Borrowing Into Debt Crises*

Radoslaw Paluszynski
Department of Economics
University of Houston

Georgios Stefanidis
Department of Economics
York University

April 21, 2022

Abstract

Quantitative models of sovereign debt predict that governments reduce borrowing during recessions to avoid debt crises. A prominent implication of this behavior is that the resulting interest rate spread volatility is counterfactually low. We propose that governments borrow into debt crises because of frictions in the adjustment of their expenditures. We develop a model of government good production which uses public employment and intermediate consumption as inputs. The inputs have varying degrees of downward rigidity which means that it is costly to reduce them. Facing an adverse income shock, the government borrows to smooth out the reduction in public employment, which results in increasing debt and higher spread. We quantify this rigidity using the OECD government accounts data and show that it explains about 70% of the missing bond spread volatility.

Keywords: Sovereign default, long-term debt, public goods
JEL Classification Numbers: F34, G15

*E-mail: rpaluszynski@uh.edu and stefa107@yorku.ca. This paper has benefited from very helpful comments by Manuel Amador, Juan Carlos Hatchondo, Fabrizio Perri, Ananth Ramanarayanan, Tao Zha (the editor), three anonymous referees, as well as seminar participants at the University of Western Ontario, and conference participants at the 2019 SAET in Ischia.
1 Introduction

Sovereign debt crises are a recurring phenomenon in the financial markets and tend to coincide with sizable disruptions in the real economy. A recent literature has developed a class of quantitative models that are able to replicate many aspects of lending to risky sovereigns such as simultaneously high average debt-to-output ratios and spreads. However, one particularly elusive aspect of debt crises is the government inertia when faced with sudden fluctuations in borrowing costs. In the data, government expenditures are slow to adjust, interest rate spreads are volatile and high-peaking, and debt ratios often rise during crises. By contrast, quantitative models of sovereign default with long-term debt predict that governments adjust their fiscal policy fast in response to adverse income shocks, thus reducing overall debt levels. As a result, the predicted spreads are too low and not volatile enough, an observation pointed out by Aguiar et al. (2016), among others.

We propose a channel that bridges this gap in a straightforward way and quantify its importance. The idea is that government spending is imperfectly flexible, and governments are unable to adjust their fiscal policy freely when faced with negative income shocks. One aspect of government spending that tends to be particularly rigid, as we show in this paper using OECD data, is public employment. Government agencies often face barriers to laying off workers which constrains their actions. A story of the Hellenic Broadcasting Corporation (ERT) in Greece provides an illustrative example. In 2013, in the midst of its sovereign default crisis, the Greek government decided to shut down ERT, the public television company, and lay off all workers as part of its effort to regain lenders’ confidence following the 2012 default. This action sparked mass street protests and forced the government to ultimately reinstate the ERT two years later. In this paper we ask: how much do frictions in adjusting public expenditure, and in particular public employment, impede the government’s ability to respond to debt crises? More precisely, to what extent can such frictions explain the increasing debt ratios during crises as well as the high volatility of interest rate spreads observed in the data?

We develop a model of sovereign default that builds on the framework of Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). An impatient government maximizes the expected lifetime utility by borrowing in defaultable non-contingent bonds. As is standard, bonds last for many periods and are priced competitively by risk-neutral foreign lenders. The main point of departure is that we model the production of the public
good. The inputs to this production in the model are intermediate government consumption\textsuperscript{1} and public labor, following the decomposition used by the OECD Government Accounts. We assume that both inputs are persistent in their nature. That is, these expenses are subjected to an asymmetric adjustment cost. The government is always free to purchase more inputs, but incurs the adjustment cost in order to reduce the rigid expenditures. With this friction in mind, the government chooses an optimal combination of public labor and intermediate consumption, as well as the new level of debt, to maximize its expected utility over a stream of public goods.

To discipline the rigidity in both inputs, we use data on different types of government spending from the aforementioned OECD Government Accounts. We estimate the elasticity of public employment expenditure with respect to intermediate consumption controlling for its own lag around debt crises, in a panel of 36 OECD countries in years 1995-2019. Our estimates indicate that public employment expenditure is only weakly related to intermediate government consumption and instead exhibits strong persistence. We use these estimates to calibrate our model for Mexico. Mexico is a frequent case study for sovereign default models and the volatility of its interest rate spread is much too high relative to what a standard model can predict (Aguiar et al., 2016).

As a main quantitative result, we find that our model generates a standard deviation of the spread of 1.82%, compared to 0.83% in the standard quantitative model. This accounts for about 70% of the gap between the standard model’s prediction and its empirical counterpart of 2.21%. This result is mirrored by a reduced volatility of government deficit which aligns our model more closely with the data. To understand this finding, we contrast the simulated behavior of our model with the standard one around debt crises and defaults. Our main qualitative finding is that during these crises in our model the government increases its debt, “borrows into crises”, whereas in the standard model it reduces it.

We identify two channels that lead to the “borrowing into debt crises” behavior: one is a direct effect of adjustment costs, while the other is an indirect effect that operates through equilibrium bond prices. The direct effect induces the country to maintain a borrowing buffer during good times and to use it during bad times. Simply put, it “borrows into crises” to reduce the cost of expenditure adjustments. The general equilibrium channel results in the government with adjustment costs facing a more favorable price sched-

\textsuperscript{1}Examples of intermediate government consumption are non-durable supplies, building rentals, energy, and military supplies.
ule, and lower default incentives, than the government without adjustment costs during crises. This in turn allows it to further pursue the strategy of “borrowing into crises”. The reasoning behind the general equilibrium channel is that the buffer described above makes access to borrowing during downturns more valuable for a government with adjustment costs to spending than for one without them. This makes the country less willing to default which is reflected in a more favorable price schedule.\(^2\)

As a second benchmark, we compare the behavior of our model to one that features fixed public labor. We show that this benchmark is equivalent to the standard model augmented with non-homothetic preferences that feature a “minimum consumption” level.\(^3\)

We show that the standard deviation of the spread in that model amounts to 1.34%, which represents about half of the improvement that our baseline model generates. Importantly, we show that while the fixed labor model indeed causes the government deficit to be less responsive to shocks in the run-up to debt crises, it does not qualitatively affect the path of government debt. Specifically, we show the debt level still declines in anticipation of the impending debt crisis, unlike in our model where it increases.

Our results hinge on the costly adjustment of inputs which is disciplined by two key empirical moments. First, we target the low elasticity of public employment expenditure to intermediate consumption. In our model, costly adjustment of public employment accounts for this low elasticity. Second, the data suggests that intermediate consumption expenditure is over 50% more variable than public employment expenditure. We achieve this target by pinning down the degree of rigidity in intermediate consumption relative to public employment expenditure. To show this, in an extension in Section 3.7, we calibrate a restricted version of our model where public employment is the only rigid input. In this calibration, intermediate consumption expenditure is 168% more variable than public employment expenditure. The government varies intermediate consumption aggressively to mitigate debt crises. This results in spread volatility that is barely higher than in the standard model without any rigidity in spending.

\(^2\)We also find that the default cost is greater in the specification with adjustment cost. This also contributes to making the price schedule more favorable for a country subject to adjustment costs. A question that remains is, if the default cost is greater in the adjustment cost specification then why is equilibrium borrowing not greater? The answer lies in the borrowing buffer in the adjustment cost specification. This buffer leads to less borrowing during good times which lowers average debt.

\(^3\)The recent quantitative sovereign default literature has used such preferences as a shortcut to make government deficit less responsive to the underlying fundamentals, see for example Bocola and Dovis (2019) and Bianchi, Hatchondo and Martinez (2018).
An attractive feature of our model is that it allows us to identify the rigidity in government spending by targeting the elasticity of substitution inferred from the OECD data. Nevertheless, in Section 4 of the paper we show that this model also maps easily into a simpler model that uses habit formation, a standard tool in quantitative macroeconomics. We discipline this model by targeting the autocorrelation of government expenditures for Mexico, i.e., an intertemporal elasticity. All quantitative and qualitative results are similar to the ones produced by our baseline model. We also show that the calibrated parameter for the habit falls within the range of the values estimated in Fuhrer (2000). We view this result as an “external validation” as well as a desirable simplification of our model that makes the mechanism we highlight easier for practitioners to apply.

The model presented in this paper contributes to our understanding of the recent European debt crisis in two ways. First, as we show in Section 5, countries in the OECD data tend to respond to adverse revenue shocks by increasing their debt levels, a result that standard models generally struggle to replicate (Paluszynski, 2021). By contrast, this response is consistent with our model in which governments face frictions to adjusting their expenditures. Second, a number of European countries experienced high interest rate spreads, in excess of 10%. We show that the distribution of simulated spreads in our model has a fat upper tail, with positive mass extending up to the spread of 20%, while the two benchmark models we consider fail to generate any spreads higher than 8% on the equilibrium path.

1.1 Literature review

This paper is closely related to the quantitative sovereign default literature, in particular one building on the seminal works of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008). Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) introduce long-duration bonds to these models and show that it is an important element in accounting for the amounts of debt and average spreads observed in the data. However, Aguiar et al. (2016) point out that such models (with long-term debt) still fall short of replicating the interest rate spread volatility observed in the data for most sovereign defaulters other than Argentina.4

Our calibration relates to Bocola and Dovis (2019) and Bocola, Bornstein and Dovis (2019).

---

4Argentina is a notable exception, as evidenced by the success of the Chatterjee and Eyigungor (2012) model, because it has an unusually volatile income process. See Aguiar et al. (2016) for more details.
Similarly to their work, we model the government budget constraint. We do so to highlight the role of government expenditure rigidities. In contrast to their work, which studies Eurozone countries, our application is for Mexico, a developing country. For this set of countries the overwhelming majority of the quantitative literature has calibrated to external public debt. As a result, we also target external public debt. Our calibration target reproduces the low spread volatility in the standard model highlighted in the literature, albeit slightly more volatile. Therefore, we view our calibration as giving the standard model the best chance to achieve the empirical spread volatility. While our model features a default driven by shocks to the country’s own income, the empirical literature has pointed out that global factors are an important source of volatility in sovereign spreads (González-Rozada and Levy Yeyati, 2008). In Appendix C we show that the standard model augmented with shocks to creditors’ risk aversion alone is unable to elevate the bond spread volatility. As such, the source of the shocks is less important than the way governments respond to them, which is our primary object of interest.

Our paper is naturally not the first attempt to raise the volatility of the spread. Aguiar et al. (2021) revisit a model with rollover crises and propose a new equilibrium selection mechanism to justify why bonds are often sold at large discounts. Chatterjee and Eyigun-gor (2019) obtain volatile spreads in a model with political frictions, while Paluszynski (2021) generates high standard deviation of the spread (especially relative to low mean) for Eurozone countries in a model with learning about rare disasters. Relative to these studies, we view the mechanism proposed in our paper as complementary and quantify its contribution to generating a volatile bond spread. In addition, other studies such as Bocola and Dovis (2019), Bocola, Bornstein and Dovis (2019) or Bianchi, Hatchondo and Martinez (2018), also accomplish this goal, but they all use the “subsistence consumption” utility which is a special case of our model. As such, our paper contributes by generalizing (and quantifying) this increasingly popular modeling technique.

Our paper is also related to a number of studies that highlight why an indebted government might fail to deleverage when facing a crisis. Conesa and Kehoe (2017) show in a model with sudden stops that, under certain conditions, the government may “gamble for redemption” by optimally increasing its debt in a recession. Further, Lorenzoni and Werning (2019) show that a model with equilibrium multiplicity à la Calvo (1988) can

---

4 Implicitly these two calibration targets correspond to two extremes on government tax policy. On the one extreme, targeting debt to GDP corresponds to the case where the government is able to use lump sum taxes. On the other extreme, debt to government revenues corresponds to the case where the tax rate is fixed to some value. Lump sum taxes offer more flexibility to the government leading to less variable spreads.
also produce borrowing into crises. Corsetti and Maeng (2020) contrast these two types of multiplicity and show that the incentives to leverage during crises are stronger with the multiplicity à la Calvo (1988). Müller, Storesletten and Zilibotti (2019) achieve debt accumulation during a recession in a model with stochastic default cost, renegotiation, and hidden effort to conduct structural reforms. In contrast to these papers, the quantitative sovereign default literature cited above assumes the government is impatient. This assumption, while matching the data better, leads to countries deleveraging in debt crises. Notably, Bocola, Bornstein and Dovis (2019) achieve an increasing debt-to-GDP ratio at the beginning of a recession by using the “subsistence consumption” preferences. Finally, several recent papers, for example Tirole (2015), Gourinchas, Martin and Messer (2020), or Corsetti, Erce and Uy (2019), explain “borrowing in debt crises” with the presence of official lending by international financial institutions (IFIs) such as the IMF or the European Stability Mechanism. Our model abstracts from IFIs to isolate the role of “sticky” expenditures.

The remainder of this paper is structured as follows. Section 2 introduces our model. Section 3 presents the quantitative analysis of our model, along with main results and extensions. Section 4 compares our main results to a simplified model with habit formation. Section 5 provides some direct evidence for our main result from the panel consisting of OECD countries. Section 6 concludes.

2 Model

In this section we present the main environment of our analysis.

2.1 Economic environment

Endowment process Each period the economy receives a stochastic endowment $Y_t$. This process has the following autoregressive structure:

\[
\log Y_{t+1} = \rho \log Y_t + \epsilon_{t+1}.
\]

Müller, Storesletten and Zilibotti (2019) extend their model to include government impatience and GDP fluctuations. In this quantitative extension they achieve debt accumulation during crises. Their quantitative exercise differs from the quantitative sovereign default literature in two notable ways. First, it emphasizes low frequency GDP fluctuation. Second, the main source of default risk comes from stochastic default cost. As a result, it is not clear how their results would translate to an environment with high frequency GDP fluctuation. However, the driving forces behind their result could further improve the fit of borrowing dynamics in quantitative sovereign default models.
The innovation term, $\epsilon_{t+1}$, is iid and is drawn from a normal distribution with zero mean and $\sigma_\epsilon$ standard deviation. Parameter $\rho$ is the usual autoregressive coefficient. Finally, the unconditional mean of the endowment process $\mu_y$ is normalized to 1. Tax revenues are proportional to the endowment with tax rate $\tau$. The history of endowments in period $t$ is denoted $Y^t = (Y_0, Y_1, ..., Y_t)$.

**Preferences**  The government values an uncertain stream of public goods $\{G_t(Y^t)\}_{t=0}^\infty$ using a utility function, given by:

$$E E\sum_{t=0}^\infty \beta^t U(G_t(Y^t)).$$

$E$ denotes the expectations on endowment process $Y_t$ implied by the autoregressive structure specified above. We assume the function $U(\cdot)$ is strictly increasing, concave and twice continuously differentiable. The discount factor is given by $\beta \in (0, 1)$.

**Production Technology**  The public good $G_t$ is produced using public labor $L_t$, and an intermediate government consumption good $C_t$, as inputs. The production function, denoted $G(L_t, C_t)$, takes the Cobb-Douglas form:

$$G_t = G(C_t, L_t) = C_t^\alpha L_t^{1-\alpha}.$$

where the weight in the production function, $\alpha$, is calibrated in the quantitative analysis.

**Inputs Adjustment Friction**  We assume that government expenditures have a degree of persistence, captured by the function

$$H_t = \phi_0 C_t + (1 - \phi_0) w L_t$$

$H_t$ can be thought of as a pool of legacy contracts consisting of both the employment contracts with public officials, as well as delivery and subscription contracts for intermediate government consumption. We assume a resource adjustment cost for $H_t$ which takes the following functional form:

$$\phi_1 \min \left\{ \frac{H_t}{H_{t-1}} - 1, 0 \right\}.$$

---

6This decomposition is guided by the classification in the OECD Government Accounts data. We elaborate on it, and use it in our quantitative analysis in Section 3.
That is, the cost applies only when input expenditure \( H_t \) is reduced. Then, the cost is proportional to the rate of decline of \( H_t \). The degree of proportionality, \( \phi_1 \), along with the weight on the intermediate consumption expenditure, \( \phi_0 \), are calibrated in the quantitative analysis.

**Debt and Default** The country enters each period with debt \( B_t \). A \( \delta \) fraction of the debt matures and has to be repaid. Outstanding debt receives coupon \( \kappa \). Finally, the government decides on debt issuance \( B_{t+1} - (1 - \delta)B_t \). The price schedule in the recursive formulation, denoted \( Q \), depends on borrowing \( B_{t+1} \), weighted input cost \( H_t \), and endowment \( Y_t \). Parameters \( \delta \) and \( \kappa \) are specified in the quantitative analysis.

Default allows the country to entirely erase debt \( B_t \). However, there are two costs associated with default. First, there is resource cost \( Y - Y^d(Y) \) where \( Y^d(Y) = \min(Y, \bar{Y}) \).

That is, endowment can at most be \( \bar{Y} \) as a result of the default (Arellano, 2008). The cost is linearly increasing in \( Y \) for values of \( Y \) larger than \( \bar{Y} \) and zero for values of \( Y \) less than \( \bar{Y} \). Parameter \( \bar{Y} \) is calibrated in the quantitative analysis. Second, the government is temporarily excluded from financial markets. Re-entry occurs stochastically with per period probability \( \theta \).

### 2.2 Decision Problem

In this section we formalize the economic environment by stating the problem faced by market participants in recursive form. The government enters a period with debt \( B \), legacy contracts \( H_{t-1} \), and endowment realization \( Y \).

**Government** The government that is current on its debt obligations decides between repayment or default. The value function is given by:

\[
W(B, H_{t-1}, Y) = \max_{d \in \{0,1\}} \left\{ d \ V^D(H_{t-1}, Y) + (1 - d) V^R(B, H_{t-1}, Y) \right\}
\]  

(2)

Convex default cost is necessary to generate realistic average bond spreads in the model and has been given some empirical support by Mendoza and Yue (2012). Further, to generate realistic standard deviation of the spread the literature has utilized convex default costs with more curvature than the one offered by the Arellano (2008) one, see for example Chatterjee and Eyigungor (2012). We choose the Arellano (2008) default cost since understanding the determinants of the standard deviation of the spread is the main objective of this paper.
Repayment \((d = 0)\) allows the government to borrow. The value function is given by:

\[
V^R(B, H_{-1}, Y) = \max_{B' \geq 0, C \geq 0, L \geq 0} \left\{ U(C^a L^{1-a}) + \beta E_{Y'|Y} W(B', H, Y') \right\}
\]

subject to

\[
C + wL = \tau Y - B \left( \delta + (1 - \delta) \kappa \right) + Q(B', H, Y) \left( B' - (1 - \delta) B \right) - \phi_1 \min \left\{ \frac{H}{H_{-1}} - 1, 0 \right\},
\]

\[
H = \phi_0 C + (1 - \phi_0) wL.
\]

A sovereign who defaults \((d = 1)\) is excluded from international credit markets and has probability \(\theta\) of being readmitted every subsequent period. The associated value is:

\[
V^D(H_{-1}, Y) = \max_{C \geq 0, L \geq 0} U(C^a L^{1-a}) + \beta E_{Y'|Y} \left[ \theta W(0, H, Y') + (1 - \theta) V^D(H, Y') \right]
\]

subject to

\[
C + wL = \tau Y d(Y) - \phi_1 \min \left\{ \frac{H}{H_{-1}} - 1, 0 \right\},
\]

\[
H = \phi_0 C + (1 - \phi_0) wL.
\]

**International Lenders** The lenders are assumed to be risk-neutral and perfectly competitive. The actuarially fair bond price that compensates them for default risk is:

\[
Q(B', H, Y) = \frac{1}{1 + r E_{Y'|Y} \left[ (1 - d(B', H, Y')) \left( \delta + (1 - \delta) \kappa + (1 - \delta) Q(B'', H', Y') \right) \right]}
\]

where

\[
B'' = B'(B', H, Y')
H' = H(B', H, Y')
\]

**Definition 1** A Markov Perfect Equilibrium for this economy consists of the government value functions \(W(B, H_{-1}, Y), V^R(B, H_{-1}, Y), V^d(H_{-1}, Y)\); policy functions \(C(B, H_{-1}, Y), L(B, H_{-1}, Y), B'(B, H_{-1}, Y), H(B, H_{-1}, Y), d(B, H_{-1}, Y)\); and bond price schedule \(Q(B', H, Y)\) such that:

1. Policy function \(d\) solves the government’s default-repayment problem.
3. Bond price function \(Q\) is such that international lenders make zero profit in expectation.
3 Quantitative analysis

In this section we take the model to data by choosing parameter values. We calibrate the model to Mexico, which is a common subject of interest in the sovereign default literature (Aguiar et al., 2016), and at the same time is a member of the OECD, providing us with rich data on different subcategories of government spending. As is common in the literature, some parameters are set externally to standard values, while others are selected to match certain empirical moments.

3.1 Parameters set externally

Preferences  Each period is assumed to be one year. We assume a CRRA utility function of the form $U(G) = \frac{G^{1−γ}}{1−γ}$, with the risk aversion parameter $γ$ set to 2.

Endowment  The persistence $ρ$ of Mexico’s annual GDP is 0.65, estimated using data from 1980, while the standard deviation of innovations $σ$ is 0.03. The tax rate $τ$ is set at 0.1 which is the average central government tax revenue to GDP as reported by the World Bank’s WDI database.

Sovereign Debt  The risk-free interest rate is set to 4% (annual value) and the probability of re-entry after default is fixed at 0.33, following Richmond and Dias (2009) who find that the median time to re-enter the credit market was 3 years in 1980-2005. To select the values for parameters that describe Mexico’s debt structure we adhere closely to the calibration in related papers, such as Aguiar et al. (2016) or Bianchi, Hatchondo and Martinez (2018). The maturing probability $δ$ is set to 0.285, while the (annual) coupon rate $κ$ is 5%.

Table 1 summarizes the calibration of parameters selected outside the model equilibrium.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$γ$</td>
<td>Risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>$θ$</td>
<td>Prob. of exiting excl.</td>
<td>0.333</td>
</tr>
<tr>
<td>$δ$</td>
<td>Bond maturity prob.</td>
<td>0.285</td>
</tr>
<tr>
<td>$κ$</td>
<td>Coupon rate</td>
<td>0.05</td>
</tr>
<tr>
<td>$r$</td>
<td>Risk-free rate</td>
<td>0.04</td>
</tr>
<tr>
<td>$τ$</td>
<td>Tax rate</td>
<td>0.1</td>
</tr>
<tr>
<td>$w$</td>
<td>Wage rate</td>
<td>1</td>
</tr>
</tbody>
</table>
Notice that we assume for simplicity that the wage rate is fixed at 1, which is an extreme form of wage rigidity as in Bianchi, Ottonello and Presno (2021).

3.2 Solving the model

Sovereign default models with long-term debt have substantial difficulties in achieving convergence. The literature proposes a number of solution methods including interpolation of borrowing decision (Hatchondo and Martinez, 2009), noise in the endowment process (Chatterjee and Eyigungor, 2012), and noise in defaulting and borrowing (Dvorkin et al., 2021). This paper follows Dvorkin et al. (2021) by considering the discrete choices of defaulting and borrowing and introducing nested logistic shocks in these decisions (see Appendix A for the detailed formulation of our model with noise). The correlation of these shocks is fixed at a low value and the variance is set to the smallest value that allows for computation of the Markov Perfect Equilibrium. To make sure that the noise is not the driver of our main results, we hold these parameters constant for all variants of the model considered in the paper. In addition, Appendix B shows that, at least for the frictionless version of our model, the noise does not distort any of the quantitative results.

3.3 Calibrating the structural parameters

The remaining five parameters \((\beta, \bar{Y}, \alpha, \phi_0, \phi_1)\) are calibrated jointly using the simulated method of moments. The economy’s endowment is simulated for 2 million periods, with the first 100 observations dropped. We also drop the observations for periods where the country is either in default or was in default less than 5 years prior. Five moments are used to identify the parameters. Two of them are related to the quantity and pricing of Mexico’s debt. The other three are based on statistical relationships between the two sub-components of government expenditure inferred from the OECD Government Accounts.

Debt Moments We match Mexico’s average debt to GDP and bond spread.\(^8\) Aguiar et al. (2016) report Mexico’s average external debt/GDP in the data as 16%.\(^9\) Further, Cruces and Trebesch (2013) report the average haircut in the 1978-2010 period as 29.72%. Because our model does not account for the possibility of debt renegotiation, we follow Chatterjee and Eyigungor (2012) and only consider the “unsecured” portion of government debt

\(^8\)Measured as \(\frac{B}{Y} \) and \(\frac{(\lambda + (1 - \lambda)z)}{Q - \lambda - r'}\), where \(Q\) is current period borrowing price, respectively.

\(^9\)This corresponds to the average public and publicly guaranteed external debt stocks since 2000 (WDI data). We focus on external debt, as well as the decomposition of the final consumption expenditure of the government, because our model is not suited to address the questions of transfers and redistribution.
which we round to 4.8% of GDP or 48% of government revenues (in Appendix D, we explore the sensitivity of our results to this moment by assuming a higher average debt target). Average Mexican bond spread, as measured by EMBI, from 1994 to 2019 was 3% which we set as our target.\textsuperscript{10}

**Government Consumption Moments** To inform parameters that determine the government’s choices over the two types of expenditure, we use the OECD Government Accounts. This source provides us with annual data on the subcomponents of central government spending for 36 OECD member countries in years 1995-2019. In particular, we focus on two major components of Final Government Consumption (by transaction):

1. Intermediate government consumption, which we denote $C_{i,t}$,

2. Compensation of public employees, which we denote $\overline{wL}_{i,t}$.\textsuperscript{11}

We focus on these two components because they constitute on average over 90% of the Final Government Consumption Expenditure across OECD countries (Appendix E provides more details on the decomposition of government consumption, along with some summary statistics). These time series are made real using GDP deflator. Further, we de-trend them using each respective country’s log-linear real GDP trend. We consider all OECD countries, rather than Mexico’s data alone.\textsuperscript{12} This is because Mexican data series are short making the moments less informative.

To inform $\alpha$, the parameter representing weight on intermediate consumption in the production function for the public good, we target the mean share of public employment expenditure in total government expenditure, averaged across countries:

$$\text{average labor share} = \frac{1}{N_c} \sum_{i=1}^{N_c} \left( \frac{1}{T_i} \sum_{t=1}^{T_i} \frac{1}{wL_{i,t}} + \frac{1}{T_i} \sum_{t=1}^{T_i} \overline{wL}_{i,t} \right).$$

In the OECD panel, public employment spending accounts for 63% of government consumption on average. For the 17 years of Mexican data this number is close to it, at 69%.

\textsuperscript{10}EMBI is a composite index of the performance of foreign-denominated bonds of emerging economies, relative to those of developed markets. Because we focus on external debt, and because almost all of Mexico’s foreign denominated debt is externally held, EMBI is the most appropriate measure of the spread in our case.

\textsuperscript{11}Due to data limitations, we are unable to separately observe the changes in public sector wages and employment for sufficiently many countries over long enough time period. For this reason, we treat them as a joint compensation variable throughout this analysis.

\textsuperscript{12}We exclude Chile and Colombia due to the missing Government Accounts data.
To calibrate $\phi_0$ and $\phi_1$, the parameters that drive the adjustment cost for long-term government contracts, we jointly use two separate moments that are informative about the relationship between the two inputs into the production of government good. The first moment is the ratio of standard deviations of logged inputs, averaged across countries:

\[
\text{ratio of input standard deviations (avg)} = \frac{1}{N_c} \sum_{i=1}^{N_c} \left( \frac{\text{st. dev.}(\log(C_{i,t}))}{\text{st. dev.}(\log(wL_{i,t}))} \right).
\]

In our sample, detrended logged intermediate consumption is on average 64% more variable than detrended logged public employment expenditure.

The second moment is the elasticity of public employment expenditure with respect to intermediate public consumption which arises from running the following regression:

\[
\log(wL_{i,t}) = \hat{\alpha}_0 + \hat{\alpha}_1 \log(wL_{i,t-1}) + \hat{\alpha}_2 \log(C_{i,t}) + \hat{\alpha}_3 \log(wL_{i,t-1}) \times \text{crisis}_{i,t} + \hat{\alpha}_4 \log(C_{i,t}) \times \text{crisis}_{i,t} + \hat{\alpha}_5 \text{crisis}_{i,t} + u_i + e_{i,t}
\]  

We use a form of indirect inference to inform the adjustment cost parameters. To do so, we pose an auxiliary specification (6) which captures the observed statistical relationship between compensation of public employees and intermediate consumption. Our model naturally generates persistence in the compensation of employees variable, which is why we use a dynamic regression that controls for its lagged value. It is also important to distinguish between adjustments to government spending around crises and normal times. For this reason, our specification includes a crisis dummy variable, by itself as well as interacted with the main two regressors. We will target the elasticity of public employment expenditure with respect to intermediate consumption around debt crises, $\hat{\alpha}_2 + \hat{\alpha}_4$.

We estimate the regression equation (6) jointly for the 36 OECD countries using country fixed effects $u_i$. The first column in Table 2 presents the results of the fixed effects estimation. A 1% increase in lagged public employment expenditure during normal times is associated with a 0.79% increase in contemporaneous public employment expenditure. Public employment expenditure is therefore quite persistent in our data. Interestingly, in-

\[\text{We associate a crisis with a peak of the bond spread (local maximum). In addition, we require that the level of the spread be at least one standard deviation above the mean for the given country (the results are very similar for the case of two standard deviations). Finally, we identify a crisis episode as } \pm 1 \text{ period around the peak.}\]
Table 2: Estimated law of motion for public employment

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>Fixed Effects</th>
<th>Arellano-Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log(\overline{wL}_{i,t-1}) )</td>
<td>0.788***</td>
<td>0.680***</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>( \log(C_{i,t}) )</td>
<td>0.130***</td>
<td>0.212***</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>( \log(\overline{wL}<em>{i,t-1}) \times crisis</em>{i,t} )</td>
<td>-0.045***</td>
<td>-0.037***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>( \log(C_{i,t}) \times crisis_{i,t} )</td>
<td>0.024**</td>
<td>0.031***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>( crisis_{i,t} )</td>
<td>-0.054</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.186***</td>
<td>-0.224***</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.085)</td>
</tr>
</tbody>
</table>

Observations 761 726  
Number of countries 36 36

Standard errors in parentheses  
*** p<0.01, ** p<0.05, * p<0.1

Intermediate consumption does not co-move strongly with public employment spending. Specifically, a 1% increase in intermediate government consumption during normal times is associated with a 0.13% increase in contemporary public employment expenditure.

We furthermore find that the effect of crisis times on these coefficient estimates is relatively modest, albeit statistically significant. As is intuitive, a debt crisis reduces the persistence of public employment expenditure and magnifies the effect of contemporary spending on intermediate consumption (i.e. a reduction in intermediate consumption is associated with higher reduction in contemporary public wage bill).

Dynamic panel regressions, i.e. specifications that contain at least one lag of the dependent variable, suffer from a well-known endogeneity problem. To correct for this endogeneity, we also use the estimator proposed by Arellano and Bond (1991). The second column in Table 2 presents the results of running this specification. The elasticity of public employment expenditure with respect to its lag during normal times becomes slightly
weaker, at 68%. By contrast, the elasticity with respect to intermediate government consumption is considerably higher, at 21%. A debt crisis affects these estimates with the same direction and similar magnitude as in the first specification. Due to the potential endogeneity problem, we use the Arellano-Bond estimated coefficient as a target for our model. Specifically, as the subsequent section shows, our model will replicate the elasticity of public employment expenditure with respect to intermediate consumption around the crisis times, equal to 0.24.

3.4 Results

In this section we present the main results from our calibrated model. We do so by simulating the ergodic distribution of the main variables, as well as averaging their behavior around default episodes. To understand the main novelties of our model, we compare our baseline model to two benchmarks: a flexible version similar to Chatterjee and Eyigungor (2012), as well as a model with fixed labor which introduces a “minimum consumption” in the utility function (Bocola and Dovis, 2019).

3.4.1 Baseline v. Flexible

Table 3 summarizes the calibration of our baseline model, along with a fully flexible version of it which is analogous to Chatterjee and Eyigungor (2012). The achieved fit to the data is good. The main two moments - average debt and average spread - are targeted for both versions of the model and come out very close. Notice in particular that the value of the discount factor needed to achieve this fit is lower in our baseline model than in the flexible one. This is because in the presence of resource costs to adjust spending the government needs to be more impatient in order to take on the same level of debt. In terms of the new parameters (which are calibrated only in our baseline model), the weight on intermediate consumption $\alpha$ is 0.45 which pins down the labor share of just under two thirds. The parameters of the adjustment cost function, $\phi_0$ and $\phi_1$, are set to 0.44 and 0.47, respectively. This calibration arises from achieving the ratio of standard deviations of the two inputs of around 150% and the elasticity of public employment expenditure with respect to intermediate consumption of 0.21 simultaneously.

Table 4 analyzes the simulated behavior of our model by presenting a set of untargeted moments and comparing them to their counterparts from the literature benchmark (flexible) and the data. The first two rows convey our main quantitative result: in the baseline model, standard deviation of the spread is 1.82%, up from 0.83% in the flexible model.
Table 3: Calibration of structural parameters: baseline v. flexible

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Flexible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor, $\beta$</td>
<td>0.724</td>
<td>0.797</td>
</tr>
<tr>
<td>Max default endowment, $\hat{Y}$</td>
<td>0.846</td>
<td>0.832</td>
</tr>
<tr>
<td>Interm. consumption weight, $\alpha$</td>
<td>0.448</td>
<td>0.448</td>
</tr>
<tr>
<td>Adjustment weight, $\phi_0$</td>
<td>0.442</td>
<td>0.000</td>
</tr>
<tr>
<td>Adjustment scale, $\phi_1$</td>
<td>0.474</td>
<td>1.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Baseline</th>
<th>Flexible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. debt/revenues (%)</td>
<td>48.00</td>
<td>48.70</td>
<td>48.78</td>
</tr>
<tr>
<td>Avg. spread (%)</td>
<td>3.03</td>
<td>3.01</td>
<td>3.01</td>
</tr>
<tr>
<td>Avg. labor share (%)</td>
<td>63.00</td>
<td>55.20</td>
<td>55.19</td>
</tr>
<tr>
<td>Elasticity of $wL$ w.r.t. $C$ in crises</td>
<td>0.24</td>
<td>0.21</td>
<td>-</td>
</tr>
<tr>
<td>Avg. ratio st. dev. of inputs (%)</td>
<td>164.00</td>
<td>150.74</td>
<td>-</td>
</tr>
</tbody>
</table>

and compared with 2.21% in the data. This is mirrored by the fact that the standard deviation of total government deficit\(^{14}\) relative to the standard deviation of output is lower in our baseline model, 0.41, than in the flexible model, 0.6, and much closer to its empirical counterpart of 0.26. Notice in addition that the government deficit exhibits a considerably weaker correlation with revenues relative to the standard model, bringing it more in line with the data.\(^{15}\) This implies that the government’s responses to shocks in our model

Table 4: Untargeted moments: baseline v. flexible

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mexico Data</th>
<th>Baseline</th>
<th>Flexible</th>
</tr>
</thead>
<tbody>
<tr>
<td>$std(S)$</td>
<td>2.21</td>
<td>1.82</td>
<td>0.83</td>
</tr>
<tr>
<td>$std(D) / std(Y)$</td>
<td>0.26</td>
<td>0.41</td>
<td>0.60</td>
</tr>
<tr>
<td>$corr(S, D)$</td>
<td>-0.58</td>
<td>-0.34</td>
<td>-0.79</td>
</tr>
<tr>
<td>$corr(Y, D)$</td>
<td>0.00</td>
<td>0.36</td>
<td>0.55</td>
</tr>
<tr>
<td>$corr(Y, S)$</td>
<td>-0.42</td>
<td>-0.73</td>
<td>-0.83</td>
</tr>
<tr>
<td>$std(C + L) / std(Y)$</td>
<td>1.57</td>
<td>1.36</td>
<td>1.40</td>
</tr>
<tr>
<td>$corr(Y, Cost)$</td>
<td>-</td>
<td>-0.40</td>
<td>-</td>
</tr>
<tr>
<td>$corr(S, Cost)$</td>
<td>-</td>
<td>0.75</td>
<td>-</td>
</tr>
<tr>
<td>avg cost (% of avg revenues)</td>
<td>-</td>
<td>0.69</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: the empirical moments are calculated for Mexico’s data covering 1994-2019. The bond spread is the EMBI index, while government final consumption expenditure and output are taken from National Accounts. Government deficit data is acquired from Banco de Mexico.

\(^{14}\)For our baseline model, we define total deficit as $C + wL + \phi_1 \min \{ H/H_{-1} - 1, 0 \} - \tau Y$.

\(^{15}\)In the Mexican data for 1994-2019, government deficit appears to be essentially acyclical, as opposed
are both *attenuated* and *desynchronized*. As a result, the government is exposed to higher spreads for more time periods.

In order to visualize our quantitative results, Figure 1 plots an excerpt from the simulated time series for deficit and spread. Notice that the spread in our baseline model is generally more volatile, and the difference is especially pronounced around debt crises, i.e. when the spreads are high. Again, this result is mirrored by the behavior of government deficit which is less volatile and moves more slowly in our baseline model during such episodes.

Figure 1: Simulated behavior of deficit and spread

Figure 2 presents our main qualitative result by focusing on the paths of endogenous variables in the model during debt crisis, averaged out across the simulations. In particular, an episode is selected if its peak spread is at least one standard deviation above the mean and if it is not accompanied by a default in the five periods before and after.\textsuperscript{16} We also conducted our analysis for two standard deviations above the mean, and the results are very similar. We settled on the case of one standard deviation above the mean because that is also a measure we use to identify crises in the data and it allows us to include more episodes in the sample.

\textsuperscript{16}We also conducted our analysis for two standard deviations above the mean, and the results are very similar. We settled on the case of one standard deviation above the mean because that is also a measure we use to identify crises in the data and it allows us to include more episodes in the sample.
As government revenues gradually decline leading to a trough in period 0 (Figure 2a), both kinds of government expenditures drop (Figures 2c-2d) and spreads go up (Figure 2b). In the flexible model, there is no difference between expenditure on public employment or intermediate consumption, so the two decline proportionally. This is not the case for our baseline model, however. Because public employment contributes much more to the pool of legacy contracts of the government, $H_{-1}$, its decline is quantitatively smaller and slower to recover following the peak. On the other hand, this sluggishness is offset by a steeper drop, and faster recovery, in the intermediate consumption. Because the government chooses to reduce its labor force by much less, it maintains a higher deficit for a longer time and never allows for an increase in surplus as drastic as in the benchmark flexible model (Figure 2e). As a result, the government debt actually increases in the run-up to the crisis, rather than declines as predicted by the standard model (Figure 2f). This choice of higher debt during the crisis naturally translates into a tolerance for higher spreads. In our baseline model, the average peak spread during debt crises is over 7%, which in the standard model it is below 5% (Figure 2b). Section 3.6 investigates this last point more in depth.

Figure 3 shows that our qualitative result similarly holds during debt crisis episodes that culminate with a sovereign default. A default is always triggered by falling government revenues (Figure 3a) and results in exploding spreads (Figure 3b). As revenues are falling, the public employment in our baseline model declines more sluggishly than in the flexible one (Figure 3d), while intermediate consumption declines faster (Figure 3c). This results in the government deficit falling much more slowly in the run-up to default in our baseline model than in the standard model (Figure 3e), and consequently the government debt increases (Figure 3f). Not surprisingly, the rise in the spread is higher in the baseline model than in the standard flexible one (Figure 3b).

3.4.2 Baseline v. Fixed Labor

We now compare our baseline model with a second benchmark case which exhibits fixed public employment and fully flexible intermediate consumption. This specification boils down essentially to a standard sovereign default model with non-homothetic preferences that feature “minimum consumption”. Such preferences have recently been used by Bocola and Dovis (2019) or Bianchi, Hatchondo and Martinez (2018) to slow down the government’s actions. Table 5 summarizes the calibration of that model along with our baseline case. Similarly as with the fully flexible model, the only relevant targeted moments are average debt and average spread, and we match them well. The
Figure 2: Behavior of the model around default crises: baseline v. flexible
Figure 3: Behavior of the model in the run-up to defaults: baseline v. flexible
discount factor required for that is slightly lower than for the standard model.

Table 5: Calibration of structural parameters: baseline v. fixed labor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Fixed Labor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor, $\beta$</td>
<td>0.724</td>
<td>0.785</td>
</tr>
<tr>
<td>Max default endowment, $\hat{Y}$</td>
<td>0.846</td>
<td>0.851</td>
</tr>
<tr>
<td>Interm. consumption weight, $\alpha$</td>
<td>0.448</td>
<td>1.000</td>
</tr>
<tr>
<td>Adjustment weight, $\phi_0$</td>
<td>0.442</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Adjustment scale, $\phi_1$</td>
<td>0.474</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 6 compares the untargeted moments produced by the two models. As expected, the variant with fixed labor features a higher standard deviation of the spread than the fully flexible one (1.34% v. 0.83%), but still way below the one in our baseline model of 1.82%. On the other hand, the volatility of government deficit falls considerably with fixed labor relative to the flexible model, but it is still slightly higher than in our baseline model. Notice also that the correlation of government deficit with revenue does not fall (in absolute value) in the fixed labor model relative to the flexible model. This suggests

Table 6: Simulated behavior: baseline v. fixed labor

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mexico Data</th>
<th>Baseline</th>
<th>Fixed Labor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{std}(S)$</td>
<td>2.21</td>
<td>1.82</td>
<td>1.34</td>
</tr>
<tr>
<td>$\text{std}(D)/\text{std}(Y)$</td>
<td>0.26</td>
<td>0.41</td>
<td>0.42</td>
</tr>
<tr>
<td>$\text{corr}(S, D)$</td>
<td>-0.58</td>
<td>-0.34</td>
<td>-0.86</td>
</tr>
<tr>
<td>$\text{corr}(Y, D)$</td>
<td>0.00</td>
<td>0.36</td>
<td>0.67</td>
</tr>
<tr>
<td>$\text{corr}(Y, S)$</td>
<td>-0.42</td>
<td>-0.73</td>
<td>-0.90</td>
</tr>
<tr>
<td>$\text{std}(C + L)/\text{std}(Y)$</td>
<td>1.57</td>
<td>1.36</td>
<td>1.29</td>
</tr>
<tr>
<td>$\text{corr}(Y, \text{Cost})$</td>
<td>-</td>
<td>-0.40</td>
<td>-</td>
</tr>
<tr>
<td>$\text{corr}(S, \text{Cost})$</td>
<td>-</td>
<td>0.75</td>
<td>-</td>
</tr>
<tr>
<td>Avg Cost (% of avg revenues)</td>
<td>-</td>
<td>0.69</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: the empirical moments are calculated for Mexico’s data covering 1994-2019. The bond spread is the EMBI index, while government final consumption expenditure and output are taken from National Accounts. Government deficit data is acquired from Banco de Mexico.
that while using a model with preferences that feature “minimum consumption” goes a long way in attenuating the government’s response to shocks, it does nothing to desynchronize it with the fundamentals and other macroeconomic variables. By contrast, our baseline model achieves both of these objectives.

Figure 4 presents the behavior of endogenous variables in the two models focused around debt crisis episodes (without default) analogous to that in Figure 2. As revenues fall (Figure 4a), the government reduces its intermediate consumption while the expenditures on public employment remain fixed (Figures 4c-4d). It is worth noting that in the run-up to the crisis, the intermediate consumption for the fixed-labor government tends to be higher than for our baseline model which results in a steeper and faster decline in government deficit (Figure 4e). Notice also that, as previously mentioned in our discussion of Table 6, the government’s response in the fixed labor model is attenuated but still well-synchronized with the peak of the crisis, just as in the flexible model. Finally, Figure 4f shows that the government with fixed labor also tends to reduce its total debt in the run-up to debt crisis episodes which contrasts with the debt accumulation pattern generated by our baseline model.

Figure 5 compares the behavior of our baseline model with the fixed-labor model ahead of actual defaults. Analogously to Figure 3, government revenues are falling and spreads are increasing continuously. In response, the government is slashing intermediate consumption at a similar pace in the two models, although it is higher to begin with in the one with fixed labor (Figure 5c). Consequently, the government deficit is falling more slowly in our baseline model and government debt increases, resulting in a higher rise of the spread (Figure 5b).

3.4.3 Taking stock

We now summarize our main results by providing a direct comparison of the three model variants. Table 7 illustrates our main quantitative result: the bond spread is much more volatile in the data than what is predicted by a standard fully flexible model of sovereign default. Our baseline model is able to bridge 72% of this gap in standard deviations, while an alternative variant with fixed labor (equivalent to a “preference for minimum consumption” model commonly used in the recent literature) can only close 36%. This result is mirrored by the opposite pattern in the volatilities of government deficit.

Figure 6 illustrates the main qualitative takeaway from our analysis by comparing the av-
Figure 4: Behavior of the model around default crises: baseline v. constant labor
Figure 5: Behavior of the model in the run-up to defaults: baseline v. fixed labor
Table 7: Main quantitative results

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mexico Data</th>
<th>Baseline</th>
<th>Flexible</th>
<th>Fixed labor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{std}(S)$</td>
<td>2.21</td>
<td>1.82</td>
<td>0.83</td>
<td>1.34</td>
</tr>
<tr>
<td>$\text{std}(D)/\text{std}(Y)$</td>
<td>0.26</td>
<td>0.41</td>
<td>0.60</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Average dynamics of government deficit and debt around crisis episodes. When faced with plunging revenues, the government responds by reducing its deficit (Panel 6a) abruptly in the standard sovereign default model. This response is attenuated slightly in a fixed labor variant of the model, but nevertheless well-synchronized with the trough of the income processes. In our baseline model, by contrast, the response is dampened further and also delayed relative to the peak of the crisis. As a consequence of this dynamics, the government ends up borrowing into debt crises (Panel 6b), instead of deleveraging.

Finally, it is important to understand the limitations of our mechanism. Appendix D shows, in particular, that under a calibration with much higher debt, the borrowing into debt crises behavior becomes weaker due to the fact that the government is much more impatient. On the other hand, our baseline mechanism still generates more than double the standard deviation of the spread compared to the flexible one.

3.5 Cost of borrowing into debt crises

One of the most striking results of the baseline model is the increasing borrowing during debt crises. At first glance this result may appear to be a direct byproduct of costly
expenditure adjustment. That is, governments increase borrowing during downturns to prevent costly expenditure adjustment. In this section, we highlight an additional channel that leads to borrowing during debt crises in the baseline model. In particular, we show that the baseline government faces weaker default incentives and lower borrowing costs during downturns. Therefore, in addition to having a stronger desire to borrow in recessions the baseline government also finds it less costly to do so.

Let us return to the crises, as defined in Section 3.4.1. The question we ask is: how costly would it be for the flexible government to borrow like the baseline one? In particular, in each period we construct a counterfactual interest rate spread using the price schedule of the flexible government. This counterfactual spread would prevail if the flexible government borrowed exactly like the baseline one, i.e. maintained higher borrowing during debt crises as in Figure 2. The counterfactual spread as well as the spreads for the flexible and baseline specifications can be seen in Figure 7a. Figure 7a paints a clear picture of a flexible government that would face exorbitant borrowing costs if it chose to “borrow into debt crises”. In particular, at the peak of the crisis the flexible government would face double the spread of the baseline government.

The counterfactual spread is slightly higher than the flexible spread even 5 periods before the peak of the crisis. This may seem surprising since as seen in Figure 2 borrowing in the baseline specification is, on average, slightly lower than in the flexible specification. However, the price schedule is quite non-linear.  

Figure 7: Spreads and default sets: baseline vs. flexible model
Panel 7b plots the default sets in the baseline model, separately for a high and low value of the legacy contracts $H$, along with the one from the flexible model. Because debt crises tend to occur when $H$ is high (and when a series of bad shocks force the economy into recession), the figure makes it clear that our baseline government is less likely to default than the flexible one.

The “discount” the baseline government receives in its borrowing stems precisely from the frictions in adjusting its expenditure. That is, during downturns the baseline government relies heavily on external borrowing to smooth its expenditure reduction. To do so it maintains a borrowing buffer during good times.\textsuperscript{19} This borrowing behavior makes financial markets more valuable for the sovereign which lowers the likelihood of default. This in turn reduces the cost of borrowing. This mechanism reinforces the government’s desire to borrow during downturns.

### 3.6 Highest-peaking spreads

As evident from Figures 2b and 4b, a common feature in standard sovereign default models with long-term debt is the fact that spreads do not achieve realistically high levels during debt crises. In reality, during debt crises countries typically face bond spreads well in excess of 10%.\textsuperscript{20} We will now show that our model is easily capable of generating such values, in contrast to the benchmark models.

Figure 8 presents the histograms of simulated spreads across the three analyzed models (along with the habit formation model, to be formally introduced in Section 4). Notice that the distribution of bond spreads is generally right-skewed, with a mode of just above 2%. Importantly, in both the flexible model and in the model with fixed labor, it is virtually impossible to observe a realized spread higher than 6% or 8%, respectively. By contrast, the upper right tail in our baseline model (as well as its simplified habit version) extends much further, and there is non-negligible mass of the distribution for spreads above 10%. This is due to the mechanism explained in Section 3.4: government prefers to reduce public employment gradually and thus tolerates higher-peaking spreads more often.

\textsuperscript{19}This buffer can be seen in Figures 2f and 4f where, in the early periods, borrowing is lower in the baseline specification. The borrowing buffer behavior discussed here provides a rationale for maintaining reserves beyond the ones highlighted in Bianchi, Hatchondo and Martinez (2018).

\textsuperscript{20}For example, during the European debt crisis of 2010-2012, the Greek spread on 10-year bonds achieved almost 25%, while the Portuguese spread reached 12%.
3.7 The role of rigid intermediate consumption

In this extension, we consider a version of our model in which public employment is rigid while intermediate consumption expenditure is fully flexible. This exercise highlights the importance of imposing rigidity to both inputs. In particular, with fully flexible intermediate consumption spreads end up being only slightly more volatile than in the flexible specification.

Table 8 presents our calibration of this model, along with its performance in terms of fit to untargeted moments. Since \( \phi_0 \) was used to match the ratio of standard deviations of the inputs, we no longer target this moment. As a result, the model produces a significantly higher ratio of 268.18%, i.e. intermediate consumption varies much more relative to public employment expenditure, compared to the data of 164%. Because of this excess
volatility of intermediate consumption, the government deficit also varies much more (with standard deviation of 0.51 compared to 0.26 in the data) and it exhibits strong correlation with output and bond spread, similar to the benchmark fully flexible model. By the same token, the standard deviation of the bond spread is only slightly higher in this model (1.03%) than in the standard one (0.83%). Finally, it is worth pointing out that the average adjustment cost incurred by the government is an order of magnitude lower in this model (0.06) compared to our baseline model in which both inputs feature some degree of rigidity (0.69). This implies that by varying intermediate consumption, the government is able to smooth out the decline in public employment expenditure while at the same time reducing debt to avoid a potential default.

Table 8: Calibration and behavior of the model with flexible intermediate consumption

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor, $\beta$</td>
<td>0.789</td>
<td>Avg.</td>
<td>48.00</td>
<td>48.50</td>
</tr>
<tr>
<td>Max default endowment, $\hat{Y}$</td>
<td>0.835</td>
<td>Avg.</td>
<td>3.03</td>
<td>3.09</td>
</tr>
<tr>
<td>Intern. consumption weight, $\alpha$</td>
<td>0.430</td>
<td>Avg.</td>
<td>63.00</td>
<td>56.87</td>
</tr>
<tr>
<td>Adjustment weight, $\phi_0$</td>
<td>0.000</td>
<td>Elast.</td>
<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>Adjustment scale, $\phi_1$</td>
<td>0.081</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. debt/revenues (%) 48.00</td>
<td>48.50</td>
<td></td>
</tr>
<tr>
<td>Avg. spread (%) 3.03</td>
<td>3.09</td>
<td></td>
</tr>
<tr>
<td>Avg. labor share (%) 63.00</td>
<td>56.87</td>
<td></td>
</tr>
<tr>
<td>Elasticity of $w_L$ w.r.t. $C$ in crises 0.24</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Untargeted</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. ratio st. dev. of inputs (%) 164.00</td>
<td>268.18</td>
<td></td>
</tr>
<tr>
<td>$std(S)$ 2.21</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>$std(D)/std(Y)$ 0.26</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>$corr(S,D)$ -0.58</td>
<td>-0.83</td>
<td></td>
</tr>
<tr>
<td>$corr(Y,D)$ 0.00</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>$corr(Y,S)$ -0.42</td>
<td>-0.84</td>
<td></td>
</tr>
<tr>
<td>$std(C + L)/std(Y)$ 1.57</td>
<td>1.36</td>
<td></td>
</tr>
<tr>
<td>$corr(Y, Cost)$ -</td>
<td>-0.30</td>
<td></td>
</tr>
<tr>
<td>$corr(S, Cost)$ -</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>Avg Cost (% of avg revenues) -</td>
<td>0.06</td>
<td></td>
</tr>
</tbody>
</table>

Note: the empirical moments are calculated for Mexico’s data covering 1994-2019. The bond spread is the EMBI index, while government final consumption expenditure and output are taken from National Accounts. Government deficit data is acquired from Banco de Mexico.
4 Habit formation

In this section, we show that a standard habit formation model produces results quantitatively similar to our baseline specification. In particular, we now assume that the government only chooses total government expenditure. To this specification we introduce habit formation as in Fuhrer (2000). We draw two conclusions from this exercise. First, the results produced by a simple habit formation model are consistent with our more “sophisticated” and carefully calibrated mechanism. Therefore, any researcher interested in utilizing our mechanism can do so in an environment that is far less challenging to implement. Second, in contrast to our baseline mechanism, a framework with habit entirely dispenses of resource costs of expense adjustment. This allows us to quantify the importance of these costs.

4.1 Recursive problem

We begin by presenting the problem in the recursive form.

**Government**  The government that is current on its debt obligations decides between repayment or default. The value function is given by:

\[
W(B, C_{-1}, Y) = \max_{d \in \{0,1\}} \left\{ d \ V^D(C_{-1}, Y) + (1 - d) V^R(B, C_{-1}, Y) \right\}
\] (7)

where \( C_{-1} \) denotes the previous period consumption. Repayment \( (d = 0) \) allows the government to borrow, and the value associated with it is given by

\[
V^R(B, C_{-1}, Y) = \max_{B' \geq 0, C \geq 0} \left\{ U\left( \frac{C}{C_{-1}} \right) + \beta \mathbb{E}_{Y'|Y} W(B', C, Y') \right\}
\] (8)

subject to

\[
C = \tau Y - B \left( \delta + (1 - \delta) \kappa \right) + Q(B', C, Y) \left( B' - (1 - \delta) B \right)
\]

In formula (7), \( \chi > 0 \) is the standard habit-formation parameter (Fuhrer, 2000). A sovereign who defaults \( (d = 1) \) is excluded from international credit markets and has probability \( \theta \) of being readmitted every subsequent period. The associated value is:

\[
V^D(C_{-1}, Y) = U\left( \frac{\tau Y^d(Y)}{C_{-1}^\chi} \right) + \beta \mathbb{E}_{Y'|Y} \left[ \theta W(0, \tau Y^d(Y), Y') + (1 - \theta) V^D(\tau Y^d(Y), Y') \right]
\] (9)
**International Lenders** The lenders are assumed to be risk-neutral and perfectly competitive. The actuarially fair bond price that compensates them for default risk is:

\[
Q(B', C, Y) = \frac{1}{1 + r} E_{Y'|Y} \left[ \left( 1 - d(B', C, Y') \right) \left( \delta + (1 - \delta) \kappa + (1 - \delta) Q(B'', C', Y') \right) \right]
\]

(10)

where

\[
B'' = B'(B', C, Y')
\]

\[
C' = C'(B', C, Y')
\]

### 4.2 Quantitative analysis

We now turn to the quantitative analysis of this model. We adopt the same functional forms for the utility function and the default cost as in Section 3. Further, we assume the same “external” parameters as in Table 1. Table 9 summarizes the moment-matching exercise in this model. In addition to the two usual parameters \((\beta, \hat{Y})\), which are jointly identified using average debt and average spread, we also calibrate the habit-formation parameter \(\chi\). We do so by targeting the autocorrelation of final government consumption expenditure of 0.72.\(^{21}\) As a result, we arrive at the value of \(\chi = 0.89\). This is well within the confidence interval of the estimate in Fuhrer (2000).

Table 10 summarizes the selected untargeted moments generated by this model. The standard deviation of the spread is very close to the one in the baseline specification. Overall,

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Habit model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor, (\beta)</td>
<td>0.837</td>
</tr>
<tr>
<td>Max default endowment, (\hat{Y})</td>
<td>0.810</td>
</tr>
<tr>
<td>Habit parameter, (\chi)</td>
<td>0.891</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Habit model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. debt/revenues (%)</td>
<td>48.00</td>
<td>48.88</td>
</tr>
<tr>
<td>Avg. spread (%)</td>
<td>3.03</td>
<td>2.99</td>
</tr>
<tr>
<td>Autocorrelation of cons.</td>
<td>0.72</td>
<td>0.69</td>
</tr>
</tbody>
</table>

\(^{21}\)Calculated using the yearly series “General government final consumption expenditure (constant LCU)” from the World Bank’s WDI from 1994 to 2019 for Mexico. We use the WDI series, rather than OECD as in the previous sections, because the latter only start from 2003 for Mexico.
the moments in the two models are quite similar. We conclude that the main quantitative result from our baseline model, a significantly increased volatility of the bond spread, can also be achieved with a standard habit formation friction.

Table 10: Untargeted moments: habit formation model

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mexico Data</th>
<th>Habit model</th>
</tr>
</thead>
<tbody>
<tr>
<td>std($S$)</td>
<td>2.21</td>
<td>1.86</td>
</tr>
<tr>
<td>std($D$)/std($Y$)</td>
<td>0.26</td>
<td>0.50</td>
</tr>
<tr>
<td>corr($S, D$)</td>
<td>-0.58</td>
<td>-0.37</td>
</tr>
<tr>
<td>corr($Y, D$)</td>
<td>0.00</td>
<td>0.29</td>
</tr>
<tr>
<td>corr($Y, S$)</td>
<td>-0.42</td>
<td>-0.80</td>
</tr>
</tbody>
</table>

Note: the empirical moments are calculated for Mexico’s data covering 1994-2019. The bond spread is the EMBI index, while government final consumption expenditure and output are taken from National Accounts. Government deficit data is acquired from Banco de Mexico.

Finally, Figure 8d in Section 3.6 shows that the model with habit formation is equally capable, just as our baseline model, of generating high-peaking spreads in equilibrium, well in excess of 10% (in fact, the highest spread we obtain in the simulations is 22% for the habit model and 16% for the baseline model).

4.3 Qualitative analysis

We now use the calibrated habit-formation model to examine the behavior of government borrowing and spreads around the crisis episodes. The two models produce similar crises. A point of departure can be seen in Figure 9. In particular, in the habit specification borrowing is higher coming into the peak of the debt crisis. However, at the peak borrowing declines slightly. This departure raises the question of the extent to which realized adjustment costs may lead to increased borrowing during the peak of the debt crisis. We further investigate this question in Appendix F.

5 Evidence on “Borrowing into Debt Crises"
than the mean plus one standard deviation; and 3) it had not defaulted in that given year. In other words, we are identifying episodes in which the bond spread peaks at a high enough level, but the government has not defaulted yet.

Figure 10 presents average paths of bond spreads and external government debt around the crisis episodes defined as above. Panel 10a verifies that the bond spread spikes, by construction, at the peak of the crises. Panel 10b shows that government debt tends to increase throughout the episode, with the pace of the increase accelerating at the height of the crisis, only to start declining three years after the peak.

6 Conclusion

This paper revisits several common issues with standard models of sovereign default. Quantitatively, such models struggle to generate the levels of bond spread volatility in line with what we observe in the data for most emerging countries. Qualitatively, the government in such models typically reduces debt sharply in anticipation of a looming

---

22Our model simulations in Figure 2 are conditional on the country not being in default.
23Similarly to the model analysis, we adopted one standard deviation above the mean spread and verified that the results are similar for the case of two standard deviations above the mean.
24In Appendix G, we investigate the behavior of the remaining variables of interest in the OECD data.
Figure 10: Behavior of endogenous variables around debt crises: spread and debt

debt crisis, while in reality many countries struggle to deleverage effectively in response to adverse income shocks. We offer a solution to these problems by considering rigid government expenditure. When faced with negative income shocks, the government finds it costly to adjust its spending, in particular the public employment expenditure. As a result, it is optimal for the government to respond to debt crises slowly and tolerate high interest rate spreads for longer time periods while often actually borrowing into debt crises.

We quantify this channel using the OECD Government Accounts data and show that our preferred calibration for Mexico delivers a much higher volatility of the bond spread, able to close about 70% of the gap in standard deviations between the data and the prediction of the standard model. This is achieved by a government whose actions are also desynchronized relative to the income shocks and who on average ends up increasing its debt in anticipation of a looming debt crisis.

References


