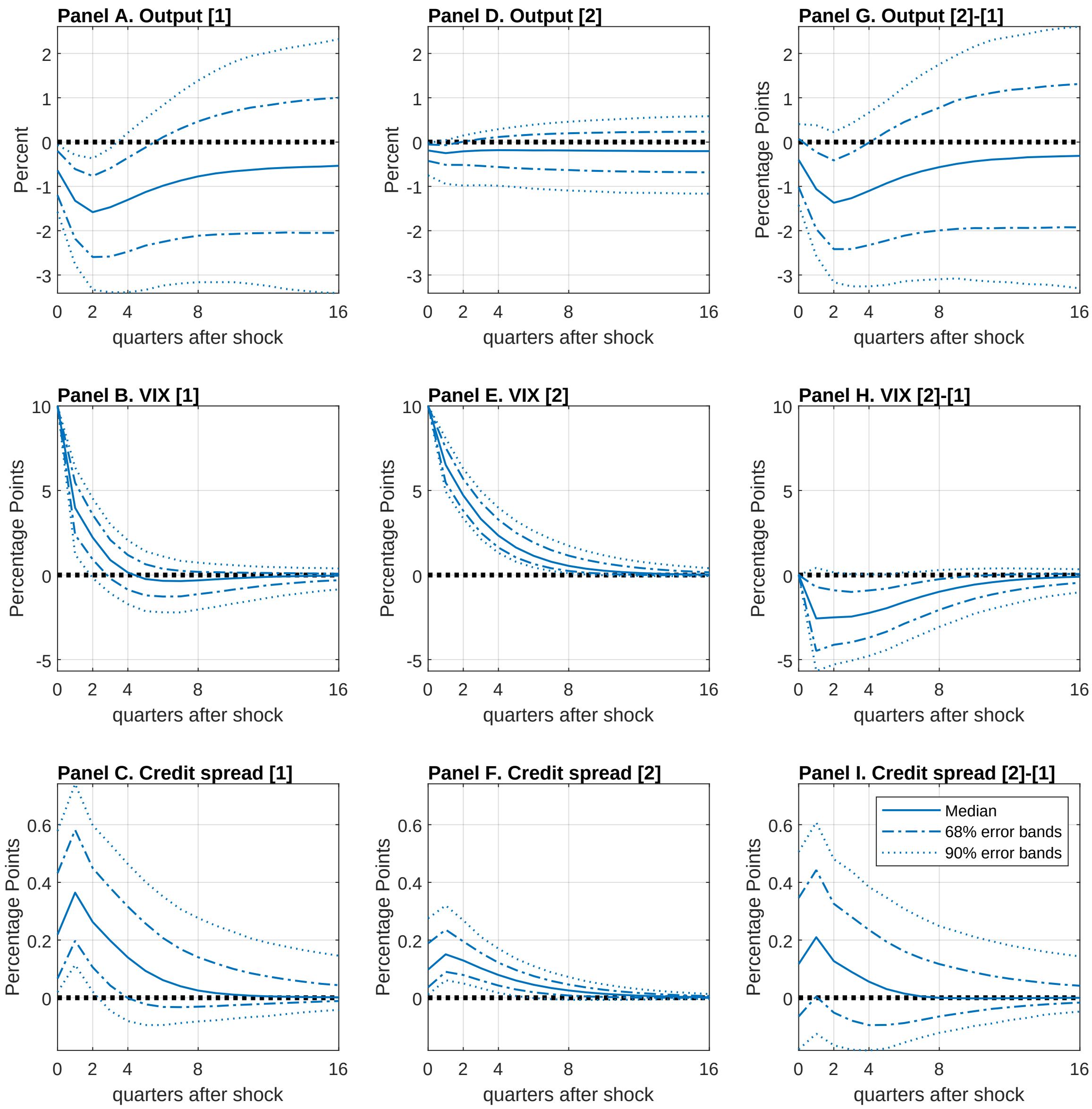

FIGURE 8. Impulse-response functions to uncertainty shock. Threshold of 70%.

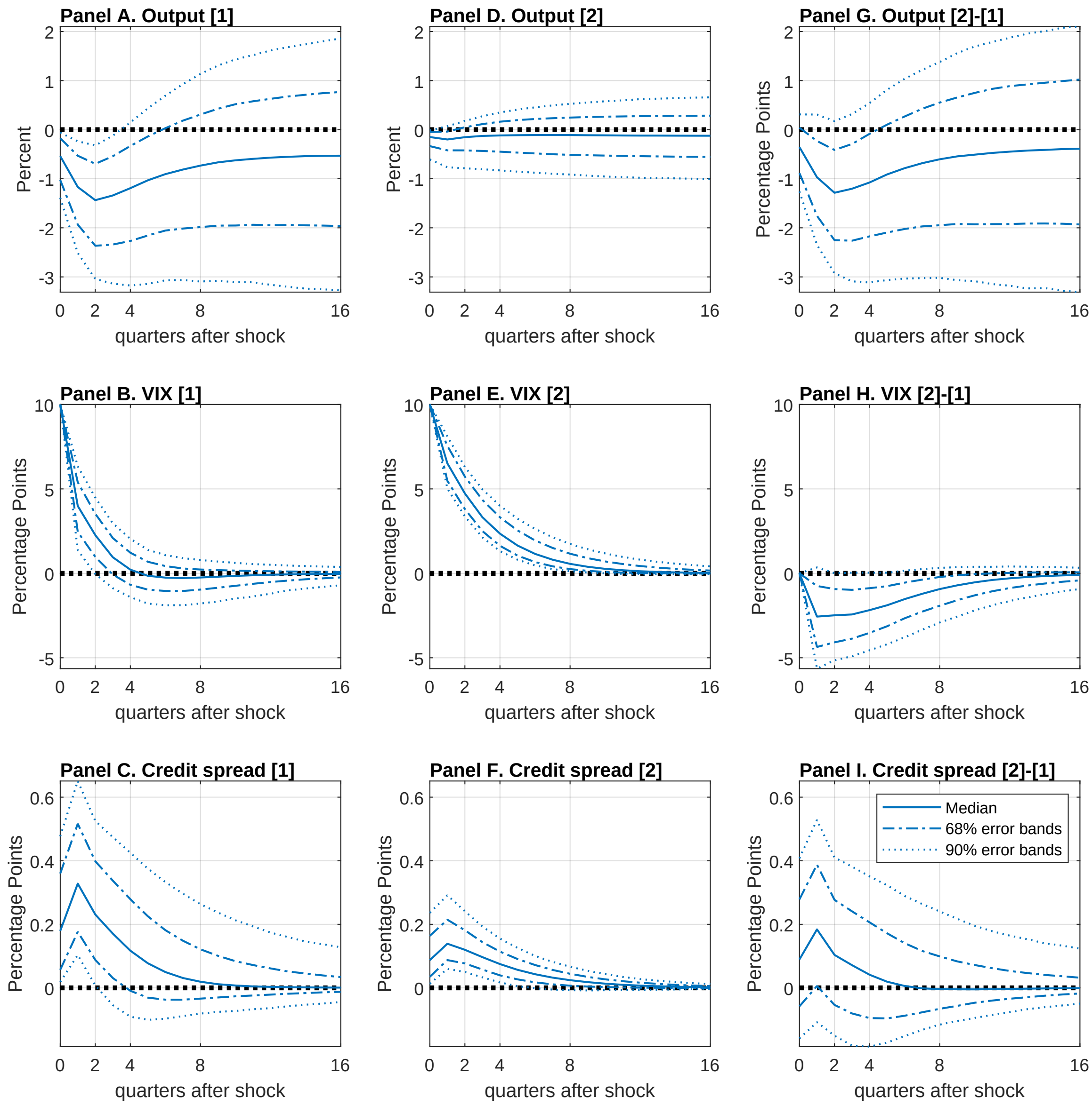

FIGURE 9. Impulse-response functions to uncertainty shock. Threshold of 80%.

C.2. **Alternative restrictions horizon.** Restrictions imposed for the next two quarters.

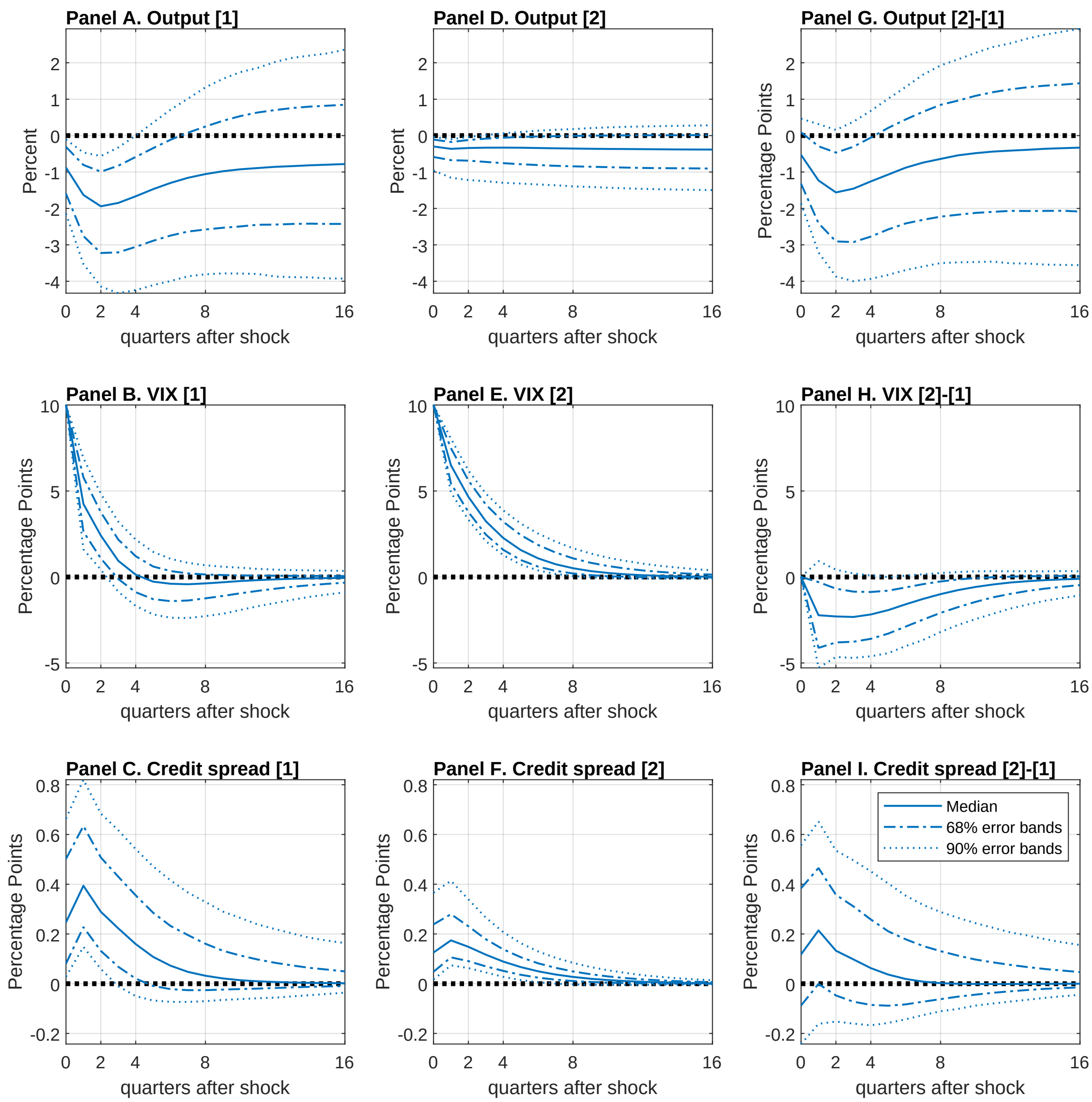

FIGURE 10. Impulse-response functions to uncertainty shock.

C.3. **Alternative restriction on forecast error variance.** Identification strategy in the spirit of Uhlig (2003).

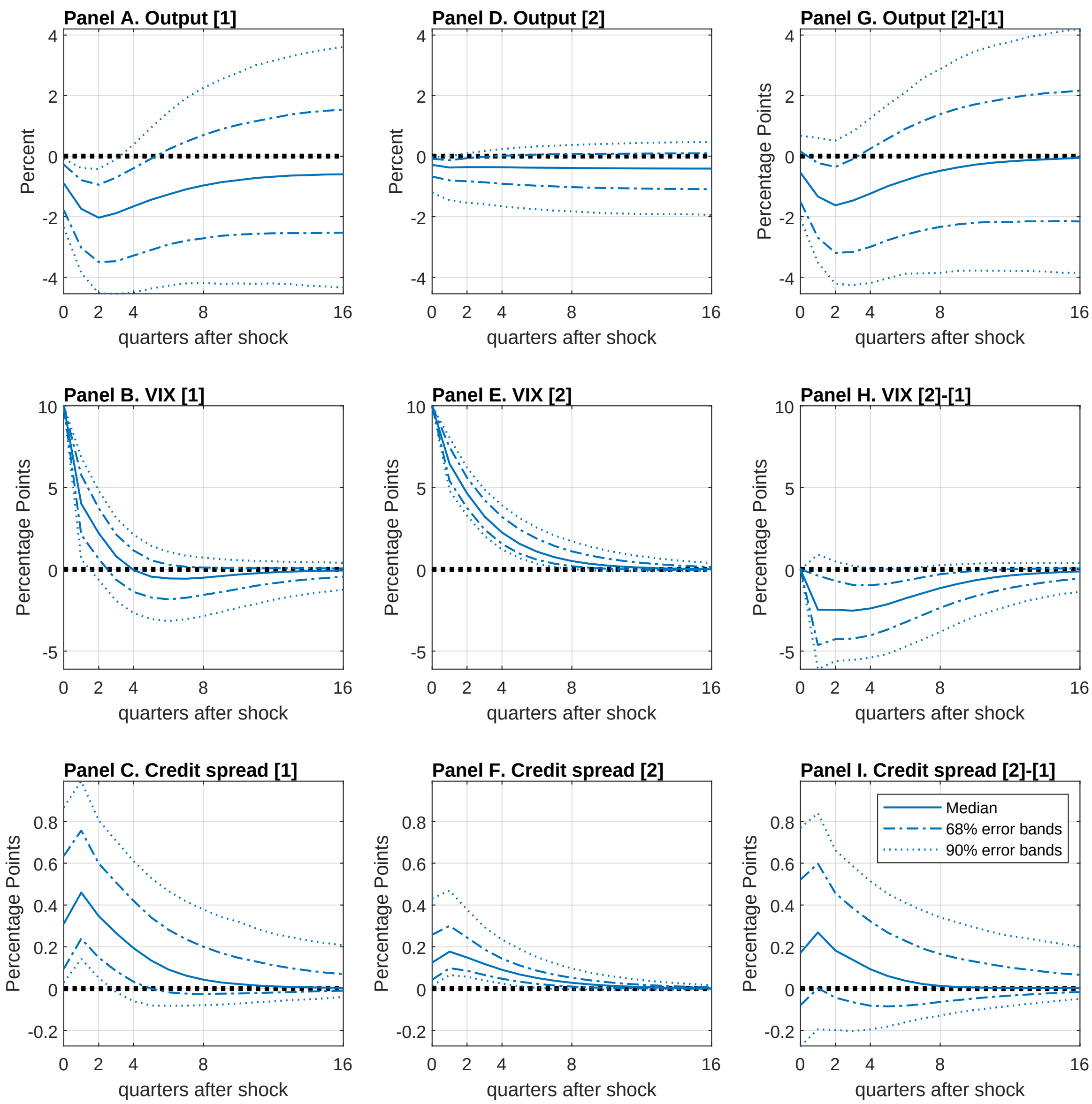

FIGURE 11. Impulse-response functions to uncertainty shock.

Appendix D. Equilibrium Conditions of MS-DSGE model

To solve our model, we require that the variables be stationary. The level of neutral and investment-specific technology have a unit root. The composite trend is then $z_t^* = z_t \Upsilon^{(\frac{\alpha}{1-\alpha})t}$ with the following steady state growth rate:

$$z^* = z\Upsilon^{(\frac{\alpha}{1-\alpha})}. \tag{D.1}$$

Several variables are then transformed to induce stationarity as follows:

$$c_t = \frac{C_t}{z_t^*}, \qquad y_t = \frac{Y_t}{z_t^*}, \qquad i_t = \frac{I_t}{z_t^* \Upsilon^t}, \qquad k_t = \frac{K_t}{z_{t-1}^* \Upsilon^{t-1}}, \qquad n_t = \frac{N_t}{P_{t-1} z_{t-1}^*} \tag{D.2}$$

$$g_t = \frac{G_t}{z_t^*}, \qquad w_t = \frac{W_t}{z_t^* P_t}, \qquad \mu_{z,t}^* = \frac{z_t*}{z_{t-1}^*}, \qquad gdp_t = \frac{GDP_t}{z_t^*}, \qquad w_t^e = \frac{W_t^e}{P_t z_t^*}. \tag{D.3}$$

The nominal rental rate on capital ($\tilde{r}_t^k P_t$) and the market price of capital are transformed to induce stationarity as well

$$r_t^k = \Upsilon^t \tilde{r}_t^k, \qquad q_t = \Upsilon^t \frac{Q_{K,t}}{P_t}. \tag{D.4}$$

We now re-write the model in a stationary form. The pricing equation by monopolistic producers is

$$p_t^* = \left((1-\xi_p) \left(\frac{K_{p,t}}{F_{p,t}} \right)^{\frac{\lambda_f}{1-\lambda_f}} + \xi_p \left(\frac{\tilde{\pi}_t}{\pi_t} p_{t-1}^* \right)^{\frac{\lambda_f}{1-\lambda_f}} \right)^{\frac{1-\lambda_f}{\lambda_f}} \tag{D.5}$$

with

$$F_{p,t} = \lambda_{z,t} y_t + \beta \mathrm{E}_t \left\{ F_{p,t+1} \, \xi_p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} \right\}, \quad \text{and} \tag{D.6}$$

$$K_{p,t} = y_t \lambda_{z,t} \lambda_f s_t + \beta \mathrm{E}_t \left\{ \xi_p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} \right\}, \tag{D.7}$$

which satisfy

$$K_{p,t} = F_{p,t} \left(\frac{1-\xi_p \left(\frac{\tilde{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p} \right)^{1-\lambda_f} \tag{D.8}$$

The inflation indexation rule is

$$\tilde{\pi}_t = \pi^{*\iota} \, \pi_{t-1}^{1-\iota} \tag{D.9}$$

The wage equation setting by labor contractor is

$$w_t^* = \left((1-\xi_w) \left(\frac{1-\xi_w \left(\mu_z \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right)^{\lambda_w} + \xi_w \left(\mu_z \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} w_{t-1}^* \right)^{\frac{\lambda_w}{1-\lambda_w}} \right)^{\frac{1}{\frac{\lambda_w}{1-\lambda_w}}} \tag{D.10}$$

with

$$F_{w,t} = \frac{h_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} \lambda_{z,t} \left(1-\tau_l\right)}{\lambda_w} + \beta \mathrm{E}_t \left\{ F_{w,t+1} \frac{\xi_w \mu_z^{*\frac{\lambda_w}{1-\lambda_w}} \tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} \left(\frac{1}{\tilde{\pi}_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right\}, \tag{D.11}$$

$$K_{w,t} = \left(h_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} \right)^{1+\sigma_L} + \beta \mathrm{E}_t \left\{ \xi_w \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \mu_z^* \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \right\}, \tag{D.12}$$

which satisfy

$$K_{w,t} = \left(\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mu_{z,t}^* \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right)^{1-\lambda_w(1+\sigma_L)} \tilde{w}_t \frac{F_{w,t}}{\psi_L}. \tag{D.13}$$

The wage inflation equation is

$$\pi_{w,t} = \pi_t \mu_z^* \frac{\tilde{w}_t}{\tilde{w}_{t-1}}, \tag{D.14}$$

and the indexation rule is

$$\tilde{\pi}_{w,t} = \left(\pi_t^{target}\right)^{\iota_w} \pi_{t-1}^{1-\iota_w}. \tag{D.15}$$

The efficiency condition for setting capital utilization is

$$r_t^k = \exp\left(\sigma_a \left(u_t - 1\right)\right) \bar{r}^k, \tag{D.16}$$

where the rental rate on capital is

$$r_t^k = \alpha \left(\frac{\Upsilon \mu_z h_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}}}{u_t k_{t-1}} \right)^{1-\alpha} s_t. \tag{D.17}$$

The capital utilization costs are given by

$$a_t = \frac{\bar{r}^k \left(\exp\left(\sigma_a \left(u_t - 1\right)\right) - 1 \right)}{\sigma_a}, \tag{D.18}$$

and the capital adjustment costs are as follows

$$S_t = \exp\left[\sqrt{\frac{S''\left(\chi_t\right)}{2}} \left(\Upsilon \mu_z \frac{i_t}{i_{t-1}} - \Upsilon \mu_z \right) \right] + \exp\left[-\sqrt{\frac{S''\left(\chi_t\right)}{2}} \left(\Upsilon \mu_z \frac{i_t}{i_{t-1}} - \Upsilon \mu_z \right) \right] - 2 \tag{D.19}$$

The level of output is given by

$$y_t = p_t^{*\frac{\lambda_f}{\lambda_f - 1}} \left(\left(\frac{u_t k_{t-1}}{\Upsilon \mu_z} \right)^{\alpha} \left(h_t w_t^{*\frac{\lambda_w}{\lambda_w - 1}} \right)^{1-\alpha} - \phi \right) \tag{D.20}$$

and the marginal cost of production is

$$s_t = \left(\frac{r_t^k}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha} \right)^{1-\alpha} \tag{D.21}$$

The stationary household first-order conditions are

$$\lambda_{z,t} = \beta \mathrm{E}_t \left\{ \frac{\lambda_{z,t+1}}{\mu_{z,t}^* \pi_{t+1}} (1 + R_t) \right\}, \tag{D.22}$$

$$(1+\tau_c)\lambda_{z,t} = \frac{\mu_{z,t}^*}{\mu_{z,t}^* c_t - b c_{t-1}} - \beta \mathrm{E}_t \left\{ \frac{b}{\mu_{z,t}^* c_{t+1} - b c_t} \right\}, \tag{D.23}$$

$$\lambda_{z,t} = \lambda_{z,t} q_t \left(-S_t' \frac{i_t}{i_{t-1}} \Upsilon \mu_{z,t}^* + 1 - S_t \right) + \beta \mathrm{E}_t \left\{ \frac{\lambda_{z,t+1} q_{t+1} S_{t+1}'}{\Upsilon \mu_{z,t}^*} \left(\frac{\Upsilon \mu_{z,t}^* i_{t+1}}{i_t} \right)^2 \right\}, \tag{D.24}$$

where (D.22), (D.23), and (D.24) are with respect to risk-free bonds, consumption, and investment, respectively.

Regarding the entrepreneurs, the zero profit condition is as follows

$$\frac{q_t k_{t+1}}{n_{t+1}} \frac{R_{t+1}^k}{R_t} \left[\Gamma_t(\omega_{t+1}) - \mu(\chi_t) G_t(\omega_{t+1}) \right] - \frac{q_t k_{t+1}}{n_{t+1}} + 1 = 0, \tag{D.25}$$

where

$$G_t(\omega_{t+1}) \equiv \int_0^{\omega_{t+1}} \omega dF_t(\omega), \qquad \text{and} \qquad \Gamma_t(\omega_{t+1}) \equiv \omega_{t+1} \left[1 - F_t(\omega_{t+1}) \right] + G_t(\omega_{t+1}). \tag{D.26}$$

The stationary entrepreneur first-order condition with respect to capital is given by

$$\begin{aligned} 0 = & \mathrm{E}_t \left\{ \frac{(1 - \Gamma_t(\omega_{t+1})) R_{t+1}^k}{R_t} \right. \\ & \left. + \frac{\Gamma_t(\omega_{t+1})'}{\Gamma_t(\omega_{t+1})' - \mu(\chi_t) G_t(\omega_{t+1})'} \left(\frac{R_{t+1}^k}{R_t} \left(\Gamma_t(\omega_{t+1}) - \mu(\chi_t) G_t(\omega_{t+1}) \right) - 1 \right) \right\}, \end{aligned} \tag{D.27}$$

where $\Gamma_t(\omega_{t+1})' = 1 - F_t(\omega_{t+1})$.

The return of capital for entrepreneurs is

$$R_t^k = \frac{(1-\tau_k)\left(r_t^k u_t - a_t \right) + (1-\delta) q_t}{\Upsilon q_{t-1}} \pi_t + \delta \tau_k, \tag{D.28}$$

The law of motion of entrepreneurial net worth is given by

$$n_{t+1} = q_t k_{t-1} \frac{\gamma}{\pi_t \, \mu_{z,t}^*} \left\{ R_t^k - R_{t-1} - \mu(\chi_t) \int_0^{\omega_t} \omega dF_{t-1}(\omega) R_t^k \right\} + n_t \left(\frac{R_{t-1}}{\pi_t \mu_{z,t}^*} \right) \gamma + W^e. \quad \text{(D.29)}$$

The monetary policy rule is as follows

$$\begin{aligned} \log\left(\frac{R_t}{R}\right) &= \rho_R \log\left(\frac{R_{t-1}}{R}\right) + \phi_\pi \left(1-\rho_R\right) \frac{\pi^*}{R} \log\left(\frac{\pi_{t+1}}{\pi^*}\right) \\ &\quad + \phi_y \left(1-\rho_R\right) \frac{\mu_z}{4R} \left(\frac{c_R \log\left(\frac{c_t}{c_{t-1}}\right) + i_R \log\left(\frac{i_t}{i_{t-1}}\right)}{\left(c_R + i_R\right) / \left(1-\eta_g\right)} \right), \end{aligned} \quad \text{(D.30)}$$

where c_R and i_R coefficients account for the share of consumption and investment in GDP.

The stationary resource constraint of the economy is

$$y_t = c_t + g_t + i_t + a_t \frac{k_t}{\Upsilon \mu_z^*} + d_t + \Theta \frac{(1-\gamma)}{\gamma} \left(n_{t+1} - W^e\right). \quad \text{(D.31)}$$

where Θ is the share of assets consumed by dying entrepreneurs and d_t the monitoring costs, which are given by

$$d_t = \frac{\mu(\chi_t) G\left(\overline{\omega}_t\right) R_t^k q_{t-1} k_t}{\pi_t \mu_z^*}. \quad \text{(D.32)}$$

The law of motion for physical capital is

$$k_{t+1} = (1-\delta) \frac{k_t}{\Upsilon \mu_{z,t}^*} + \left(1 - S\left(\frac{i_t \mu_{z,t}^* \Upsilon}{i_{t-1}}, \chi_t\right)\right) i_t, \quad \text{(D.33)}$$

The law of motion for the uncertainty shock is as follows

$$\log \sigma_t = (1 - \rho_\sigma(\chi_t)) \log \sigma + \rho_\sigma(\chi_t) \log \sigma_{t-1} + \varepsilon_{\sigma,t}, \quad \text{(D.34)}$$

with $\varepsilon_{\sigma,t} = \text{normal}(\varepsilon_{\sigma,t} | 0, \sigma_\sigma(\chi_t))$.

Appendix E. Results for alternative specifications of MS-DSGE model

Table 5. Posterior distribution, alternative specifications

		Model			
		(1)	(2)	(3)	(4)
Coefficient	Description	Benchmark	Taylor rule LWZ (2011)	Nominal rigidities LWZ (2011)	Flexible prices and wages
p_{11}	Transition matrix	0.9170 [0.7971;0.9600]	0.9134 [0.7997;0.9566]	0.9139 [0.8066;0.9558]	0.9079 [0.7851;0.9440]
p_{22}	Transition matrix	0.9684 [0.8385;0.9868]	0.9794 [0.8105;0.9856]	0.9670 [0.8368;0.9867]	0.9989 [0.7851;0.9996]
κ	Measurement VIX	0.9142 [0.5849;1.5301]	0.9516 [0.6492;1.5800]	0.7720 [0.5271;1.3030]	0.7305 [0.5234;1.2774]
$S''(\chi_t = 1)$	Investment adj. costs	0.5993 [0.2820;1.2952]	0.6453 [0.2799;1.4609]	0.5337 [0.2419;1.3725]	0.9779 [0.7734;1.7522]
$S''(\chi_t = 2)$	Investment adj. costs	1.3437 [0.7985;1.9746]	1.4032 [0.8543;2.0745]	0.9059 [0.4270;1.6728]	2.0424 [1.6096;2.6620]
$\mu(\chi_t = 1)$	Monitoring costs	0.2090 [0.1125;0.3231]	0.2076 [0.1130;0.3198]	0.2169 [0.1264;0.3345]	0.1631 [0.1059;0.2656]
$\mu(\chi_t = 2)$	Monitoring costs	0.0615 [0.0326;0.1290]	0.0597 [0.0387;0.1385]	0.0662 [0.0402;0.1303]	0.0682 [0.0503;0.1326]
$\rho_\sigma(\chi_t = 1)$	Persistence shock	0.5509 [0.2962;0.6603]	0.5156 [0.2714;0.6305]	0.5980 [0.3374;0.7116]	0.5227 [0.2235;0.7610]
$\rho_\sigma(\chi_t = 2)$	Persistence shock	0.7689 [0.6902;0.8103]	0.7493 [0.6846;0.7971]	0.7730 [0.7011;0.8095]	0.7425 [0.6211;0.7972]
$\sigma_\sigma(\chi_t = 1)$	Std. Dev. shock	0.4096 [0.3197;0.5682]	0.4003 [0.3213;0.5650]	0.4018 [0.3118;0.5418]	0.3805 [0.2809;0.4944]
$\sigma_\sigma(\chi_t = 2)$	Std. Dev. shock	0.4204 [0.3045;0.5588]	0.4227 [0.3041;0.5389]	0.4528 [0.3316;0.5708]	0.3130 [0.2369;0.3620]
Log Marginal Likelihood		−22.5363	−22.6416	−22.4009	−44.1524

Note: Posterior modes and 90% probability intervals are reported. LWZ (2011) stands for Liu, Waggoner, and Zha (2011).