Appendix A

Further Generalizations the Baseline Model

In this section we further generalize the Baseline model to admit (1) incomplete depreciation, (2) increased risk aversion, (3) habit formation, (4) the addition of a labor-leisure choice, and (5) factor share uncertainty. We describe in detail the consequences of modification (1) as it has substantial implications for the observed ACT-correlation patterns. Because their effects are largely marginal, we only offer a brief summary of the results arising from generalizations (3) – (6)[1].

A.1 Incomplete Depreciation

In this section we modify the model of Section 5.1 to admit partial depreciation. As a result, the equation of motion on capital stock becomes:

$$k_{t+1} = \left(1 - \Omega\right) k_t + i_t \quad .$$

With this change, closed form expressions for the risk-free bond price and its rate of return are not available. We therefore rely exclusively on numerical simulation. Table 7.1 summarizes the results.

[1] In a private communication, Parantap Basu has pointed out that the introduction of a cost of adjustment function in conjunction with time varying volatility of the productivity shock may overturn the generally observed mean aversion in equity and risk free returns. This modification is left for future work.

Table A.1
Autocorrelations, ACTs
Baseline Case: $u(c)=\log(c)$ **,** $\alpha=.36$, $\beta=.96$, $\sigma_{\varepsilon}^{2}=.00712$
Various Ω, ρ

Panel A: Autocorrelations

	$\Omega=1$			$\Omega=.4$			$\Omega=.025$		
	$\rho=0$	$\rho=.4$	$\rho=.95$	$\rho=0$	$\rho=.4$	$\rho=.95$	$\rho=0$	$\rho=.4$	$\rho=.95$
$\mathrm{corr}\left(\tilde{p}_t^e,\tilde{p}_{t+1}^e\right)$	0.39	0.68	0.98	0.67	0.84	0.99	0.97	0.98	1.00
$\mathrm{corr}\left(\tilde{p}_t^b,\tilde{p}_{t+1}^b\right)$	0.39	0.80	0.46	0.67	0.87	0.67	0.97	0.60	0.93
$\mathrm{corr}\left(\tilde{r}_t^e,\tilde{r}_{t+1}^e\right)$	-0.30	-0.02	0.35	-0.13	0.20	0.63	0.02	0.41	0.93
$\mathrm{corr}\left(\tilde{r}_t^b,\tilde{r}_{t+1}^b\right)$	0.39	0.80	0.46	0.67	0.87	0.67	0.97	0.60	0.93
$\mathrm{corr}\left(\tilde{c}_t,\tilde{c}_{t+1}\right)$	0.39	0.68	0.98	0.67	0.82	0.98	0.97	0.98	1.00
$\mathrm{corr}\left(\tilde{k}_t,\tilde{k}_{t+1}\right)$	0.39	0.68	0.98	0.67	0.84	0.99	0.97	0.98	1.00

Panel B: ACTs

$\mathrm{ACT}\left(\tilde{p}_t^e\right)$	2.696	3.827	13.89	3.77	5.49	19.92	11.31	16.89	74.63
$\mathrm{ACT}\left(\tilde{p}_t^b\right)$	2.697	4.876	2.84	3.78	6.11	3.81	11.19	3.35	8.51
$\mathrm{ACT}\left(\tilde{r}_t^e\right)$	1.658	1.968	2.61	1.83	2.32	3.58	2.01	2.737	8.33
$\mathrm{ACT}\left(\tilde{r}_t^b\right)$	2.691	4.898	2.84	3.77	6.07	3.81	11.22	3.35	8.52

The message of Panels A and B of Table 7.1 is unambiguous: lower depreciation rates (smaller Ω) increase autocorrelations for all price and return series. Compatible results are found in the ACT measurements. When $\Omega = .025$ all series become positively autocorrelated, even in the case of $\left\{\tilde{r}_t^e\right\}$ when $\rho = 0$. This result stands in contrast to the conclusions of Proposition 3.3 (which applies only to the $\Omega = 1$ case)

Why is this observed? When the depreciation rate declines, ceteris paribus, the period t+1 capital stock becomes more similar to its period t predecessor. At the same time, investment, which is volatile, shrinks as a proportion of period t+1 capital. As a result, capital stock becomes more highly autocorrelated as indicated at the bottom of Table 7.1 Panel A. If capital stock becomes more highly autocorrelated so must the series $\left\{\tilde{p}_t^e\right\}$ and $\left\{\tilde{r}_t^e\right\}$. The consumption series is similarly affected. If consumption becomes more highly autocorrelated, so will the risk-free bond price series and the risk-free return.

A.2 Other Parameter Changes

As mentioned earlier, we also explored the consequences of increasing risk aversion (directly or indirectly, via external habit formation), the addition of endogenous labor/leisure choice etc. These results are summarized below. In all cases the conclusions reported are based on the values (entry by entry) associated with the same parameters ($\alpha, \beta, \rho, \Omega, \sigma_t^2$) as employed in Table 5.1.

A.2.1 Greater Risk Aversion[2]

Entry by entry higher risk aversion, ceteris paribus, increases autocorrelations and ACTs across all return and price series. Greater

[2] Here we expand the basic model to include period preference orderings captured by $u\left(c_t\right) = c_t^{1-\gamma} / 1 - \gamma$, for various $\gamma > 1$.

representative agent risk aversion translates into the desire for a smoother intertemporal consumption stream, which leads to higher consumption autocorrelation. It directly follows that the price of one unit of consumption next period, the risk-free bond price, and its associated return will become more highly autocorrelated as well.

On the equity side, in order to promote a smoother consumption path, the path of the capital stock, $\{\tilde{k}_t^e\} = \{\tilde{p}_t^e\}$ must be made intertemporally more stable – more positively autocorrelated at the expense of greater investment volatility (to which the representative agent is indifferent). Accordingly, the equity price and return series becomes more highly autocorrelated as well. In summary, within the CRRA class of preference orderings, greater risk aversion promotes Property I mean aversion for all financial series; ACT patterns follow in tandem.

It is well known that habit formation causes the agent to behave in a more risk averse fashion[3]. Following our earlier observations we find that the addition of habit formation increases autocorrelations and ACTs across the board (all cases of returns and prices). The logic behind this effect is also unchanged from our earlier explanation. Higher degrees of risk aversion further compound the effect.

[3] In the Baseline case, this means modifying the representative agent's period utility function to be of the form $u\left(c_t - \psi c_{t-1}\right) = \log\left(c_t - \psi c_{t-1}\right)$; with higher risk aversion, CRRA utilities are modified similarly.

A.2.2 Adding a Labor/Leisure Choice[4]

First, if $\Omega = 1$, then the addition of a labor/leisure choice has no impact on the autocorrelations or ACTs for any of the financial series we study. If $\Omega < 1$, then the addition of a labor/leisure choice slightly diminishes the autocorrelations and ACTs for all the series.

In the cases where $\Omega = 1$, the equilibrium level of hours worked, n_t is independent of the shock and capital stock values. The extent of hours represents a level effect alone, the ACTs are thus unaffected. In the cases where $\Omega < 1$, the fact that the ACTs are all somewhat diminished indicates that the addition of a labor decision variable tends to pull the capital stock and consumption series back towards their means, relative to an environment in which it is absent. The effect is very modest, however, and greatest in the $\Omega = .025$ cases where the decline in ACT magnitude is about 10%. This is a way of saying that variations in the supply of the agent's labor assist in stabilizing both the economy's capital stock series (and thus reduce the ACTs of $\left\{\tilde{p}_t^e\right\}$ and $\left\{r_t^e\right\}$), and its consumption series (and thus reduce the ACTs of $\left\{\tilde{p}_t^b\right\}$ and $\left\{r_t^b\right\}$), a fact well known in the business cycle literature. The effect is small, however, not only because the agent also prefers low variation in leisure, $\left(1-n_t\right)$, but also for the fact that the capital

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[4] We do this in two related ways by specifying the representative agent's period utility function to be either

$$(1) \qquad u\left(c_t, 1-n_t\right) = \log c_t + A\log\left(1-n_t\right)$$

or

$$(2) \qquad u\left(c_t, 1-n_t\right) = (c_t^{\delta}\left(1-n_t\right)^{1-\delta})^{1-\gamma} / 1-\gamma$$

where, n_t is the hours of labor supplied in period *t*. In either case the production function is generalized to be of the form

$f\left(k_t, n_t\right)e^{\tilde{\lambda}_t} = k_t^{\alpha}\left(n_t\right)^{1-\alpha} e^{\tilde{\lambda}_t}$.

stock and hours series are themselves very highly positively correlated. The ACTs thus exceed 2.

We summarize these observations as follows:

1. For the representative agent class of models, mean aversion, as defined by Properties I and II, in equity and risk free returns is the norm, except in the Baseline case where $\Omega = 1$, and shock correlation is low.

2. Conditional on the same levels of Ω and ρ, the previous sections demonstrate that the addition (individually or collectively) of a wide variety of model features to the Baseline paradigm only serves to increase the Property I--II mean *aversion* in the time series of interest. If the source of uncertainty is a multiplicative productivity shock, we venture to suggest that this feature will generally be observed, whatever additional features are imposed.

In short, Property I or II "mean aversion" appears to rule for all financial time series within this family of DSGE models if the models are to have empirical relevance.

A.3 Factor Share Uncertainty

The decision to introduce factor share uncertainty arises from the results reported in Guvenen (2009). He observes a mild negative autocorrelation in the equity premium at lags of 1, 2, 3, 5 and 7 years arising in an incomplete markets model where shareholders trade both equity and debt, but workers trade only debt securities, a restriction that generates time varying income shares to these two groups. Following Lansing (2015), we capture variation in factor shares by a reduced form model where uncertainty arises via a stochastic production parameter $\tilde{\alpha}$. In the family of complete market models we have been considering, $\{\alpha_t\}$ represents the share of income to capital[5].

For all indicated parameter combinations, both the correlations and the corresponding ACT's are, by and large, similar to those in Table 5.1 except for the mild negative autocorrelation of $\left\{\tilde{r}_t^p\right\}$ as per Guvenen's (2009) results. In addition, the return on equity is very slightly mean averting for all ρ values unlike in the Baseline case, and the monotonicity in equity return autocorrelations and ACTs also observed in the Baseline case is lost. Otherwise, if uncertainty in the capital share parameter is introduced into model (6), it does not radically alter the correlation and ACT structure characteristic of the Baseline formulation.

[5] In particular, we explore the extent of mean reversion in the simple complete markets model identified by the following optimum formulation:

$$\max_{\{z_t\}} E\left(\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)\right)$$

$$c_t + z_t \leq k_t^{\tilde{\alpha}_t}$$

$$k_{t+1} = \left(1-\Omega\right)k_t + z_t$$

$$\tilde{\alpha}_t = \rho\,\alpha_{t-1} + \tilde{\varepsilon}_t,\ \tilde{\varepsilon}_t \sim N\left(0,\sigma_\varepsilon^2\right)$$

Note that the optimal decision rules for the model of this footnote are identical to those for the Baseline formulation.

Appendix B

List of Variables

. $\tilde{p}_t^e$.: the price of equity (residual claims) in period t.

$\tilde{p}_t^b$: the price of a one-period zero coupon bond in zero net supply in period t.

r_t^e : the one period real rate of return on the equity security from period t-1 to t.

r_t^b : the one period real rate of return on the one period zero coupon bond from period t-1 to t.

r_t^b : the one period equity premium, r_t^e - r_t^b, from period t-1 to t.

d_t : the representative firms's dividend in period t.

i_t : the representative firms's investment in period t, which becomes productive capital in period t+1.

k_t : the representative firms's capital stock at the start of period t, just prior to production taking place.

c_t : the representative shareholder-worker's real consumption in period t.

Ω : the period depreciation rate of capital stock during period t.

β : the subjective discount factor of the representative shareholder-worker.

α : constant returns to scale production function parameter identifying the share of income to capital when the capital market and the labor market are competitive.

n_t : the representative shareholder-worker's period t labor supplied to the representative firm.

λ_t: the total factor productivity shock to the representative firm's technology.

$u(\ )$: period utility function of the representative shareholder-worker.

$f(\cdot,\cdot)$: the representative firm's constant-returns-to-scale production function.

ε_t: the i.i.d. component of the representative firm's total factor productivity shock.

ρ: persistence parameter when the firm's total factor productivity shock follows an AR-1 process.