Supplement to "Social Distancing and Supply Disruptions in a Pandemic"

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A Additional Details for the Two-Sector Model

This appendix derives the equilibrium conditions for the two-sector model and the steady-state condition. Finally, it also shows the derivation of the elasticity of substitution between the two factor inputs in the production function for final output goods.

A.1 Equilibrium conditions

Households maximize

$$\max_{c_t,\lambda_{c,t},i_t,k_t,\lambda_{k,t},u_t,\lambda_{i,t}} E_t \sum_{i=0}^{\infty} \beta^i \left[\log(c_{t+i} - \kappa c_{t+i-1}) \right.$$
$$+\lambda_{c,t+i}\left(-c_{t+i} - i_{t+i} + w_{1,t+i}l_{1,t+i} + w_{2,t+i}l_{2,t+i} + r_{k,t+i}u_t k_{t+i-1} - \nu_0 \tfrac{u_t^{1+\nu}}{1+\nu} - \tfrac{\zeta}{2}\tfrac{(i_t-i_{t-1})^2}{i_{t-1}}\right)$$
$$\left. +\lambda_{k,t+i}\left(-k_{t+i} + (1-\delta)k_{t+i-1} + i_{t+i}\right) + \lambda_{i,t+i}\left(i_t - \phi i^{ss}\right)\right].$$

These are the first-order conditions from the households' problem:

$$\frac{1}{c_t - \kappa c_{t-1}} - \beta\kappa E_t \frac{1}{c_{t+1} - \kappa c_t} = \lambda_{c,t}, \tag{16}$$

$$c_t + i_t = w_{1,t}l_{1,t} + w_{2,t}l_{2,t} + r_{k,t}k_{t-1} - \nu_0 \frac{u_t^{1+\nu}}{1+\nu} - \frac{\zeta}{2}\frac{(i_t - i_{t-1})^2}{i_{t-1}}, \tag{17}$$

$$\lambda_{c,t}\left[1 + \zeta\frac{(i_t - i_{t-1})}{i_{t-1}}\right] - \beta\lambda_{c,t+1}\left[\zeta\frac{(i_{t+1} - i_t)}{i_t} + \frac{\zeta}{2}\frac{(i_{t+1} - i_t)^2}{i_t^2}\right] = \lambda_{k,t} + \lambda_{i,t+i}, \tag{18}$$

$$E_t\lambda_{c,t+1}r_{k,t+1}u_{t+1} - \lambda_{k,t} + \beta(1-\delta)E_t\lambda_{k,t+1} = 0, \tag{19}$$

$$k_t = (1-\delta)k_{t-1} + i_t, \tag{20}$$

$$\lambda_{c,t}r_{k,t}k_{t-1} = \lambda_{c,t}\nu_0 u_t^{\nu}, \tag{21}$$

and the complementary slackness condition

$$\lambda_{i,t+i}\left(i_t - \phi i^{ss}\right) = 0. \tag{22}$$

Firms in Sector 1 solve this cost-minimization problem

$$\min_{l_t^1} w_t l_t^1 + p_t^1\left[v^1 - \gamma\left(l_t^1 - \nu\right)\right]$$

And from the production function, we also have that

$$v_t^1 = \max\left[\gamma\left(l_t^1 - \chi\right), 0\right] \tag{23}$$

and that

$$w_t = \gamma p_t^1. \tag{24}$$

Firms in Sector 2 solve this cost-minimization problem

$$\min_{u_t k_{t-1}, l_t^2, v_t^1} r_{k,t} u_t k_{t-1} + w_{2,t} l_{2,t} + p_{1,t} v_{1,t}$$
$$+\left[y_t - \left((1-\omega)^{\frac{\rho}{1+\rho}}\left(v_{1,t}\right)^{\frac{1}{1+\rho}} + \omega^{\frac{\rho}{1+\rho}}\left(\left(u_t k_{t-1}\right)^{\alpha}\left(l_{2,t}\right)^{1-\alpha}\right)^{\frac{1}{1+\rho}}\right)^{1+\rho}\right]$$

Notice that firms choose $u_t k_{t-1}$ as if it were a single input, representing capital services. The first-order conditions for this problem are:

$$r_{k,t} - (1+\rho)\left((1-\omega)^{\frac{\rho}{1+\rho}}\left(v_{1,t}\right)^{\frac{1}{1+\rho}} + \omega^{\frac{\rho}{1+\rho}}\left(k_{t-1}^{\alpha} l_{2,t}^{1-\alpha}\right)^{\frac{1}{1+\rho}}\right)^{\rho}$$
$$\frac{1}{1+\rho}\omega^{\frac{\rho}{1+\rho}}\left(\left(u_t k_{t-1}\right)^{\alpha} l_{2,t}^{1-\alpha}\right)^{\frac{1}{1+\rho}-1}\alpha\left(u_t k_{t-1}\right)^{\alpha-1} l_{2,t}^{1-\alpha} = 0.$$

Notice that $y^x = \left((1-\omega)^{\frac{\rho}{1+\rho}}\left(v_{1,t}\right)^{\frac{1}{1+\rho}} + \omega^{\frac{\rho}{1+\rho}}\left(\left(u_t k_{t-1}\right)^{\alpha}\left(l_{2,t}\right)^{1-\alpha}\right)^{\frac{1}{1+\rho}}\right)^{(1+\rho)x}$. Find x, such that $x(1+\rho) = \rho$. That is $x = \frac{\rho}{1+\rho}$. Accordingly,

$$r_{k,t} - y^{\frac{\rho}{1+\rho}}\omega^{\frac{\rho}{1+\rho}}\left(v_{2,t}\right)^{-\frac{\rho}{1+\rho}}\alpha\frac{v_{2,t}}{u_t k_{t-1}} = 0.$$

Which can be further simplified as

$$r_{k,t} = \alpha\left(\omega\frac{y_t}{v_{2,t}}\right)^{\frac{\rho}{1+\rho}}\frac{v_{2,t}}{u_t k_{t-1}}. \tag{25}$$

$$w_{2,t} = (1-\alpha)\left(\omega\frac{y_t}{v_{2,t}}\right)^{\frac{\rho}{1+\rho}}\frac{v_{2,t}}{l_{2,t}}. \tag{26}$$

$$p_{1,t} - (1+\rho)\left((1-\omega)^{\frac{\rho}{1+\rho}}\left(v_{1,t}\right)^{\frac{1}{1+\rho}} + \omega^{\frac{\rho}{1+\rho}}\left(\left(u_t k_{t-1}\right)^{\alpha}\left(l_{2,t}\right)^{1-\alpha}\right)^{\frac{1}{1+\rho}}\right)^{\rho}\frac{1}{1+\rho}(1-\omega)^{\frac{\rho}{1+\rho}}\left(v_{1,t}\right)^{\frac{1}{1+\rho}-1} = 0.$$

Simplifying

$$p_{1,t} - y^{\frac{\rho}{1+\rho}}(1-\omega)^{\frac{\rho}{1+\rho}}\left(v_{1,t}\right)^{-\frac{\rho}{1+\rho}} = 0.$$

$$p_{1,t} = \left(\frac{(1-\omega)\,y}{v_{1,t}}\right)^{\frac{\rho}{1+\rho}}. \tag{27}$$

And from the production function,

$$y_t = \left((1-\omega)^{\frac{\rho}{1+\rho}}\left(v_{1,t}\right)^{\frac{1}{1+\rho}} + \omega^{\frac{\rho}{1+\rho}}\left(v_{2,t}\right)^{\frac{1}{1+\rho}}\right)^{1+\rho} \tag{28}$$

and where

$$v_{2,t} = (u_t k_{t-1})^{\alpha} (l_{2,t})^{1-\alpha}. \tag{29}$$

And from the budget constraint we can derive that the goods market must clear

$$y_t = c_t + i_t + \nu_0 \frac{u_t^{1+\nu}}{1+\nu} + \frac{\zeta}{2} \frac{(i_t - i_{t-1})^2}{i_{t-1}}.$$

The 14 equations above allow us to determine 14 variables y_t, $v_{1,t}$, $v_{2,t}$, c_t, i_t, k_t, u_t, $\lambda_{c,t}$, $\lambda_{i,t}$, $\lambda_{k,t}$, $p_{1,t}$, $w_{1,t}$, $w_{2,t}$, $r_{k,t}$, with $l_{1,t}$ and $l_{2,t}$ determined by exogenous processes.

A.2 Steady-State Conditions

Set $u_t = 1$ and later set ν_0 to support this choice. Notice that the investment constraint must be slack in the steady state, so

$$\lambda_i = 0. \tag{30}$$

Using

$$\lambda_{c,t} = \lambda_{k,t} + \lambda_{i,t},$$

and $\lambda_{c,t} r_{k,t} - \lambda_{k,t} + \theta(1-\delta) E_t \lambda_{i,t+1} = 0$, we can see that

$$r_k = 1 - \theta(1-\delta). \tag{31}$$

Using

$$r_k = \alpha \left(\omega \frac{y}{v_2} \right)^{\frac{\rho}{1+\rho}} \frac{v_2}{k} \tag{32}$$

and combining it with $r_k = 1 - \theta(1-\delta)$, we can use a numerical solver to get k, given l_1 and l_2.

Knowing k, and with

$$v_1 = \eta(l_1 - \chi), \tag{33}$$

we can solve for y using the production function

$$y = \left((1-\omega)^{\frac{\rho}{1+\rho}} (v_1)^{\frac{1}{1+\rho}} + \omega^{\frac{\rho}{1+\rho}} \left(k^{\alpha} (l_2)^{1-\alpha} \right)^{\frac{1}{1+\rho}} \right)^{1+\rho} \tag{34}$$

From $k_t = (1-\delta)k_{t-1} + i_t$, we have that

$$i = \delta k \tag{35}$$

Using $\lambda_{c,t} r_{k,t} k_{t-1} = \lambda_{c,t} \nu_0 u_t^{\nu}$, find the value of ν_0 that ensures $u = 1$. Accordingly

$$\nu_0 = r_k k \tag{36}$$

And using the resource constraint, we can solve for c

$$c = y - i - \nu_0 \frac{u_t^{1+\nu}}{1+\nu} \tag{37}$$

$$\lambda_c = \frac{1}{(1-\kappa)\,c} - \theta\kappa\frac{1}{(1-\kappa)\,c}. \tag{38}$$

$$\lambda_k = \lambda_c \tag{39}$$

$$p_1 = \left(\frac{(1-\omega)\,y}{l_1}\right)^{\frac{\rho}{1+\rho}} \tag{40}$$

$$w_1 = \eta p_1 \tag{41}$$

$$v_2 = k^{\alpha}\,(l_2)^{1-\alpha} \tag{42}$$

$$w_2 = (1-\alpha)\left(\omega\frac{y}{v_2}\right)^{\frac{\rho}{1+\rho}}\frac{v_2}{l_2} \tag{43}$$

A.3 Deriving the Elasticity of Substitution for the Production Function of Sector 2

$$y_t = \left((1-\omega)^{\frac{\rho}{1+\rho}}\,(v_{1t})^{\frac{1}{1+\rho}} + \omega^{\frac{\rho}{1+\rho}}\,(v_{2,t})^{\frac{1}{1+\rho}}\right)^{1+\rho}$$

$$\frac{\partial y_t}{\partial v_{1,t}} = (1+\rho)\left((1-\omega)^{\frac{\rho}{1+\rho}}\,(\eta(l_{1,t}-\nu))^{\frac{1}{1+\rho}} + \omega^{\frac{\rho}{1+\rho}}\,(v_{2,t})^{\frac{1}{1+\rho}}\right)^{\rho}\frac{1}{1+\rho}(1-\omega)^{\frac{\rho}{1+\rho}}\,(v_{1,t})^{\frac{1}{1+\rho}-1}$$

Notice again that $y^x = \left((1-\omega)^{\frac{\rho}{1+\rho}}\,(\eta(l_{1,t}-\nu))^{\frac{1}{1+\rho}} + \omega^{\frac{\rho}{1+\rho}}\,(v_{2,t})^{\frac{1}{1+\rho}}\right)^{(1+\rho)x}$. Find x, such that $x(1+\rho)=\rho$. That is $x=\frac{\rho}{1+\rho}$. Accordingly,

$$\frac{\partial y_t}{\partial v_{1,t}} = y^{\frac{\rho}{1+\rho}}(1-\omega)^{\frac{\rho}{1+\rho}}\eta\,(v_{1,t})^{-\frac{\rho}{1+\rho}}$$

$$\frac{\partial y_t}{\partial v_{2,t}} = y^{\frac{\rho}{1+\rho}}(\omega)^{\frac{\rho}{1+\rho}}\,(v_{2,t})^{-\frac{\rho}{1+\rho}}$$

$$\frac{\frac{\partial y_t}{\partial v_{1,t}}}{\frac{\partial y_t}{\partial v_{2,t}}} = \frac{(1-\omega)^{\frac{\rho}{1+\rho}}\,(v_{1,t})^{-\frac{\rho}{1+\rho}}}{(\omega)^{\frac{\rho}{1+\rho}}\,(v_{2,t})^{-\frac{\rho}{1+\rho}}}$$

$$\log\frac{\frac{\partial y_t}{\partial v_{1,t}}}{\frac{\partial y_t}{\partial v_{2,t}}} = \log\left(\frac{(1-\omega)^{\frac{\rho}{1+\rho}}\,(v_{1,t})^{-\frac{\rho}{1+\rho}}}{(\omega)^{\frac{\rho}{1+\rho}}\,(v_{2,t})^{-\frac{\rho}{1+\rho}}}\right) = \log\left(\frac{(1-\omega)^{\frac{\rho}{1+\rho}}}{(\omega)^{\frac{\rho}{1+\rho}}}\right) + \frac{\rho}{1+\rho}\log\frac{v_{2,t}}{v_{1,t}}$$

The elasticity id given by

$$Elast = \frac{dlog(v_{2,t}/v_{1,t})}{dlog(\frac{\partial y_t}{\partial v_{1,t}}/\frac{\partial y_t}{\partial v_{2,t}})} = \frac{1+\rho}{\rho}$$

Therefore to hit a destired elasticity set ρ as

$$\rho Elast - \rho = 1$$

$$\rho = \frac{1}{Elast - 1}.$$

B Calibration of the Minimum-Scale Parameter and Additional Sensitivity Analysis

To calibrate the minimum-scale parameter for the production function of Sector 1 in the two-sector model, see Equation 12, we adopt the following strategy. We feed into the model a path of labor supply shocks that meets two restrictions: 1) it balances the decline in value added across sectors and 2) it brings about a reduction in labor inputs in line with the increase in the unemployment rate relative to the 3.5 percent mark observed in February, 2020. We then set the minimum scale parameter to match a 12 percent decline in GDP in the the second quarter of 2020. We calculated this decline relative to the consensus level of GDP in the Blue Chip forecasts published in January 2020, before private forecasters entertained the possibility of a pandemic. The resulting calibration choice for the parameter χ is $\frac{6}{10}$ times the steady state value for the labor input of Sector 1.

Figure A.1 compares one- and two-sector models that match the observed increase in unemployment from March through October 2020 relative to the level in February 2020. After October 2020, the labor supply shocks follow an auto-regressive process with a coefficent of 0.95. The figure shows sizable differences between the economic collapse that can be matched with our two-sector model and the smaller economic decline implied by the special case of a one-sector model. We conclude that our two-sector model is a more appropriate choice to study the economic consequences of the COVID-19 pandemic.

B.1 Sensitivity Analysis

Figure A.2 offers sensitivity analysis pertaining to the comparison on the economic effects of the spread of COVID-19 without any social distancing measures. We compare the economic effects using one- and two-sector models. Figure A.2 considers sensitivity to a range of values of the initial reproduction rate. It shows that the differences between the one- and two-sector models persist as long as the reproduction rate does not drop below 1.2, a level that would also curtail the spread of the disease.

Figures A.3 and A.4 complement the discussion of the cost of waiting for a vaccine in Section 5.3. They pertain, respectively, to sensitivity analysis to the effectiveness of the lockdown and to the probability of transmission of the disease for given contacts.

Figure A.1. Using the Two-Sector Model to Match the Observed Increase in the Unemployment Rate Relative to February 2020

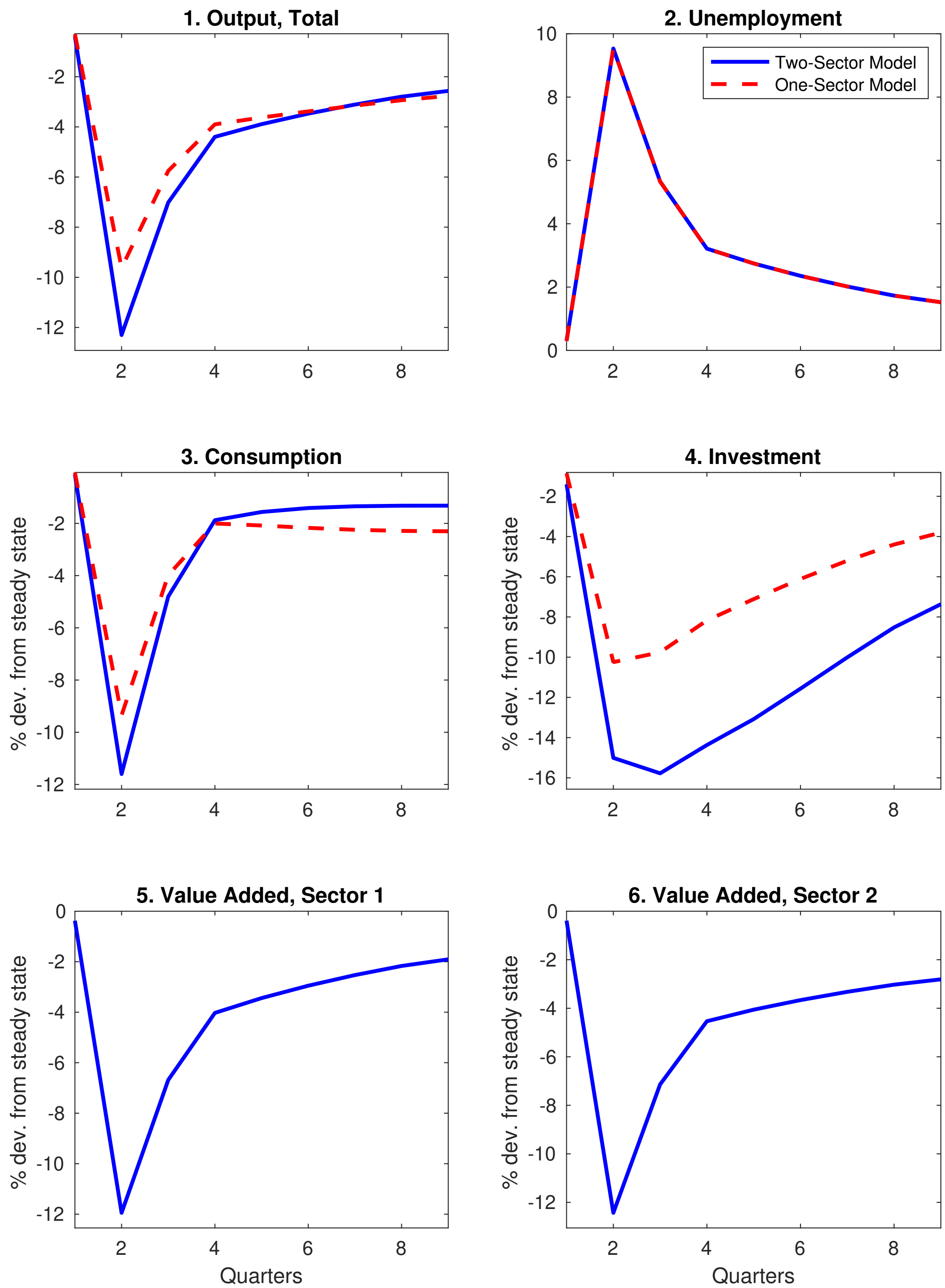
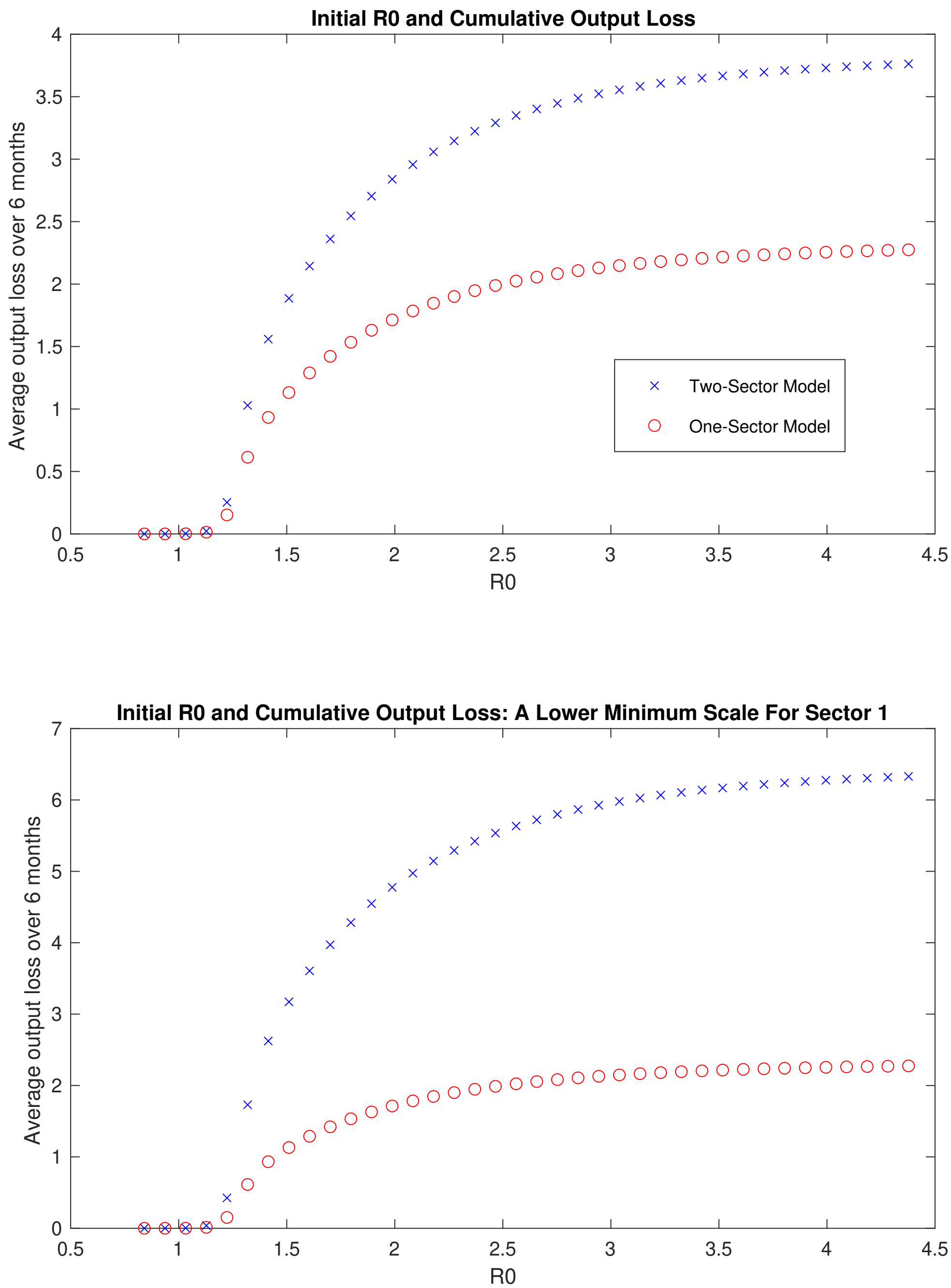


Figure A.2. Comparing the Aggregate Economic Consequences of COVID-19 Without Social Distancing in One- and Two-Sector Models: Sensitivity to the Reproduction Rate



Note: We assume that no social distancing measures are taken to reduce the spread of the disease. The output loss stems from the reduction in labor supply from symptomatic infected individuals. The figure shows the cumulative output loss over six months alternatively based on one- and two-sector models for different values of the reproduction rate (set to 2 in our baseline). The top panel keeps all other parameters at their baseline values. For the bottom panel, we have increased the minimum scale parameter for Sector 1, χ, to $\frac{8}{10}$ of the steady state labor input, as opposed to $\frac{6}{10}$ in the baseline calibration.

Figure A.3. Waiting for a Vaccine: A Lower Effectiveness of the Lockdown at Reducing Contact Rates

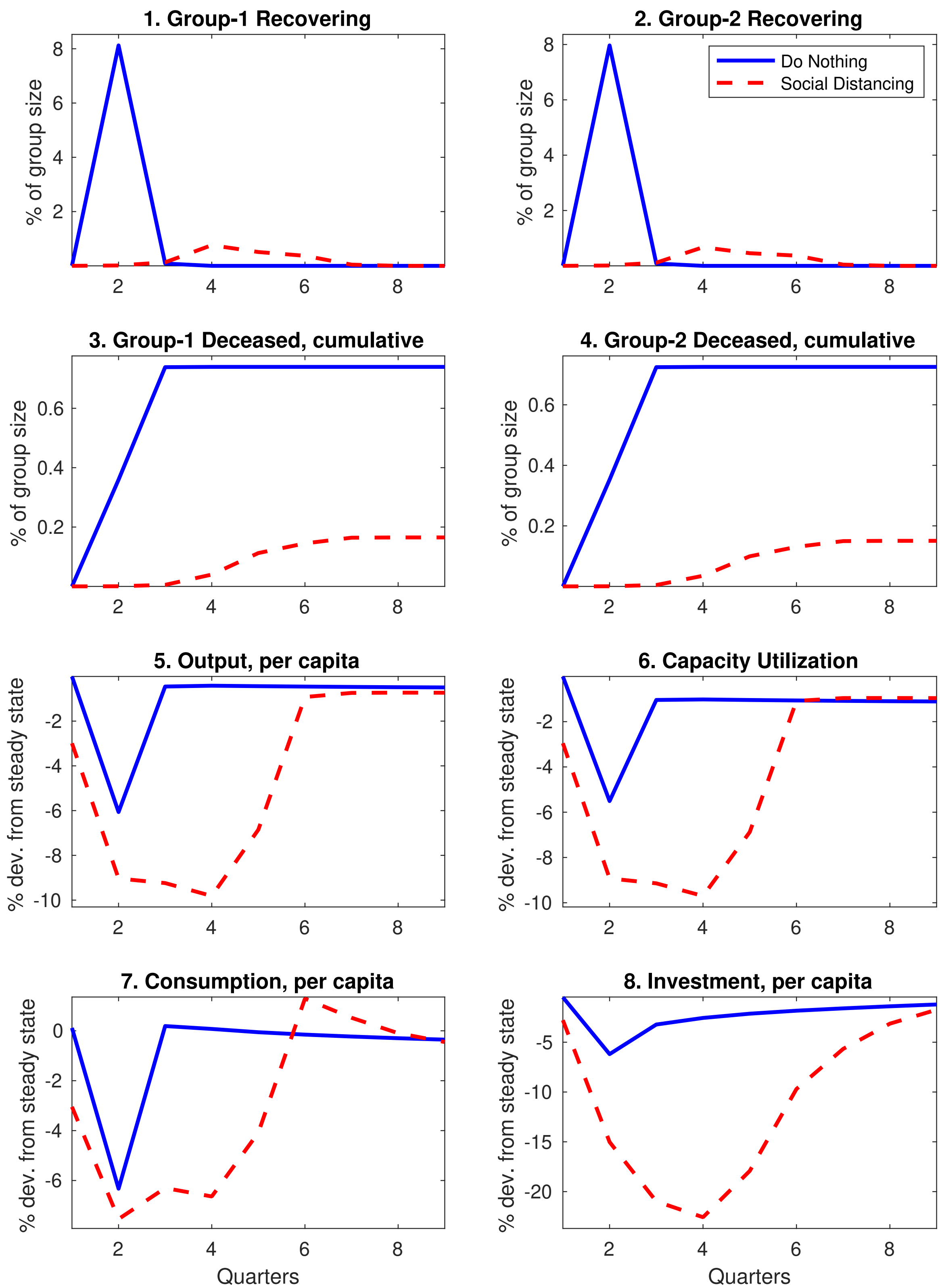


Figure A.4. Waiting for a Vaccine: A 70 Percent Increase in the Transmissibility of the Virus